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Corporate Investment Decisions and Security Values

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CORPORATE INVESTMENT DECISIONS

AND

SECURITY VALUES

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The study of the impact of the announcement of various investment decisions of firms on their security values has occupied a central role in research in the area of finance. One has merely to skim the title pages of finance journals to confirm this preeminent position. The studies, to date, have generally used the Capital Asset Pricing Model to analyze the impact of these announcements. The development of the literature in the area of contingent claim pricing provides a new methodology in tackling the above issues. This thesis uses the option pricing framework to study the effects of two specific firm investment decisions--mergers, and stock repurchases.

The first essay examines the pure financial effects of conglomerate mergers. Using the technique for valuing compound options, equations are derived for post-merger values of equity, short-term debt, and long-term debt. With the help of these valuation equations it is shown that the merger can result in wealth transfers from equity to both debts, from equity and one debt to the other debt, and from long-term debt to equity and short-term debt. The existence of these wealth transfers provide a rationale for the protective covenants against mergers that are commonly seen in debt contracts. In addition, it is shown that these protective covenants imply that the post-merger capital structure of the firm would be different from a simple pooling of the pre-merger capital structure of the individual firms.

The second essay examines the effects of an announcement by a firm to repurchase a fraction of its outstanding equity. Given the existence of protective covenants in debt contracts against repurchases, and the voting rights of shareholders, it is theoretically shown that a repurchase must convey some information about the firm's futur prospects for it to be approved by all security-holders. In addition, it is shown that the signal must be firm value increasing, and firm risk decreasing. The theoretical signalling effects of repurchases are also shown to be consistent with empirical results obtained in recent studies.

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ESSAY 1

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1. INTRODUCTION AND SUMMARY

The growth in the number of mergers since the late 50's has fueled interest in the study of the effects related to this activity. This interest is evidenced by the large number of articles concerned with mergers that have appeared in the journals of finance and economics in recent years. The early work in this area was concerned mainly with the formulation of theories aimed towards explaining the rationale behind mergers, and the study of the existence and sources of possible opportunities for profitable corporate growth through mergers. It has been argued that mergers are motivated by increased market power, diversification, and bankruptcy avoidance.[1] The focus of more recent studies has been on the theory of conglomerate mergers.[2] The theory of conglomerate mergers is concerned with the pure financial effects of merging. The first major result of this theory concerns the value of the merged firm. Myers (1968),

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[1] See Alberts (1966), Dean and Smith (1966), Gort (1966), Lewellen (1971), Higgins and Schall (1975), Steiner (1975), and Shrieves and Stevens (1979).

[2] See Myers (1968), Lewellen (1971), Higgins (1971), Rubinstein (1973), Higgins and Schall (1975), Galai and Masulis (1976), and Scott (1977).

Lewellen (1971), and others have argued that conglomerate mergers will not alter total values if the capital market is perfect. It has been argued that if corporate bankruptcy is possible and if mergers reduce the probability of default, then the value of the debt will increase, with this increase being exactly off-set by a decrease in the value of equity.[3] In addition, Rubinstein (1973), Higgins and Schall (1975), and Galai and Masulis (1976) have shown that with costless bankruptcy the bondholders of the merged firm are always better off because the risk of default always decreases.

The purpose of this paper is to examine the financial effects of a non-synergistic merger between two firms with different capital structures and riskiness. Using the techniques developed by Black and Scholes (1973), Cox and Ross (1975), and Geske (1977, 1979), for valuing securities as simple, and compound options, it is shown that the results obtained by Galai and Masulis (1976) are a direct result of their assumption that the debt of the two firms involved in the merger have the same maturity date, and that the variances of returns on both firms are

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[3] See Levy and Sarnat (1970), Lewellen (1971), Higgins (1971), Rubinstein (1973), Higgins and Schall (1975), Galai and Masulis (1976) and Scott (1977).

the same. It is shown here that in a merger between two firms containing only pure coupon bonds and equity in their capital structure, the possible wealth transfer effects are:

(i) from stockholders to short and long-term debtholders

(ii) from stockholders and long-term debtholders to short-term debtholders

(iii) from stockholders and short-term debtholders to long-term debtholders

and (iv) from long-term debtholders to stockholders and short-term debtholders.

The above wealth-expropriation effects are shown to carry through even when the bonds receive coupon payments and when the variance of returns on the merged firm changes after retirement of short-term debt.

This implies that without the existence of protective covenants restricting mergers, stockholders can engage in activities that cause the wealth of bondholders to decrease. This, in turn, provides a rationale for the existence of merger covenants in debt contracts. Finally, it is shown that the existence of these merger covenants would generally result in a capital structure for the merged firm which is different from a simple combination of the capital structures of the two individual firms involved in the merger.

The various assumptions used in this paper are set forth in Section 2. In Section 3, valuation equations for debt and equity are derived, assuming no coupon payment on debt, and that the variance of returns on the merged firm is constant. The result of simulations using these valuation equations are presented in Section 4. Section 5 studies the effect of introducing coupon payments on debt and the effect of a change in the variance of returns on the merged firm after retirement of the shorter maturity debt. The implication of the results (obtained in earlier sections) for protective covenants and methods of compensation are discussed in Section 6.

2. THE ASSUMPTIONS

As stated earlier the corporate securities considered here are valued using the option pricing framework. The valuation equations derived in later sections follow from the fact that stocks can be viewed as call options with the underlying securities being the assets of the firm. These equations are derived under the following set of assumptions:

A1. The two firms under consideration (A and B), each have common stock and one debt issue outstanding.

A2. The bonds are pure discount bonds, giving the holder the right to the face value M_1, i = A, B, at some future date T_i, i = A, B($T_A < T_B$).

A3. There are no dividend payments on the stocks.

A4. There are no synergistic effects involved in the merger. Thus the merger results in a new firm, C, with current market value $V^C = V^A + V^B$, where $V^A(V^B)$ is the current market value of firm A(B).

A5. The distribution of firm asset values at the end of any finite time interval is stationary long-normal.[4]

A6. The variance of returns on the firm is constant. Let σ^2, σ^2, and σ^2 denote the variances of returns on firms A, B, and C respectively. It is assumed that σ^2 is given by

$$\sigma^2=\alpha^2\sigma_A^2 + (1 - \alpha)^2\alpha_B^2 + 2\alpha(1 - \alpha)\rho\sigma_A\sigma_B$$

where $\alpha = V^A/V^C$

[4] The assumption that firm C value is log-normally distributed is at best an approximation, since the sum of two log-normal variables is not log-normal.

and ρ = correlation coefficient between firms A and B.

A7. There exists a riskless asset paying a known constant rate of return, r_f.

A8. Trading takes place continuously.

A9. Individuals can sell any security short and receive proceeds of the sale.

A10. Capital markets are perfect.

A11. It is assumed that the firm liquidates some of its assets to retire maturing debt. Thus, if the debt is retired, the value of the firm falls by the face value of the debt. Bankruptcy occurs when the value of the firm is less than the face value of the maturing debt. If firm C goes bankrupt at time T_A, then the two debtholders share the liquidated firm, with the holders of Bond A getting a proportion b of the firm. It is assumed that b is given by[5]

$$b = M_A/M_A + M_B e^{-r_f(T_B-T_A)}$$

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[5] The exact expression for b can take various other forms. The results presented in the paper will not change for a different bankruptcy sharing rule.

Assumption A6 is very restrictive in the sense that the variance of returns on firm C is not allowed to change after debt A is paid off. In general, given that the firm liquidates some of its assets to retire debt A (assumption A11), the variance may change at T_A. Therefore, valuation equations are derived under the following additional assumption:

A12. The variance of returns on firm C changes after retirement of debt A. The variances before and after retirement of debt A are denoted by σ^2_{C1} and σ^2_{C2} respectively, where σ^2_{C2} is given by the expression for σ^2_C in assumption A6. σ^2_{C2} could take on values less than, equal to, or greater than σ^2_{C1} .

The assumption that the bonds are of a pure discount variety is also relaxed in Section 5. Assumption A2 is replaced by

A2'. The bonds have coupon payments of r_i% per annum (i = A,B). The first payment is made in t_1 years from the present (i = A,B). It is further assumed that $t_A = t_B = t^*$. In addition, the bonds have a face value of M_1, i = A,B, and mature at some future date T_i, i = A,B($T_A < T_B$)

Table 1 presents the notation used in this paper. All equations derived in Section 3 and 4 are based on assumptions A1-A11. The first part of Section 5 requires assumptions A1-A12, while the second part uses A1, A2', A3-A11.

3. THE VALUATION EQUATIONS

a) Before Merger

For the firms described in the earlier section, the shareholders can be viewed as having an option to buy back the firm from the bondholders at an exercise price equal to the face value of the debt at the maturity date of the latter. Thus the value of the stock at the maturity date, T, is $Max[0, V_T - M]$, where V_T is the value of the firm at time T, and M is the face value of the maturity debt. This implies that the stock is a call option on the value of the firm, and can be valued using the Black-Scholes option pricing model.[6] The value of the stock of firm i, i=A,B,

$$S^i = V_i N_1(k_i + \sigma_i \sqrt{T_i}) - M_i e^{-r_f T_i} N_1(k_i) \tag{1}$$

where $N_1(.)$ = univariate normal cumulative distribution function.

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[6] See Black and Scholes (1973).

TABLE 1

THE NOTATION

VARIABLES OF THE FIRM	FIRM A	FIRM B	FIRM C
Current market value of the firm	V^A	V^B	V^C
Market value of firm at date t ($t \leq T_A$)	V_t^A	V_t^B	V_t
Market value of firm at date t ($t > T_A$)	—	V_t^B	V_t'
Current market value of stock	S^A	S^B	S^C
Market value of stock at time t	S_t^A	S_t^B	S_t^C
Current market value of debt A	D^A	—	D^{AC}
Market value of debt A at time t	D_t^A	—	D_t^{AC}
Current market value of debt B	—	D^B	D^{BC}
Market value of debt B at time t	—	D_t^B	D_t^{BC}
Variance of returns on firm	σ_A^2	σ_B^2	σ_C^2

In addition, the following notation is used:

r_f = risk-free rate

T_A = time to maturity of debt A

T_B = time to maturity of debt B

M_A = face value of debt A

M_B = face value of debt B

r_A = interest rate on debt A

r_B = interest rate on debt B

t^* = time to first interest payment on both debt A and debt B.

b = proportion of liquidated firm obtained by holders of debt A when firm C goes bankrupt at time T_A.

$$k_i = \frac{\ln(V^i/M_i) + (r_f - 0.5\sigma_i)T_i}{\sigma_i/T_i}$$

and

The current market value of debt is $V^i - S^i$, $i =$ A,B. Thus,

$$D^i = V^i[1 - N_1(k_i + \sigma_i/T_i)] + M_i e^{-r_f T_i} N_1(k_i) \quad (2)$$

b) <u>After Merger</u>

The merger results in a firm C with the following debt-equity structure:

(i) A pure discount bond with a face value M_A and maturity date T_A.

(ii) A pure discount bond with a face value M_B and maturity date T_B.

(iii) Common stock with firm A shareholders owing a proportion S^A/S^A+S^B and firm B shareholders owing a proportion S^B/S^A+S^B).[7]

The value of the new firm is just the sum of the individual firm values, as per assumption A4. At time T_A, one of the bonds matures. If the value of firm C at T_A is greater than the face value of the maturing debt M_A, the debt holders receive M_A. If the value of the firm is less

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[7] This is an assumption. There are a number of different ways to divide the stock among the two stockholders. The specification of a different stock sharing rule will not affect the conclusion drawn about the effects on stockholders as a whole.

than the face value of the maturing debt, the firm goes bankrupt and as per assumption A11, debt A receives a proportion b of the liquidated firm, while debt B receives a proportion (1 - b) of the firm. Thus, the value of the short-term debt (debt A) at time T_A is given by

$$D^{AC}_{TA} = \begin{cases} bV_{T_A} & \text{if} \quad V_{T_A} \leq M_A \\ M_A & \text{if} \quad V_{T_A} > M_A \end{cases}$$

where V_{T_A} is the value of firm C at date T_A.

Cox and Ross (1975) have shown that if one can create a riskless hedge involving the security that one is interested in pricing, then the current value of such a security can be obtained by discounting the expected value of the security at some future date by the risk-free rate. Assuming that such a riskless hedge can be created in this case, the current post-merger value of debt A is given by[8]

$$\begin{aligned} D^{AC} &= e^{-r_f T_A} E[D^{AC}_{T_A} \mid V^C] \\ &= e^{-r_f T_A} \int^{M_A} bV_T \; f(V_{T_A} \mid V^C) dV_{T_A} + \int^{-} M_A f(V_{T_A} \mid \\ &\quad V^C) dV_{T_A} \end{aligned}$$

where $f(V_{T_A} \mid V^C)$ = density function for V_{T_A} conditional on current value V^C. Evaluating the above integral yields:

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[8] If the hedge cannot be created, the solution obtained using the Cox-Ross approach is only an approximation.

$$D^{AC} = bV^C[1 - N_1(k_1 + \sigma_C/T_A)] + M_A e^{-r_f T_A} N_1(k_1) \quad (3)$$

where

$$k_1 = \frac{\ln(V^C/M_A) + (r_f - 1/2\sigma_C^2)T_A}{\sigma_C/T_A}$$

Now, consider the stock at T_A, an instant after debt A matures. If $V_{T_A} < M_A$, the firm goes bankrupt and the value of the stock of firm C at T_A is zero. If $V_{T_A} > M_A$, Bond A is paid off, and the firm now consists of bond B and common stock, with the former maturing in $(T_B - T_A)$ periods. In addition (as per assumption A11), the value of the firm after retirement of debt A is given by

$$V'_{T_A} = V_{T_A} - M_A \qquad \text{if} \qquad V_{T_A} > M_A.$$

Therefore, the stock of the merged firm is an option on an option (compound option). The stockholders own an option which expires at date T_A and is exercised if the value of the firm at this date (V_{T_A}) is greater than the face value of a maturing debt. On exercise at date T_A, the stockholders receive a call option on the value of the firm which expires in $T_B - T_A$ periods and has an exercise price of M_B.

It is now assumed that the value of the firm is still log-normally distributed and that the variance of returns on the firm is still the same as it was before

T_A.[9] Therefore, at time T_B, the value of the stock is either zero if the firm value, V_T , is less than or equal to the face value of bond B, M_B, or is equal to the difference between the firm value and the bond B face value if the former is greater than the latter.[10] Algebraically

$$S^C_{T_B} = \begin{cases} 0 & \text{if} \quad V'_{T_B} \leq M_B \\ V'_{T_B} - M_B & \text{if} \quad V'_{T_B} > M_B \end{cases}$$

where V'_{T_B} is the realization of a process that started at V'_{T_A}.

Given the above boundary condition, the value of the stock at time T_A can be obtained using the model for pricing options presented by Black and Scholes (1973). The value of the stock at T_A is given by

$$S^C_{T_A} = \begin{cases} 0 & \text{if } V_{T_A} \leq M_A \\ V'_{T_A} \; N_1(k'_2 + \sigma_C \sqrt{T_B - T_A}) - M_B e^{-r_f(T_B - T_A)} \; N_1(k'_2) & \text{if } V_{T_A} > M_A \end{cases}$$

$$\text{where } k'_2 = \frac{\ln(V_T / M_B) + (r_f - 1/2\sigma^2_C)\;(T_B - T_A)}{\sigma_C \sqrt{T_B - T_A}}$$

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9 A log-normal variable less a constant does not result in a variable that is log-normally distributed. Thus, this assumption is only an approximation. In addition, the variance of returns could change after retirement of debt A given assumption A11. The effect of a change in variance is considered in Section 5.

10 The value of firm C at time T_B is now denoted with a prime because the value of firm C at time T_A after retirement of debt is V'_{T_A}.

The value of the stock of firm C at time of merger is[11]

$$S^C = e^{-r_f T_A}\, E[S^C_{T_A}]$$

$$= e^{-r_f T_A} \int_{\bar{M}_A}^{\infty} [(V_{T_A} - M_A) N_1(k_2' + \sigma_C \sqrt{T_B - T_A})$$

$$- M_B e^{-r_f (T_B - T_A)} N_1(k_2')] f(V_{T_A} \mid V^C) dV_{T_A}$$

Evaluating the above integrals as in Geske (1979) yields[12]

$$S^C = V^C N_2(k_1 + \sigma_C \sqrt{T_A},\ k_2 + \sigma_C \sqrt{T_B};\ \sqrt{T_A/T_B}) - M_A e^{-r_f T_A} N_2$$

$$(k_1, k_3;\ \sqrt{T_A/T_B})$$

$$- M_B e^{-r_f T_B} N_2(k_1, k_2;\ \sqrt{T_A/T_B}) \qquad \textbf{(4)}$$

where

$$k_1 = \frac{\ln(V^C/M_A) + (r_f - 1/2\sigma^2) T_A}{\sigma_C \sqrt{T_A}}$$

$$k_2 = \frac{\ln(V^C/M_B) + (r_f - 1/2\sigma^2) T_B}{\sigma_C \sqrt{T_B}}$$

$$k_3 = \frac{\ln(V^C/M_B) + (r_f + 1/2\sigma^2) T_B - \sigma^2 T_A}{\sigma_C \sqrt{T_B}}$$

$$V^C_1 = V^C - M_A e^{-r_f T_A}$$

and $N_2(Z_1, Z_2; \rho)$ = bivariate normal distribution function with z_1 and z_2 as upper limits and ρ as the correlation

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[11]This again assumed that a riskless hedge can be created. If such a hedge cannot be formed the use of this approach will yield a valuation equation that is an approximation to the "true" equation.

[12]This is an approximate solution to the integral. See Appendix A and footnote 22 for details.

coefficient. Since the current values of the three outstanding securities should sum to the current value of firm C, the current values of debt B is $V^C - S_C - D^{AC}$. Therefore,

$$
\begin{aligned}
D^{BC} = V^C[1 - b + bN_1(k_1 + \sigma_C\sqrt{T_A}) - N_2(k_1 + & \\
\sigma_C\sqrt{T_B};\ \sqrt{T_A/T_B})] & \\
-M_A e^{-r_f T_A}[N_1(k_1) - N_2(k_1, k_3; \sqrt{T_A/T_B})] & \\
+M_B e^{-r_f T_B}\ N_2(k_1, k_2; \sqrt{T_A/T_B}) & \qquad (5)
\end{aligned}
$$

The valuation equations derived above collapse to the Black-Scholes equation in three special cases. One such case obtains when one of the merging firms has only equity in its capital structure (if either $M_A = 0$ or if either $M_B = 0$ or $T_B \to \infty$). Another case is when firm A's debt matures at time of merger ($T_A = 0$). A third case requires debt A and debt B to mature at the same time ($T_A = T_B$).[13] When the variance of returns on the firm goes to zero, both bonds become riskless (if $V^C \geq M_A + M_B$) and are therefore priced at their present

---

[13]In this case the valuation equation obtained for debt B is of a different form as compared to those in the earlier cases because the value of debt at maturity date T is given by

$$
\begin{array}{ll}
(1 - b)V_T & \text{if } V_T \leq M_A \\
V_T - M_A & \text{if } M_A < V_T \leq MA + M_B;\ T_A = T_B = T\ . \\
M_B & \text{if } V_T > M_B
\end{array}
$$

TABLE 2

LIMITING BEHAVIOR OF MERGED FIRM SECURITY VALUES

	D^{AC}	S^C	D^{BC}
$M_A \to 0$[a]	0	$V^C N_1(k_4+\sigma_C\sqrt{T_B}) - M_B e^{-r_f T_B} N_1(k_4)$	$V^C[1-N_1(k_4+\sigma_C\sqrt{T_B})] + M_B e^{-r_f T_B} N_1(k_4)$
$T_B \to \infty$ or $M_B \to 0$	$V^C[1-N_1(k_1+\sigma_C\sqrt{T_A})] + M_A e^{-r_f T_A} N_1(k_1)$	$V^C N_1(k_1+\sigma_C\sqrt{T_A}) - M_A e^{-r_f T_A} N_1(k_1)$	0
$T_A \to 0$[b]	M_A	$(V^C - M_A) N_1(k_5+\sigma_C\sqrt{T_B}) - M_B e^{-r_f T_B} N_1(k_5)$	$(V^C - M_A)[1-N_1(k_5+\sigma_C\sqrt{T_B})] + M_B e^{-r_f T_B} N_1(k_5)$
$T_A = T_B = T$[c]	$bV^C[1-N_1(k_1+\sigma_C\sqrt{T})] + M_A e^{-r_f T} N_1(k_1)$	$V^C N_1(k_6+\sigma_C\sqrt{T}) - (M_A+M_B) e^{-r_f T} N_1(k_6)$	$V^C[1-b-N_1(k_6+\sigma_C\sqrt{T}) + bN_1(k_1+\sigma_C\sqrt{T})] - M_A e^{-r_f T}[N_1(k_1)-N_1(k_6)] + M_B e^{-r_f T_B} N_1(k_6)$
$\sigma_C \to 0$	$M_A e^{-r_f T_A}$	$V^C - M_A e^{-r_f T_A} - M_B e^{-r_f T_B}$	$M_B e^{-r_f T_B}$
$\sigma_C \to \infty$	0	V^C	0

a. $k_4 = \dfrac{\ln(V^C/M_B) + (r_f - \frac{1}{2}\sigma_C^2)T_B}{\sigma_C\sqrt{T_B}}$ b. $k_5 = \dfrac{\ln(V^C - M_A/M_B) + (r_f - \frac{1}{2}\sigma_C^2)T_B}{\sigma_C\sqrt{T}}$ c. $k_6 = \dfrac{\ln(V^C/M_A + M_B) + (r_f - \frac{1}{2}\sigma_C^2)T}{\sigma_C\sqrt{T}}$

values.[14] The values of bonds A and B go to zero when the variance of returns on the firm becomes infinite.

The valuation equations were used to obtain security values to test their responses to changes in parameters. The signs of the responses as obtained from the simulations are given in Table 3. The value of the stock is an increasing function of the value of the firm, time to maturity of debt A, time to maturity of debt B, the variance of returns on the merged firm and the risk-free rate. It is a decreasing function of the face value of debt A and debt B. The value of bond A is an increasing function of its own face value. It is a non-decreasing function of the value of the firm and the time to maturity of debt B. Bond A value is non-increasing function of the face of debt B, and is a decreasing function of its own time to maturity, variance of returns on the merged firm and the risk-free rate. Bond B value is an increasing function of the face value of debt A, its own time to maturity, the variance of returns on the merged firm, and the risk-free rate. The response of Bond B to changes in time to maturity of debt A is ambiguous.

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[14]The bond is called riskless if $N_1(k_1) = 1$, i.e., the probability of bankruptcy is extremely low.

TABLE 3

EFFECTS OF CHANGES IN PARAMETER VALUES

X	D^{AC}	D^{BC}	S^{C}
$\frac{\partial X}{\partial V^C}$	≥ 0	> 0	> 0
$\frac{\partial X}{\partial M_A}$	> 0	< 0	< 0
$\frac{\partial X}{\partial M_B}$	≤ 0	> 0	< 0
$\frac{\partial X}{\partial T_A}$	< 0	$\gtrless 0$	> 0
$\frac{\partial X}{\partial T_B}$	≥ 0	< 0	> 0
$\frac{\partial X}{\partial \sigma_C}$	< 0	< 0	> 0
$\frac{\partial X}{\partial r_F}$	< 0	< 0	> 0

4. MERGED VS. INDIVIDUAL FIRM SECURITY VALUES: SOME COMPARISONS

The preceding results are now used to investigate the effect of the merger on the values of Bond A, Bond B, and the common stock of both firms. The difference between the post-merger and pre-merger values of securities is made up of three main components. The first is the variance effect, the second is the leverage effect, and the third is the maturity effect. The variance effect is a result of the change in the variance of returns on the firm. The leverage effect is caused by a change in the face value of debt-to-firm value ratio, and the maturity effect is caused by the different maturities of Bond A and Bond B. Since the three components do not necessarily act in the same direction, the change in value of any security is determined by the relative magnitudes of each effect. The magnitude of the effects are, in turn, determined by the values of the various parameters in the valuation equations. Initially, in order to isolate these three effects, each one is considered separately while the other two are forced to zero.

a) The Variance Effect

To analyze the variance effect, we need to consider three special cases:

(i) $\sigma_B < \sigma_C < \sigma_A$. After the merger, the securities of firm B are part of a riskier firm, while the opposite holds for the securities of firm A. Therefore, the variance effect would be positive for Bond A and the common stock of firm B, while it would be negative for Bond B and the common stock of firm A.[15]

The effect on the combined common stock will depend on the relative magnitudes of pre-merger stock A and stock B values, and is therefore ambiguous.

(ii) $\sigma_A < \sigma_C < \sigma_B$. After the merger, the securities of firm A are part of a riskier firm, while the opposite holds for the securities of firm B. Therefore, the variance effect would be positive for Bond B and the common stock of firm A, while it would be negative for Bond A and the common stock of firm B. The effect on the combined common stock is ambiguous.

(iii) $\sigma_A > \sigma_B > \sigma_C$. After the merger, the securities of both firms are part of a less risky firm.

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[15]These propositions follow from the face the bond prices are decreasing functions of variance, while stock prices are increasing functions of variances.

Therefore, the variance effect is positive for both Bonds A and B, and negative for both stocks. In this case, the effect on the combined common stock is unambiguously negative.

b) The Leverage Effect

The leverage effect is concerned with the change in bankruptcy risk caused by a ceteris paribus change in the leverage ratio (defined as the ratio of face value of debt to firm value). If the leverage ratio of firm A is greater than that for firm B, the merger results in a firm that has a leverage ratio less than that of firm A and greater than that of firm B. Therefore, the merger results in a decrease in the leverage-ratio related risk for Bond A, while the opposite holds for Bond B. This, in turn, implies an increase in the value of Bond A, and a decrease in the value of Bond B. The opposite effects obtain when the leverage ratio of firm A is less than the leverage ratio of firm B. In addition, given the no synergy assumption, the leverage effect is always negative for the combined common stock.[16] This is the result of the unambiguous increase in the leverage-ratio related risk for the combined equity.

c) <u>The Maturity Effect</u>

Even if the variance and leverage effects are zero, the merger will result in a change in security value because of the differences in debt maturity dates. From the point of view of Bond B, the merger is equivalent to the firm issuing new debt with a shorter maturity. Although it has been assumed that both bonds have equal priority, this priority clause comes into play only if the firm goes bankrupt. If the value of the firm is higher than the face value of Bond A at the time of its maturity, Bond A is paid in full and thus as a result of its shorter maturity, Bond A in some sense becomes "senior" to Bond B. Therefore, this "seniority" component of the maturity effect would be positive for Bond A and negative for Bond B. As stated earlier, Bond A is affected by the presence of Bond B, if and only if bankruptcy occurs at Bond A's

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[16]See Appendix B for proof of these propositions. The effect on each individual stock would depend on the stock-sharing rule. One could specify sharing rules which would result in wealth transfers between shareholders i.e., stockholders of one firm lose, while the stockholders of the other firm gain, even though the combined stock decreases in value.

maturity date. In that case, Bond A has to share the liquidated firm with Bond B. Therefore, the bankruptcy component of the maturity effect is negative for Bond A and positive for Bond B. The size of this effect depends both on the probability of bankruptcy and the bankruptcy sharing rule. However, it must be noted that from the point of view of B, the "new" debt issue is always accompanied by a change in the firm value, with this change being, in general, greater than the face value of the new issue. Therefore, the maturity effect of Bond B value is ambiguous. Since the negative components of the maturity effect on Bond A will, in general, be small (in absolute value), the maturity effect will be positive for Bond A. Therefore, given the no synergy assumption, the increase in value of Bond A will result in a decrease in the combined value of Bond B and the two common stocks. If Bond B increases in value, the combined stock will always decrease in value. In the situation where Bond B decreases in value, the amount of this decrease (in absolute value) is always less than the increase in Bond A value.[17] Therefore, the maturity effect is always

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[17]See Appendix C for proof of these propositions.

negative for the combined common stock.[18] A summary of the above discussion is presented in Table 4.

d) <u>The Combined Effect</u>

If the variance of firm A is greater than that of firm C, and if the leverage ratio of firm A is greater than that if firm B, Bond A would have a non-negative change in value (since all three effects are positive). If the variance of firm A is smaller than that of firm C, and (or) the leverage ratio of firm A is lower than that of firm B, the value of Bond A could fall as a result of the merger. In the simulations that follow, it will be shown that for the value of Bond A to fall, the variance of return on firm A should, in most cases, be smaller than the variance of returns on firm C. Given the positive maturity effect, it is only in some extreme cases that the

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[18] One reason for the maturity effect being negative can be see by considering the choices available to the stockholder before the merger. Before the merger, an individual who held both stocks had four mutually exclusive actions available. These actions were a) exercise both stocks, b) exercise neither, c) exercise only A, and d), exercise only B. After the merger, since the longer term option is now contingent on the short-term option, the fourth action is no longer available. Since this action must have had a non-negative value, its removal must decrease the value of the combined equity.

TABLE 4

The Effects of the Merger on Firm Security Values

		Bond A	Bond B	Common Stock[a]
Variance Effect	$\sigma_A > \sigma_C > \sigma_B$	> 0	< 0	$\gtrless 0$
	$\sigma_A < \sigma_C < \sigma_B$	< 0	> 0	$\gtrless 0$
	$\sigma_A \ \sigma_B > \sigma_C$	> 0	> 0	< 0
Leverage Effect	$M_A/V_A > M_B/V_B$	> 0	< 0	< 0
	$M_A/V_A < M_B/V_B$	< 0	> 0	< 0
Maturity Effect		> 0	$\gtrless 0$	< 0

a. The effect of the merger on each individual firm's stock would depend on the stock-sharing rule.

leverage effect is sufficiently negative for the combination of the two to be less than zero.[19]

Such extreme cases are not required for the merger to result in a decrease in Bond B value. Since the three components act in different directions and vary in magnitude with changes in parameter values, there must exist a combination of values for the parameters at which the pre-merger and post-merger values of Bond B are exactly equal. Specifically, if the time to maturity of Bond B is increased, holding all other parameters constant, there will exist a critical time to maturity, T_B^*, which has the property that for $T_B > T_B^*$, the post-merger value is less than the pre-merger value, and the opposite holds for $T_B < T_B^*$. Similarly, if the variance effect on the stock value is positive and outweighs the negative leverage and maturity effects, the stock will increase in value as a result of the merger. If the variance effect on stock value is negative, the post-merger value of stock is always less than its pre-merger value.

To analyze the behavior of security prices, we now consider a few numerical examples. Assume that the value

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[19]One such case obtains when the face value of Bond A is much greater than the face value of Bond B.

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Such extreme cases are not required for the merger to result in a decrease in Bond value. Since the three components act in different directions and vary in magnitude with changes in parameter values, there must exist a combination of values for the parameters at which the pre-merger and post-merger values of Bond B are exactly equal. Specifically, if the time to maturity of Bond B is increased, holding all other parameters constant, there will exist a critical time to maturity, T*, which has the property that for $T_B > T_B^*$, the post-merger value is less than the pre-merger value, and the opposite holds for $T_B < T_B^*$. Similarly, if the variance effect on the stock value is positive and outweighs the negative leverage and maturity effects, the stock will increase in value as a result of the merger. If the variance effect on stock value is negative, the post-merger value of stock is always less than its pre-merger value.

To analyze the behavior of security prices, we now consider a few numerical examples. Assume that the value

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[19] One such case obtains when the face value of Bond a is much greater than the face value of Bond B.

of both firm A and B is \$100.00, the risk-free rate is 6% per annum, and the correlation between firms A and B is 0.85.

The result of the first set of simulations are presented in figures 1-8 and Tables 5-7. In these simulations it was assumed that the variance of returns on firm A was 0.3, the variance of returns on firm B was 0.2, the face values of debt A and debt B were \$50.00, \$75.00 and \$90.00, and the time to maturity of debt A was 1 year.

The change in value of B ($D^{BC} - D^{B}$) is plotted as a function of its time to maturity for different Bond A and B face value in figures 1, 2 and 3. The change in value of Bond B varies anywhere from -4.63% of pre-merger value to +3.00% of pre-merger values (see Tables 5,6,7). For a majority of the cases considered in these simulations Bond B falls in value. There are only four Bond A-Bond B face value combinations (M_A = 50, M_B + 50, 75, 90; M_A = 75, M_B = 90) for which there exist values of T_B at which holders of Bond B experience a gain in value. For example, with Bond A and Bond B face values of \$50, increases in Bond B value take place only when the time to maturity of Bond B is less than 10 years (Figure 1 and Table 5). Thus, given the parameters values used in this simulation, if firm B had debt in its capital structure with a maturity greater

FIGURE 1: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER

(M_B=50)

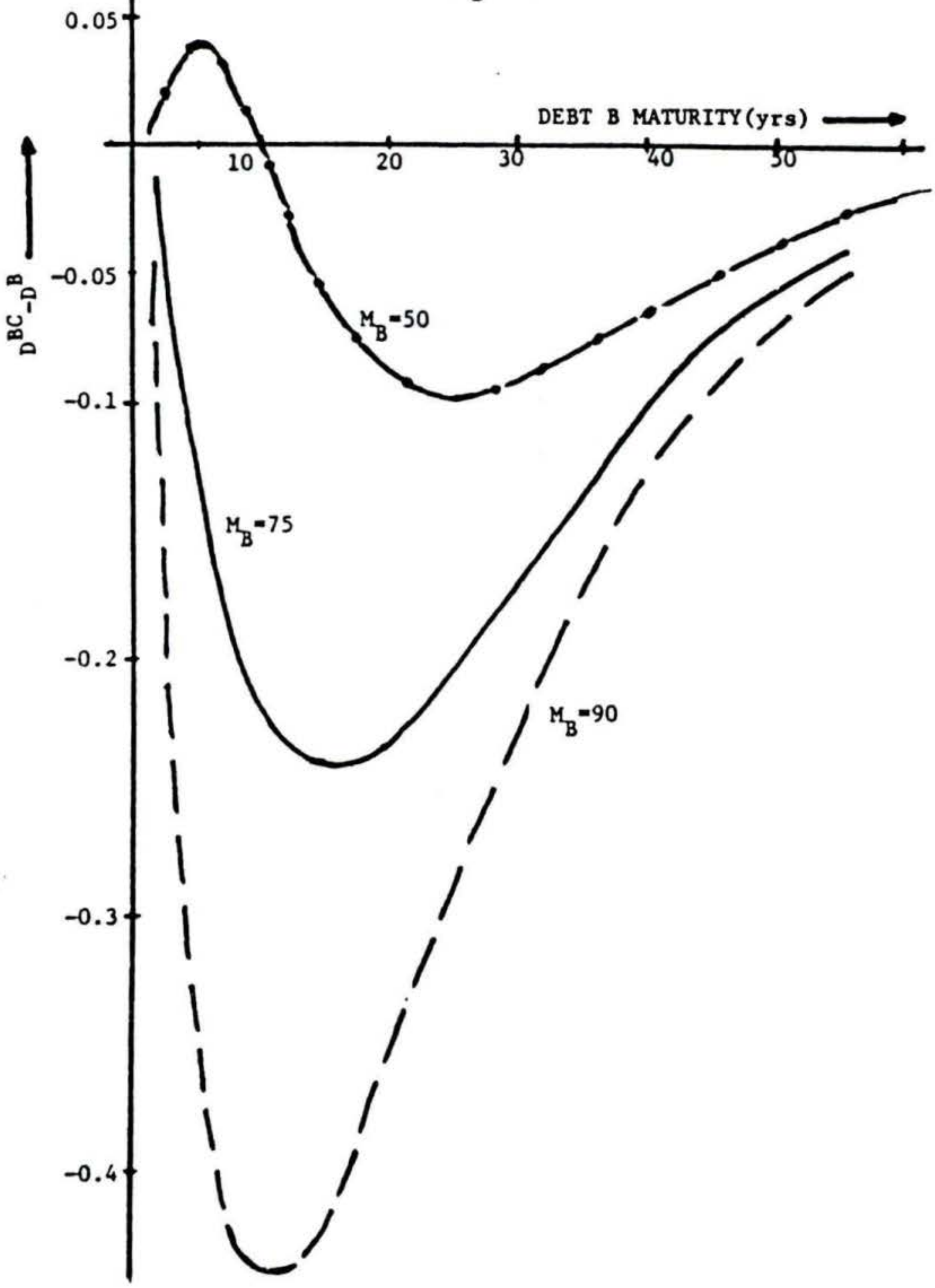

TABLE 5

CHANGE IN SECURITY VALUES[a]

$V^A = V^B = 100$, $M_A = 50$, $T_A = 1.0$, $\sigma_A = 0.3$, $\sigma_B = 0.2$, $\rho = 0.85$, $r_f = 0.06$

M_B	T_B	D^B	D^{BC}	%Change in Value of Debt B	S^A	S^B	S^C	% Change Value of Stock
	2.0	44.33	44.34	0.02		55.67	108.58	-0.05
	3.0	41.72	41.74	0.05		58.28	111.17	-0.05
50.00	5.0	36.92	36.95	0.11		63.08	115.96	-0.07
	10.0	27.20	27.20	0		72.80	125.71	-0.03
	15.0	20.09	20.03	-0.3		79.91	132.88	0.02
	20.0	14.86	14.77	-0.61		85.14	138.14	0.04
	2.0	65.75	66.31	0.84		43.25	86.61	-0.68
	3.0	61.54	62.22	1.10	52.95	38.46	90.69	-0.79
75.00	5.0	54.11	54.77	1.22		45.89	98.14	-0.71
	10.0	39.74	40.00	0.68		60.26	112.91	-0.27
	15.0	29.39	29.38	-0.03		70.61	123.53	-0.02
	20.0	21.8	21.66	-0.69		78.20	131.26	0.08
	2.0	76.79	78.98	2.85		23.21	73.93	-2.93
	3.0	71.74	73.89	3.00		28.26	79.02	-2.70
90.00	5.0	63.10	64.83	2.74		36.90	88.08	-1.96
	10.0	46.53	47.21	1.46		53.47	105.70	-0.68
	15.0	34.56	34.67	0.35		65.44	118.24	-0.13
	20.0	25.71	25.57	-0.54		74.29	127.34	0.08

a. D^A = 47.05, D^{AC} = 47.09, and % change in debt A value is 0.09 for all cases considered.

FIGURE 2: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER

(M_B=75)

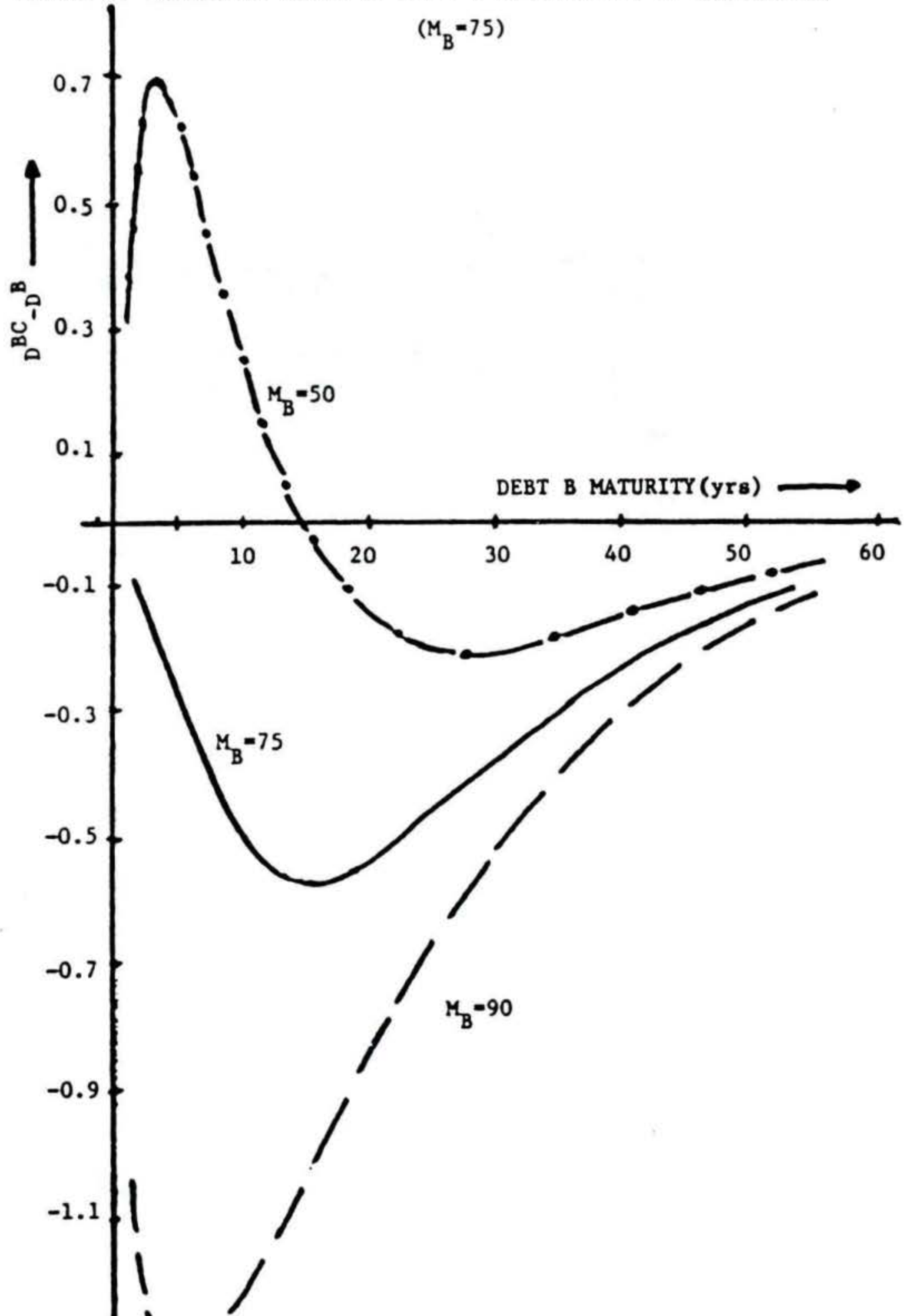

TABLE 6

CHANGE IN SECURITY VALUES[a]

$V^A = V^B = 100$, $M_A = 75$, $T_A = 1.0$, $\sigma_A = 0.3$, $\sigma_B = 0.2$, $\rho = 0.85$, $r_f = 0.06$

M_B	T_B	D^B	D^{BC}	% Change in Value of Debt B	S_A	S_B	S^C	% Change in Value of Stock
50.00	2.0	44.33	44.31	-0.05	30.85	55.67	85.05	-1.73
	3.0	41.72	41.66	-0.14		58.28	87.70	-1.65
	5.0	36.92	36.80	-0.33		63.08	92.57	-1.53
	10.0	27.20	26.98	-0.81		72.80	102.38	-1.26
	15.0	20.09	19.85	-1.19		79.91	109.52	-1.16
	20.0	14.86	14.63	-1.55		85.14	114.74	-1.12
75.00	2.0	65.75	65.62	-0.21		34.25	63.75	-2.34
	3.0	61.54	61.36	-0.29		38.46	68.01	-1.95
	5.0	54.11	53.83	-0.52		45.89	75.53	-1.63
	10.0	39.74	39.24	-1.26		60.26	90.13	-1.13
	15.0	29.39	28.83	-1.91		70.61	100.54	-0.96
	20.0	21.8	21.27	-2.43		78.20	108.09	-0.92
90.00	2.0	76.79	77.06	0.36		23.21	52.31	-3.32
	3.0	71.74	71.95	0.28		28.26	57.42	-2.92
	5.0	63.10	63.04	-0.11		36.90	66.33	-2.15
	10.0	46.53	45.96	-1.23		53.47	83.41	-1.14
	15.0	34.56	33.83	-2.11		65.44	95.54	-0.83
	20.0	25.71	25.-0	-2.76		74.29	104.36	-0.78

a. $D^A = 69.11$, $D^{AC} = 70.93$, and % change in debt A value is 2.21 for all cases considered.

FIGURE 3: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER

(M_B=90)

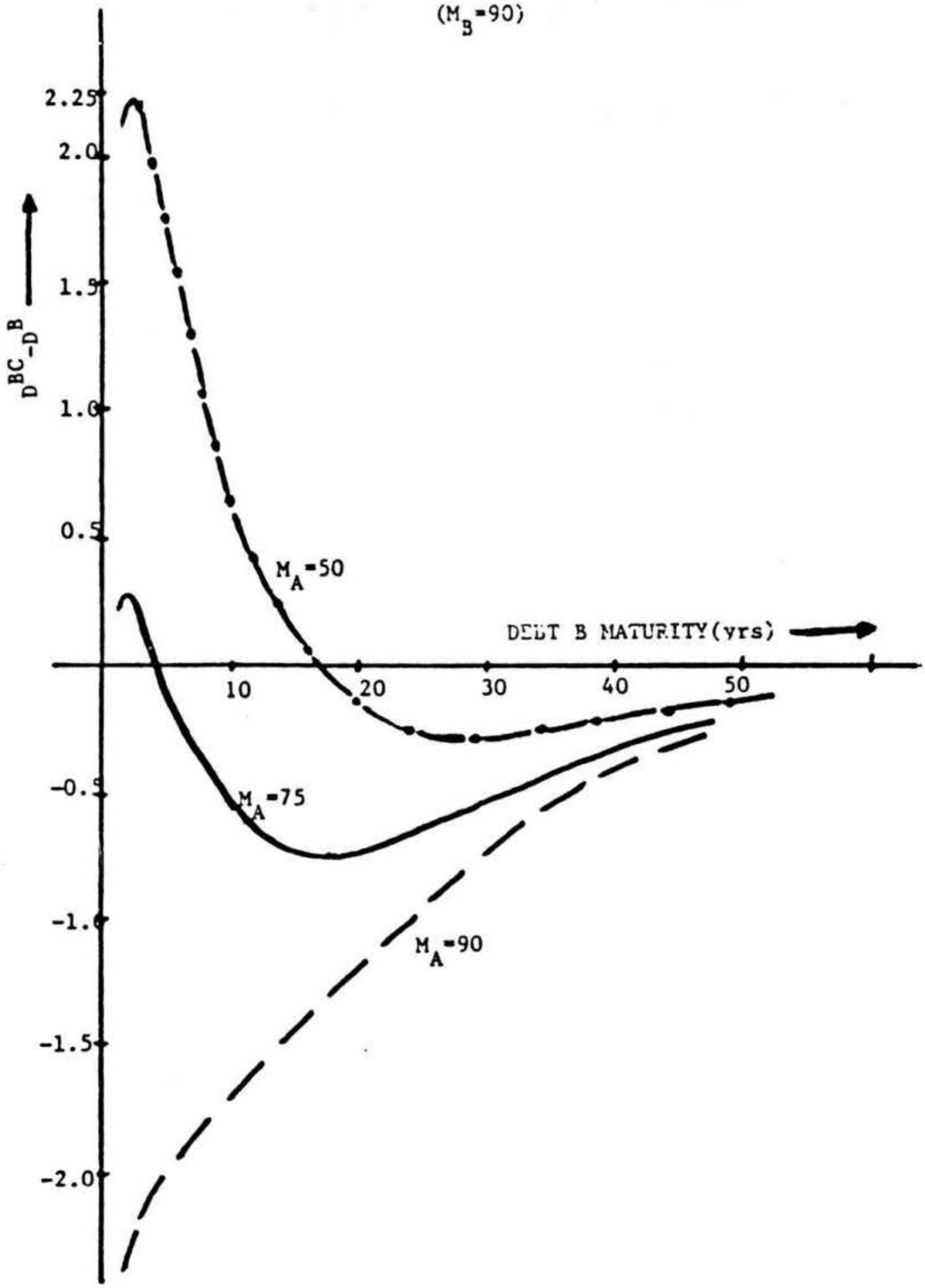

TABLE 7

CHANGE IN SECURITY VALUES[a]

$V^A = V^B = 100$, $M_A = 90$, $T_A = 1.0$, $\sigma_A = 0.3$, $\sigma_B = 0.2$, $\rho = 0.85$, $r_f = 0.06$

M_B	T_B	D^B	D^{BC}	% Change in Value of Debt B	S^A	S^B	S^C	% Change in Value of Stock
	2.0	44.33	44.24	-0.2		55.67	71.01	-6.47
	3.0	41.72	41.52	-0.49		58.28	73.73	-6.11
50.00	5.0	36.92	36.58	-0.92		63.08	78.67	-5.59
	10.0	27.20	26.76	-1.62		72.80	88.49	-4.90
	15.0	20.09	19.67	-2.09		79.91	95.58	-4.57
	20.0	14.86	14.50	-2.42		85.14	100.74	-4.41
	2.0	65.95	64.50	-1.9		34.25	50.75	-6.88
	3.0	61.54	60.23	-2.13		38.46	55.02	-6.29
75.00	5.0	54.11	52.80	-2.42	20.25	45.89	62.45	-5.58
	10.0	39.74	38.52	-3.07		60.26	76.73	-4.70
	15.0	29.39	28.35	-3.54		70.61	86.90	-4.36
	20.0	21.8	20.95	-3.90		78.20	94.30	-4.22
	2.0	76.69	74.59	-2.86		23.21	40.67	-6.44
	3.0	71.74	69.75	-2.79		28.26	45.51	-6.18
90.00	5.0	63.10	61.25	-2.95		36.90	54.00	-5.49
	10.0	46.53	44.84	-3.63		53.47	70.42	-4.49
	15.0	34.56	33.10	-4.22		65.44	82.15	-4.14
	20.0	25.71	24.53	-4.63		74.29	90.72	-4.04

a. $D^A = 79.75$, $D^{AC} = 84.75$, and % change in debt A value is 6.27 for all cases considered.

than 10 years, the merger would result in the expropriation of wealth in the debtholders of firm B. Similar results carry through to other cases considered here, with the critical time to maturity, T_B^*, (which was 10 years in the preceding example) increasing with the face value of debt B and decreasing with the face value of debt A.[20] Thus, given the above parameter values, the probability that firm B debtholders will get expropriated increases with increases in the face value of debt A and decreases in the face value of debt B.

The maximums and minimums in the figures can be explained by the fact that the rate of change in debt value with respect to time is different pre-merger versus post-merger. In the pre-merger situation the value of debt decreases with increasing time to maturity, because of the effect of time value of money. In the post-merger situation, the effect of changing time to maturity would affect debt value in two ways. The time value of money effect would reduce the value of debt, while the bankruptcy condition at date T_A would either add to or reduce the time value effect. Therefore, the post-merger

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[20] T_B is assigned the value T_A if the value of bond B always falls as a result of the merger, and a value ∞ if it always increases as a result of the merger.

value of debt can decrease at a rate faster than the pre-merger value of debt in certain ranges of time to maturity, and decrease at a slower rate in other rangers. This would result in maximums and minimums in the plot of change in debt value against time to maturity.

The value of Bond A always increases as a result of the merger or the parameter values considered in these simulations. This increase is a direct result of the fact that Bond A is not riskless before the merger. If the post-merger probability of bankruptcy at time T_A is approximately 0, then Bond A attains its maximum possible value ($M_A e^{-r_f T_A}$) after the merger. In such a case, the post-merger value of debt A is invariant to changes in the time to maturity of debt B (given $T_A < T_B$). The simulations with debt A face values of \$50 and \$75 yielded such a situation and are depicted in figure 4. With a non-zero probability of bankruptcy at time T_A, the post-merger value of Bond A increases with increasing time to maturity of debt B, with the former becoming constant once it reaches its maximum possible value. This case obtains in the simulation conducted here, when debt A has a face value of \$90 and is depicted in figure 5. The lower the face value of debt B, the faster debt A reaches its maximum possible value. Thus, for the parameter

FIGURE 4: CHANGE IN VALUE OF DEBT A AS A RESULT OF THE MERGER

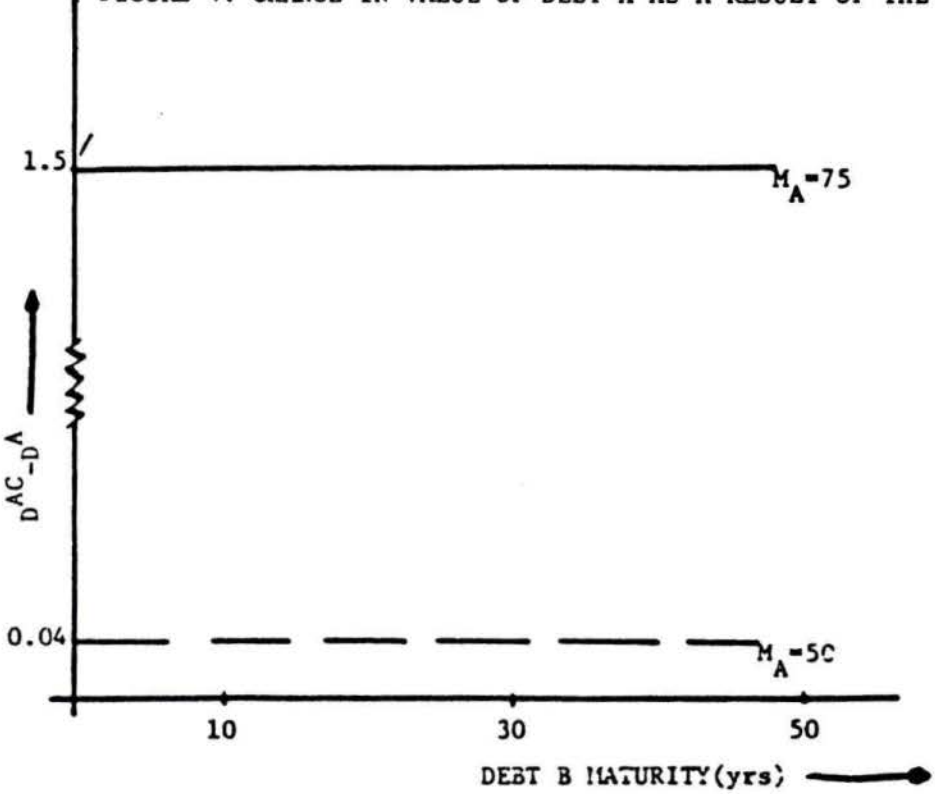
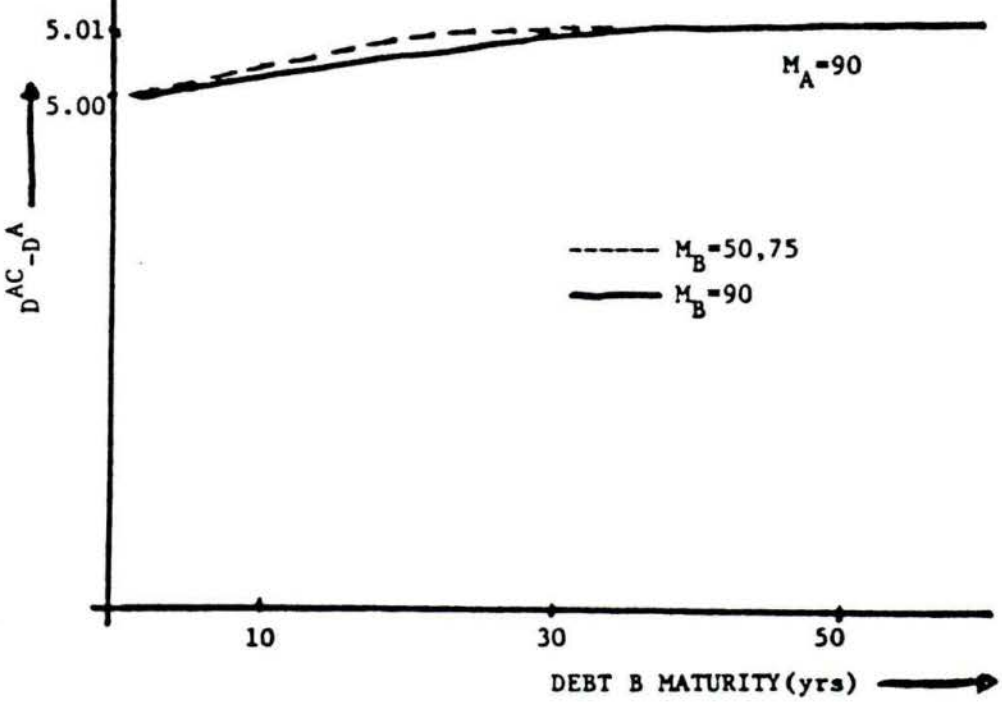


FIGURE 5: CHANGE IN VALUE OF DEBT A AS A RESULT OF THE MERGER



values considered here, holders of debt A always gain as a result of the merger. All or part of their gain results from the loss in value of debt B. Thus, the merger results in a transfer of wealth from the debt holders of firm B to the debtholders of firm A in a majority of the cases considered here.

As stated earlier in this section, the post-merger value of stock can be greater than or less than its pre-merger value. If the values of debt A and debt B increase as a result of the merger, the no synergy assumption implies that the value of common stock must fall. Therefore, as long as the time to maturity of debt B is less than the critical value T_B^*, the value of common stock always falls after the merger (given that the value of debt A is increased). For debt B maturities greater than T_B^*, the result is unambiguous if Bond A value has fallen. In this case, the value of stock would increase and therefore wealth transfers occur from both debtholders to stockholders. If there is no change in Bond a value and $T_B > T_B^*$, then the value of common stock will rise and now the wealth transfer takes place from the holders of debt B to the stockholders. However, if the value of debt A increases, the direction of change in common stock value is ambiguous. As long as the increase in value of Bond A is less than the decrease in value of Bond B, the stockholders gain and wealth transfer takes place from the holders of debt B to the holders of debt A and the stockholders. For the parameter values specified earlier, the change in stock value is plotted as a function of the time to maturity of debt B in figures 6, 7, and 8. A favorable wealth transfer from the point of view of the stockholders takes place only when the face value of debt A is \$50. The maximum gain in value is 0.08% of pre-merger value (see Table 5). In every other case, the stockholders lose as a result of the merger. These losses range from -0.02% to-6.88% of pre-merger value.

The wealth transfer effect discussed above is critically dependent on two factors. The first is the assumption that the two debts have different maturities and the second is the fact that the variance values chosen were such that the variance of returns on firm B was less than that on firm A ($\sigma_B < \sigma_C < \sigma_A$). When A and B were changed to make $\sigma_C < \sigma_B = \sigma_A$, the results obtained here are similar to those obtained by Galai and Masulis (1976). For this particular case, it was found that the values of both debt A and B increase after the merger. Therefore, wealth expropriation of holders of Bond B occurs only when the variance of returns on firm B is less than the

FIGURE 6: CHANGE IN VALUE OF STOCK AS A RESULT OF MERGER

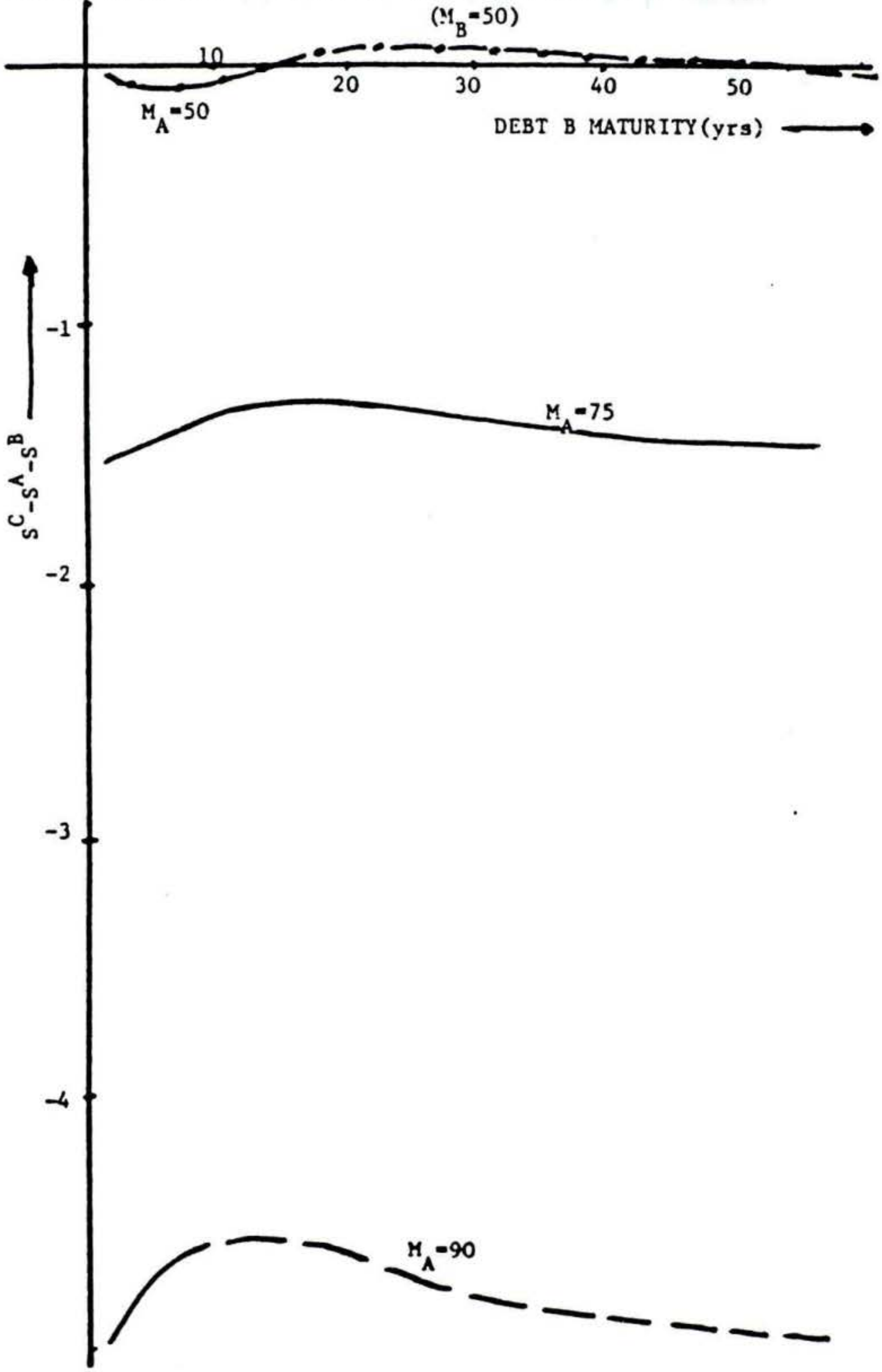


FIGURE 7: CHANGE IN VALUE OF STOCK AS A RESULT OF MERGER

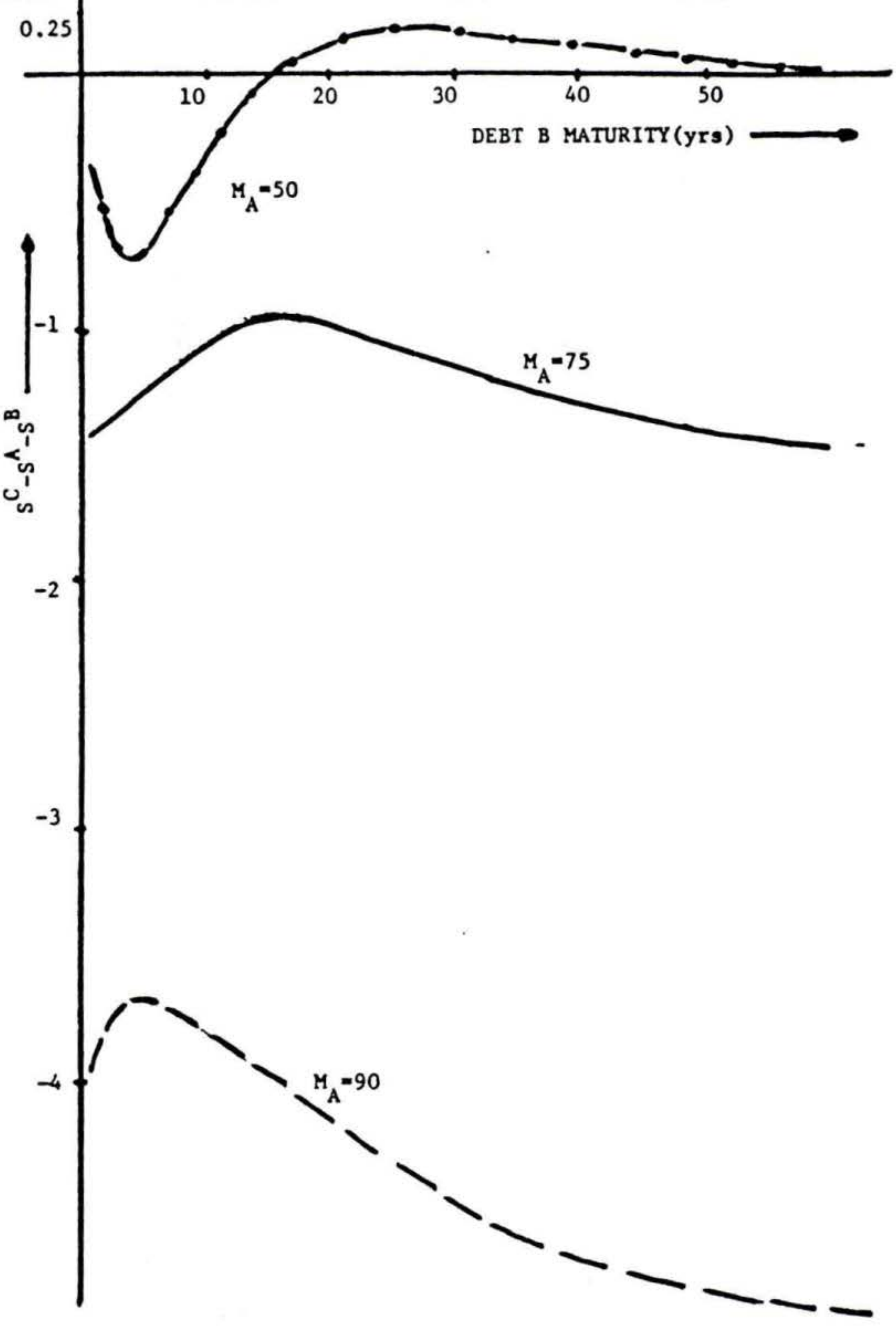
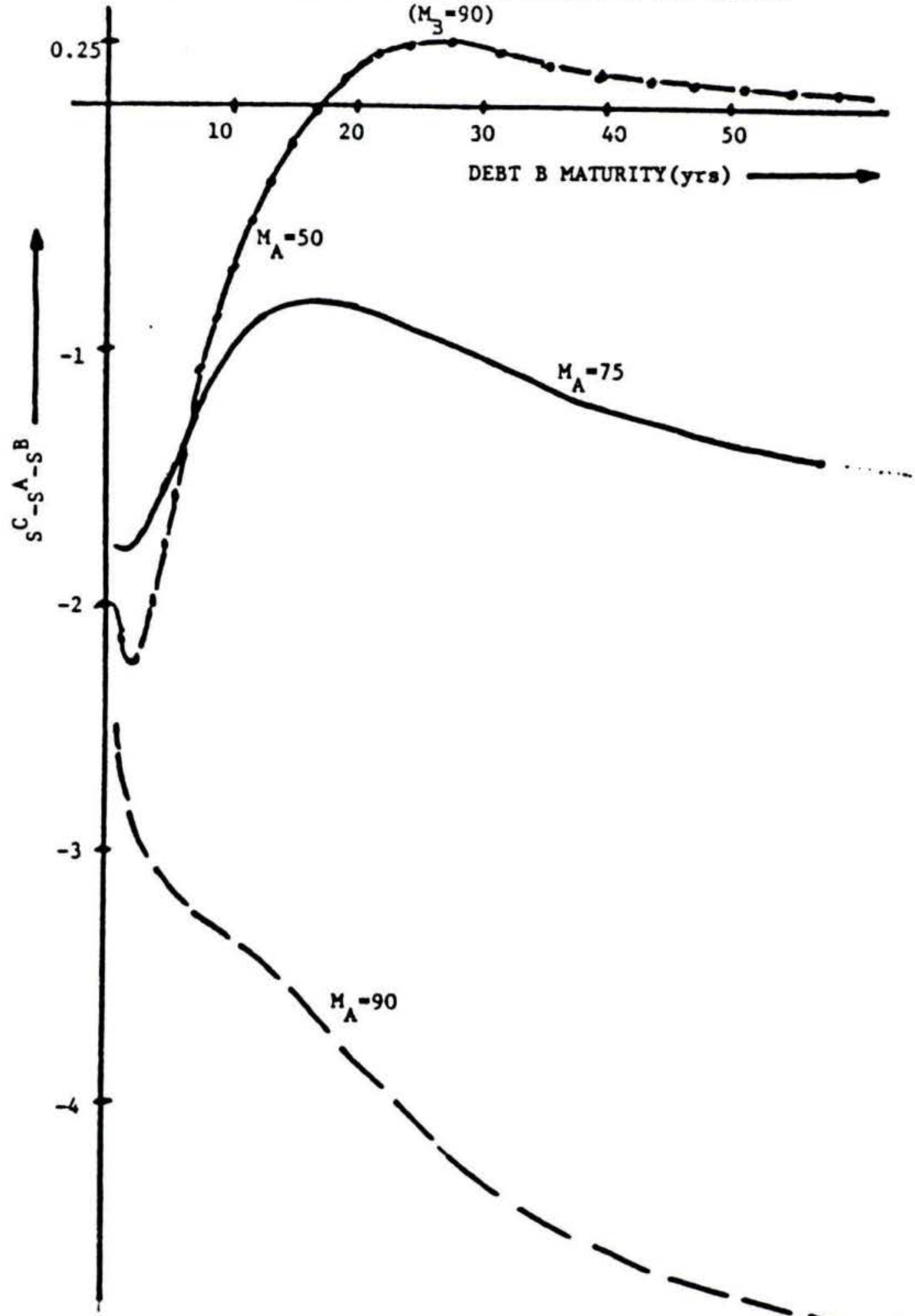


FIGURE 8: CHANGE IN VALUE OF STOCK AS A RESULT OF THE MERGER
(M_B=90)



variance of returns on firm A and debt A matures before debt B. In addition, for the example considered above the value of debt A always increased after the merger. This result, though not as sensitive to parameter changes as those obtained for debt B, depends on the fact that $\sigma_A > \sigma_B$. Even if $\sigma_A < \sigma_B$, the result follows through as long as the difference in the two variances is not very large. Figures 9, 10, and 11 present the results of a case where the variance of returns on firm A was much smaller than that on firm B ($\sigma_A = 0.2$, $\sigma_B = 0.7$).[21]

For such an extreme value for the difference between the two variances, there exist cases where the value of bond A falls as a result of the merger with the value of bond B increasing. Therefore, there exist cases where a wealth transfer can occur from the short-term debtholders to the long-term debtholders. In this example, there also exists a wealth transfer from the stockholders to the debtholders.

In summary, the various wealth transfer effects that can occur as a result of the merger are:

(i) from stockholders to debtholders

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[21] The maximums and minimums in this case can be explained by using the same analysis that was applied to Figures 1-8.

FIGURE 9: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER

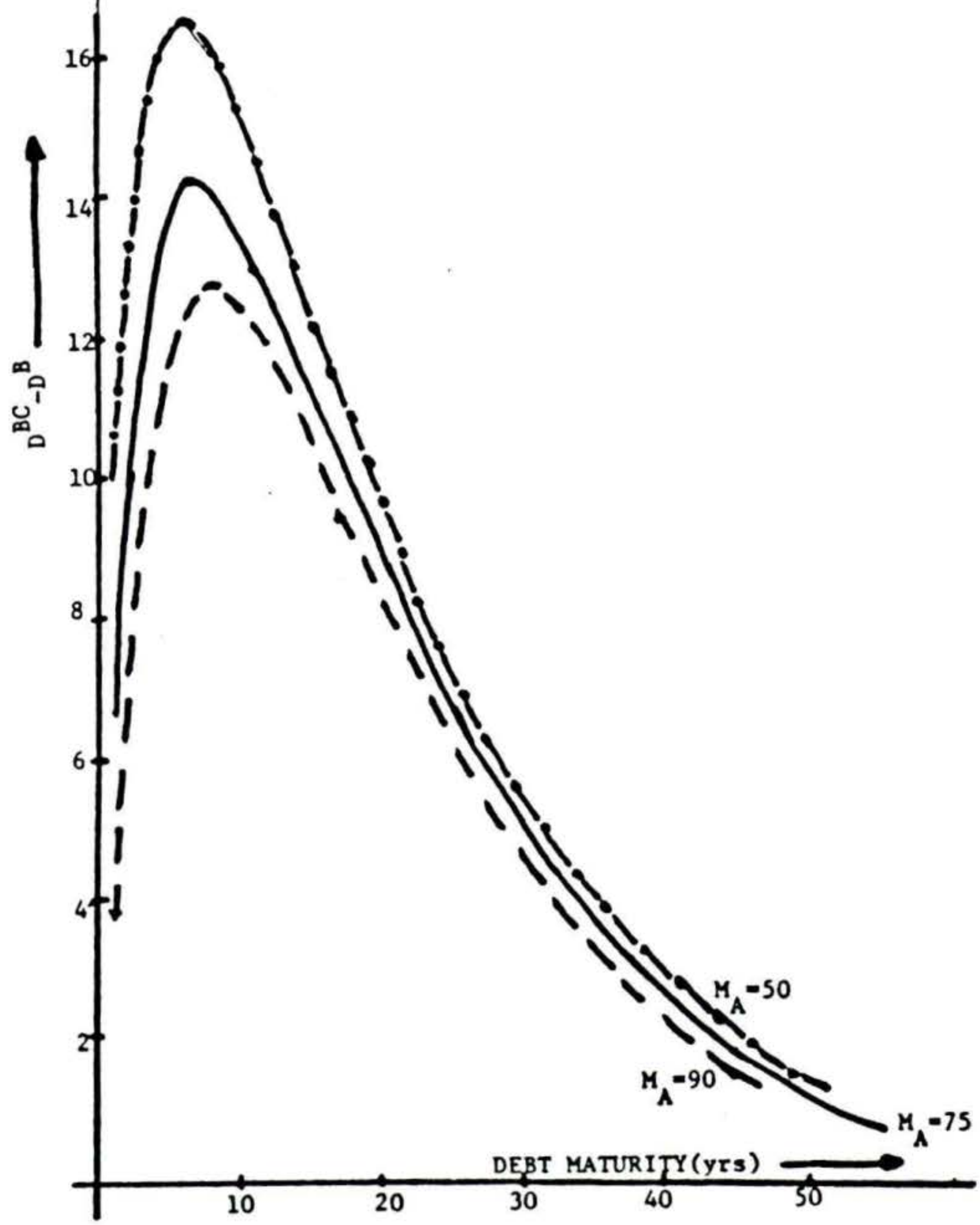


FIGURE 10: CHANGE IN VALUE OF DEBT A AS A RESULT OF THE MERGER

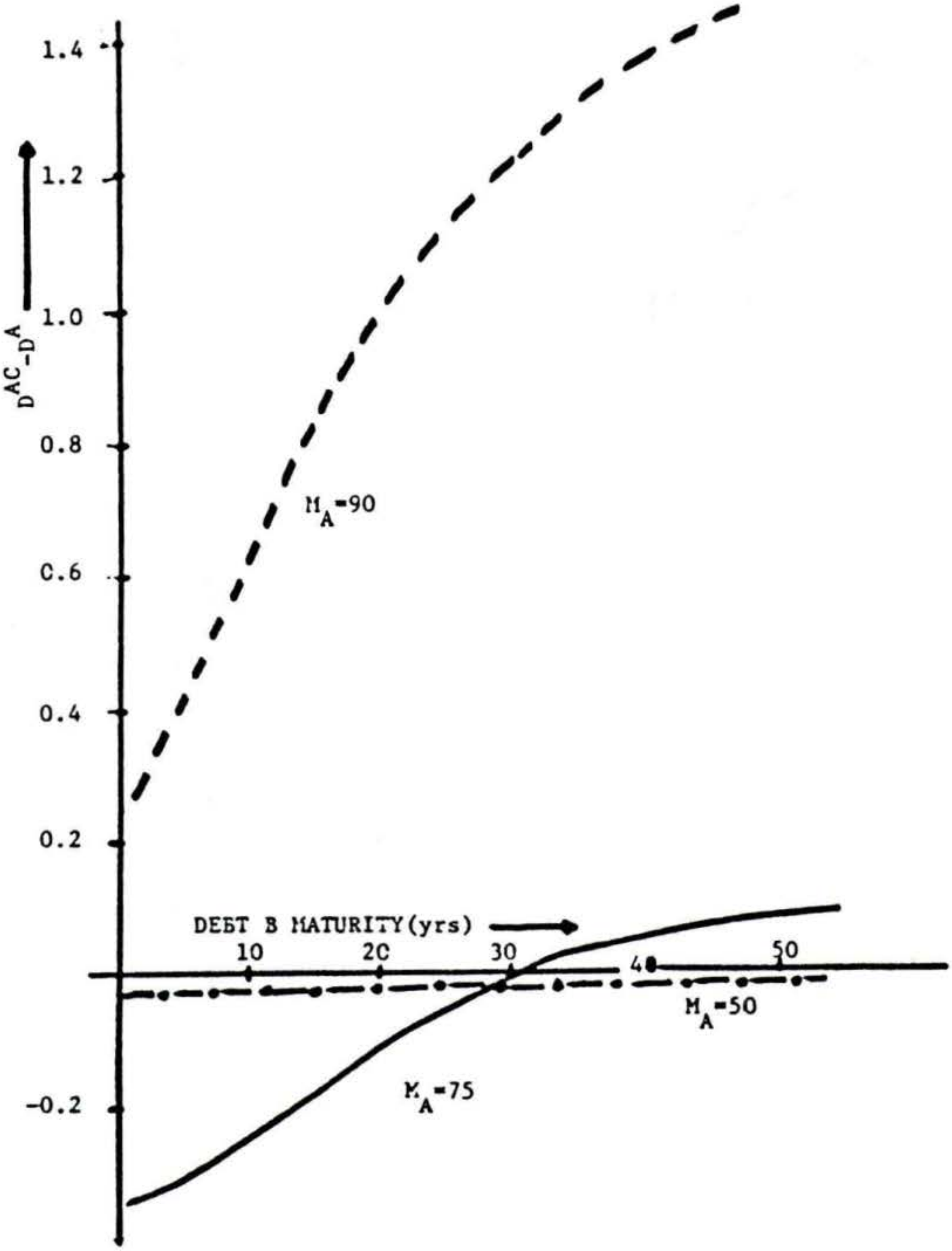
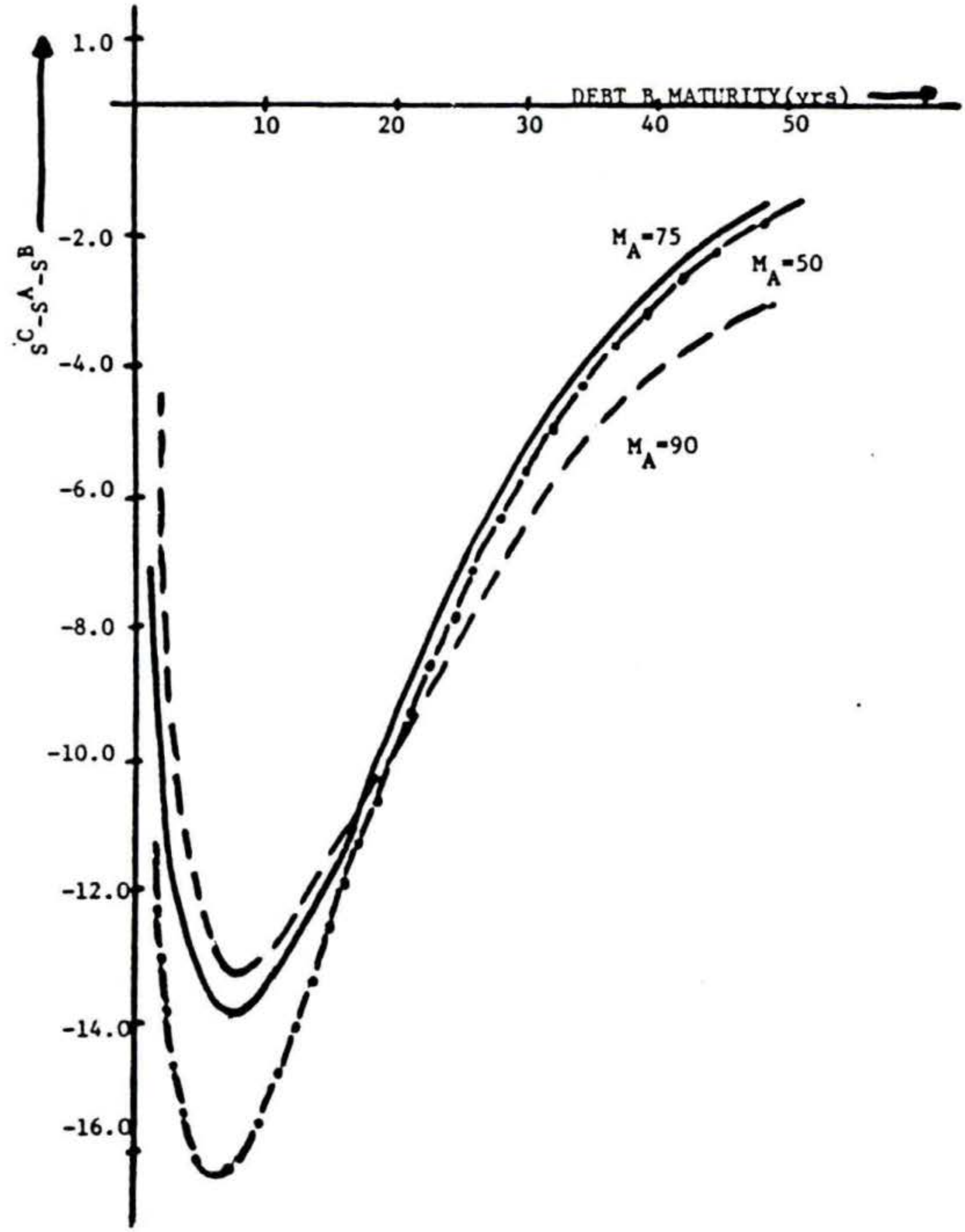


FIGURE 11: CHANGE IN VALUE OF COMMON STOCK AS A RESULT OF THE MERGER



(ii) from stockholders and holders of long-term debt to holders of short-term debt

(iii) from stockholders and holders of short-term debt to holders of long-term debt

and (iv) from holders of long-term debt to stockholders and holders of short-term debt.

The valuation equations used to obtain the above results are approximations to the "true" equations because of the assumptions required in the derivations of the former. We now test for the possibility that the conclusions drawn above are a direct result of the errors caused by some of the assumptions required in the derivation. A natural way to obtain the sign and magnitude of the errors is to use one of the many numerical solution techniques that are available for valuing options.[22] These techniques include the binomial approximation and finite difference

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[22]The numerical techniques do not yield the magnitude of the errors caused by "inaccurate" distributional assumptions. The errors referred to here are results of approximations used in solving for the valuation equations. See Appendix A and footnote 29 for further details on the approximations used.

methods.[23] The binomial approximation is used here mainly because it is computationally more efficient than the finite difference methods.

The binomial approximation was used to compute the values of all three securities for the parameter values mentioned earlier. The comparison of these values with those obtained from the valuation equations are in Table 8. Since a difference of ± 0.01 can be attributed to rounding and computation errors, it is assumed that if the absolute value of the error is less than or equal to \$01.01, the equations derived here are pricing the securities "correctly". The values obtained from the binomial approximations were identical (within ± 1 percent) to those obtained from the valuation equations for all securities except two. The two cases where the errors exists are in the post-merger value of debt B and therefore the post-merger value of the stock. The average error in the post-merger value of debt B for the 117 cases tested was -0.56%. There were 34 cases in which identical post-merger debt B values (within ± 0.01) were obtained from the binomial approximation and the valuation

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[23]See Cox, Ross and Rubinstein (1979) for details about the binomial approximation. See Schwartz (1977) and Brennan and Schwartz (1977, 1978) for details about the finite difference approximation.

TABLE 8

Errors in Security Valuation[a]

$V^A = V^B = 100,\ T_A = 1.0,\ \sigma_A = 0.3,\ \sigma_B = 0.2,\ \rho = 0.85,\ r_f = 0.06$

M_A	M_B	No. of Cases	Average % error in D^A	Average % error in D^{AC}	No. of negative errors in D^{BC}	Average % negative error in D^{BC}	No. of positive errors in D^{BC}	No. of cases where error in D^{BC} $\in$ -0.01,0.01
50	50	13	0	0	0	-	0	13
	75	13			8	0.16		5
	90	13			9	0.23		4
75	50	13			10	0.35		3
	75	13			10	0.80		3
	90	13			11	0.68		2
90	50	13			11	0.99		2
	75	13			12	1.46		1
	90	13			12	1.34		1
TOTAL		117	0	0	83		0	34

a. The error is calculated as $(C^B - C^V)/C^B$, where C^B is the value obtained from the binomial approximation and C^V is the value obtained from the valuation equations.

equations. In the remaining 83 cases, the post-merger value of debt B obtained from the binomial approximation was always lower than the one obtained from the valuation equation. Therefore, the post-merger valuation equation for debt B always tends to overvalue the security. This implies that the stock is always undervalued when valuation equation (4) is used. This, therefore, lends more strength to the conclusion that there exist cases where the wealth of the holders of bond B is reduced as a result of the merger. Table 8 also points to the fact that the errors increase in absolute value with increasing face value of debt A and decreasing time to maturity of debt B. We can, therefore, conclude that the errors caused by the assumptions required in deriving the valuation equations do not invalidate the conclusion drawn about the possible wealth transfer.

5. THE VALUATION EQUATIONS: SOME GENERALIZATIONS

The results derived to those state are based on assumptions A1-A11. The effect of two particular assumptions, namely A6 and A2 are now considered. We first consider the effect of a change in the variance of returns on firm C after retirement of debt A (Assumption

A12) and then relax A2 to include coupon payments on debt (Assumption A2').

a) <u>The Effect of a Post Debt A Retirement Variance Change</u>

In general, the retirement of debt A could cause a change in the variance of returns on firm C. Since the value of debt A is unaffected by events that occur after its maturity date, the valuation equation for short-term debt remains unchanged and is given by equation 3. The current market value of the stock of the merged firm is given by[24]

$$S^C = V^C N_2(k_1 + \sigma_{C1}\sqrt{T_A},\ k_7;\rho) - M_A e^{-r_f T_A} N_2(k_1, k_8;\rho) - M_B e^{-r_f T_B} N_2(k_1, k_9;\rho) \quad (6)$$

where $$k_1 = \frac{\ln(V^C/M_A) + (r_f - \frac{1}{2}\sigma^2)\, T_A}{\sigma_{C1}\sqrt{T_A}}$$

$$k_7 = \frac{\ln(V^C/M_B) + (r_f + \frac{1}{2}\sigma^2)T_B + \frac{1}{2}(\sigma^2 - \sigma^2)T_A}{\sqrt{T_B\sigma^2 + T_A(\sigma^2 - \sigma^2)}}$$

$$k_8 = \frac{\ln(V^C/M_B) + (r_f + \frac{1}{2}\sigma^2)T_B - \frac{1}{2}(\sigma^2 - \sigma^2)T_A}{\sqrt{T_B\sigma^2 + T_A(\sigma^2 - \sigma^2)}}$$

$$k_9 = k_7 - \sqrt{T_B\sigma^2 + T_A(\sigma^2 - \sigma\sigma^2)}$$

and $$\rho \quad \sigma_{C1}\sqrt{T_A}/\sqrt{T_B\sigma_2 + T_A(\sigma^2 - \sigma^2)}$$

The current post-merger value of debt B is given by

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[24]See Appendix A for proof.

$$D_B{}^C = V^C[1 - b + bN(k_1 + \sigma_C/T_A) - N_2(k_1 + \sigma_{C1}/T_A, k_7;\rho)] - M_A e^{-r_f T_A}[N_1(k_1) - N_2(k_1,k_8;\rho)] + M_B e^{-r_f T_B} N_2(k_1,k_9;\rho) \quad (7)$$

Equations (6) and (7) are identical to the corresponding valuation equations derived in Section III for stock and debt B when $\sigma^2_{C1}=\sigma^2_{C2}=\sigma^2$. The value of σ^2_{C2} in comparison to σ^2_{C1} would depend on a number of factors and we can have $\sigma^2_{C2} \geq \sigma^2_{C1}$, or $\sigma^2_{C2} < \sigma^2_{C1}$ (see Assumption A11, A12). If σ^2_{C2} is the same as σ^2_{C1} then the results in section 4 carry over unchanged. We, therefore, consider only two cases, the first in which the variance decreases. Since the value of debt B is inversely related to the variance of returns on the firm, the first case should yield current post-merger debt B values that are lower than those obtained in the preceding section, while the second case should yield higher debt B values. Thus, if $\sigma^2_{C2} > \sigma^2_{C1}$, the merger would lead to larger wealth transfers from the holders of bond B to the stockholders in comparison to the case where the two variances are equal. A fall in the variance would reduce the size of the wealth transfer.

The values of securities were computed for the parameter values used in the preceding section and changes in standard deviation of $\pm 1\%$, $\pm 2\%$, $\pm 5\%$, $\pm 10\%$, and $\pm 20\%$

(plotted in Figures 12-15). As stated earlier, an increase in the variance causes the size of the wealth transfer from debt B to stock to increase. For a variance increase of 21% (a standard deviation increase of 10%), the holders of debt B gain as a result of the merger for low debt A face values ($50) and times to maturity close to the date of retirement of debt A (less than 7½ years). A variance increase of 44% (20% increase in standard deviation) is almost sufficient to wipe out all cases where gain takes place as a result of the merger. A decrease in the variance does cause bond B value to be higher than the case where variance is unchanged, but there still exist a large number of cases where there is a transfer of wealth from bondholders to stockholders as a result of the merger. Even with a 36% decrease in variance (20% decrease in standard deviation) there are cases where bond B values fall after the merger. These cases include high debt A face value ($90) and debt B maturities close to debt A retirement (less than 8½ years).

The results obtained above support the conclusion that there exist a wide variety of cases under which holders of bond B would experience a fall in their wealth position as a result of the merger. They always gain only

FIGURE 12: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER
(10% INCREASE IN STANDARD DEVIATION)

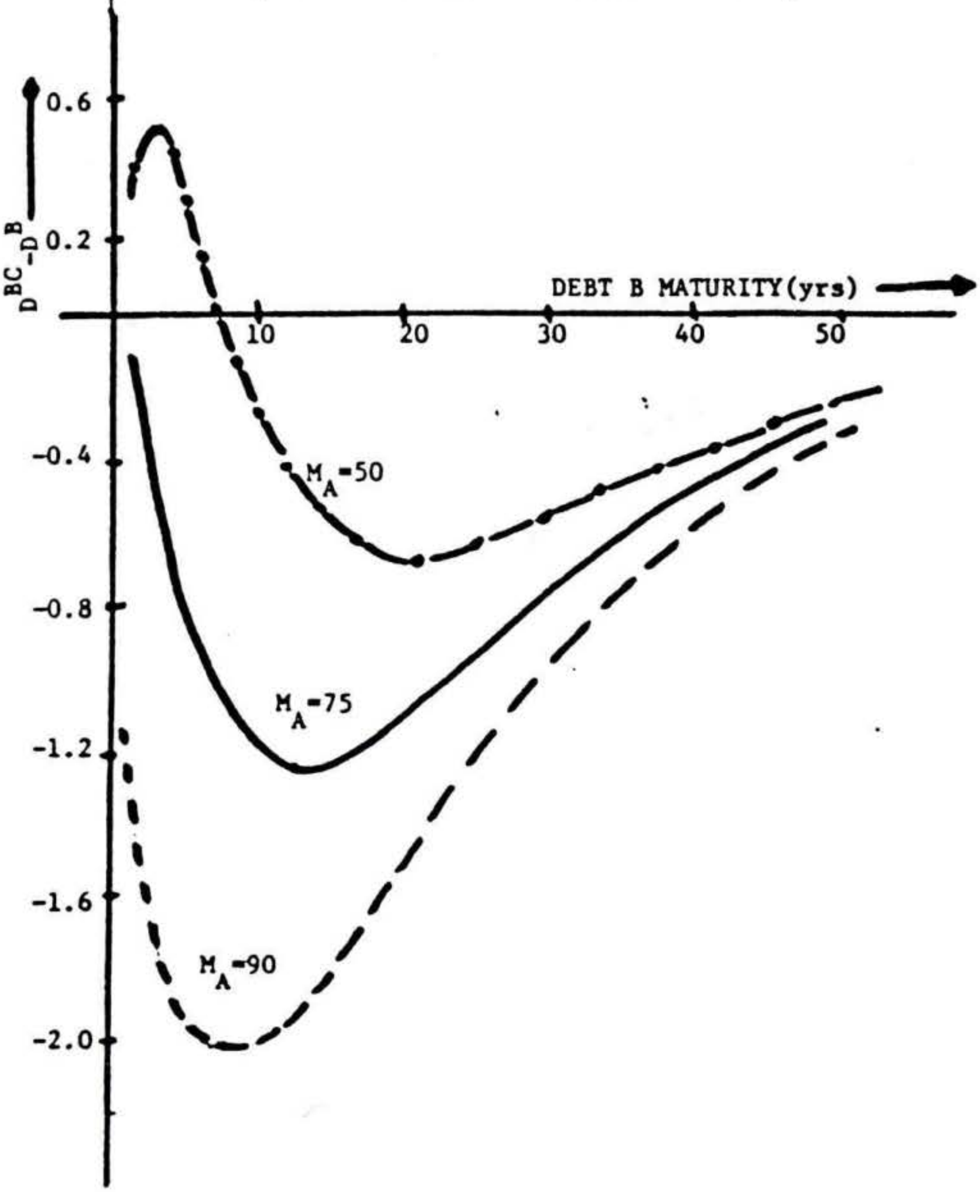


FIGURE 13: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER
(20% INCREASE IN STANDARD DEVIATION)

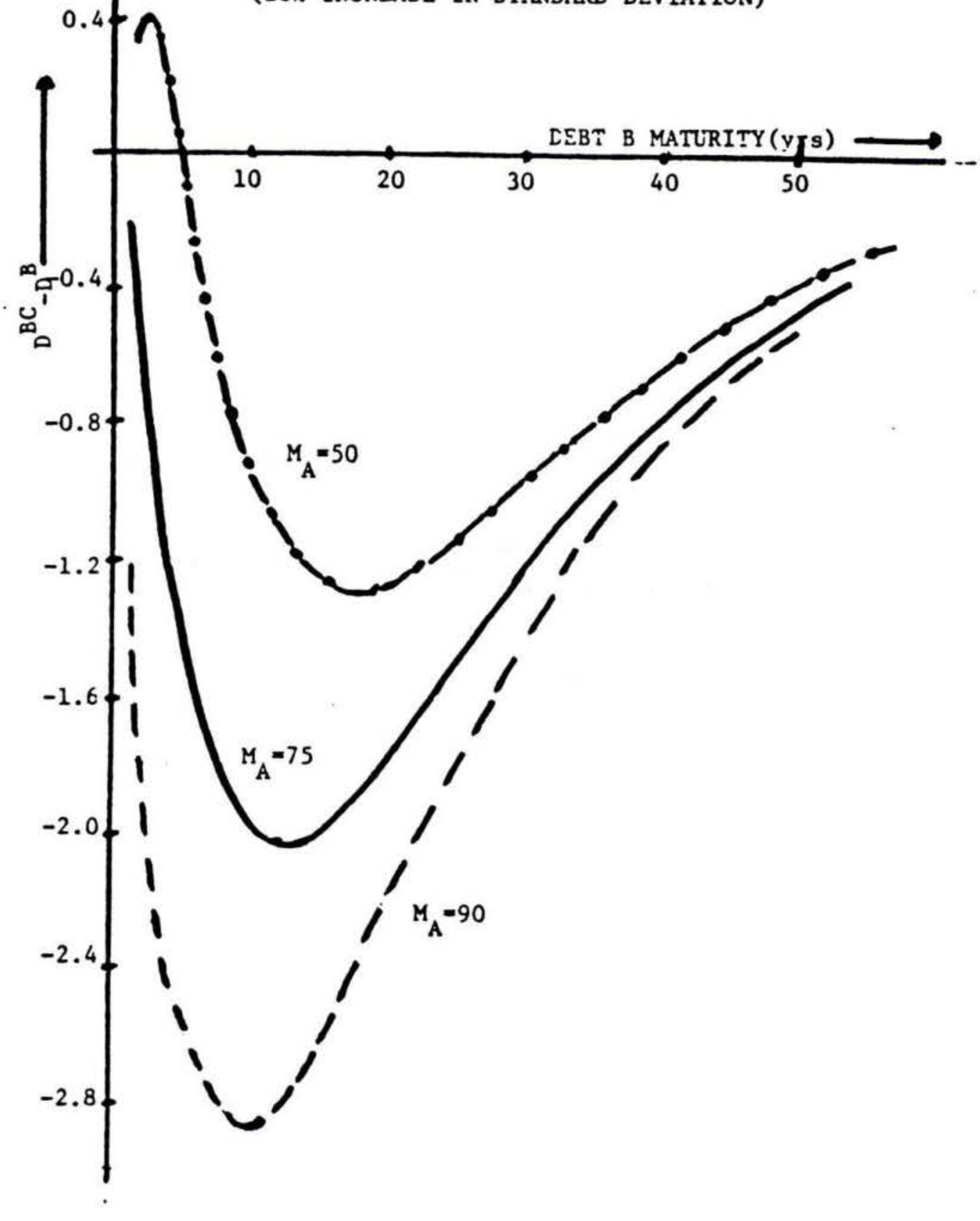


FIGURE 14: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER
(10% DECREASE IN STANDARD DEVIATION)

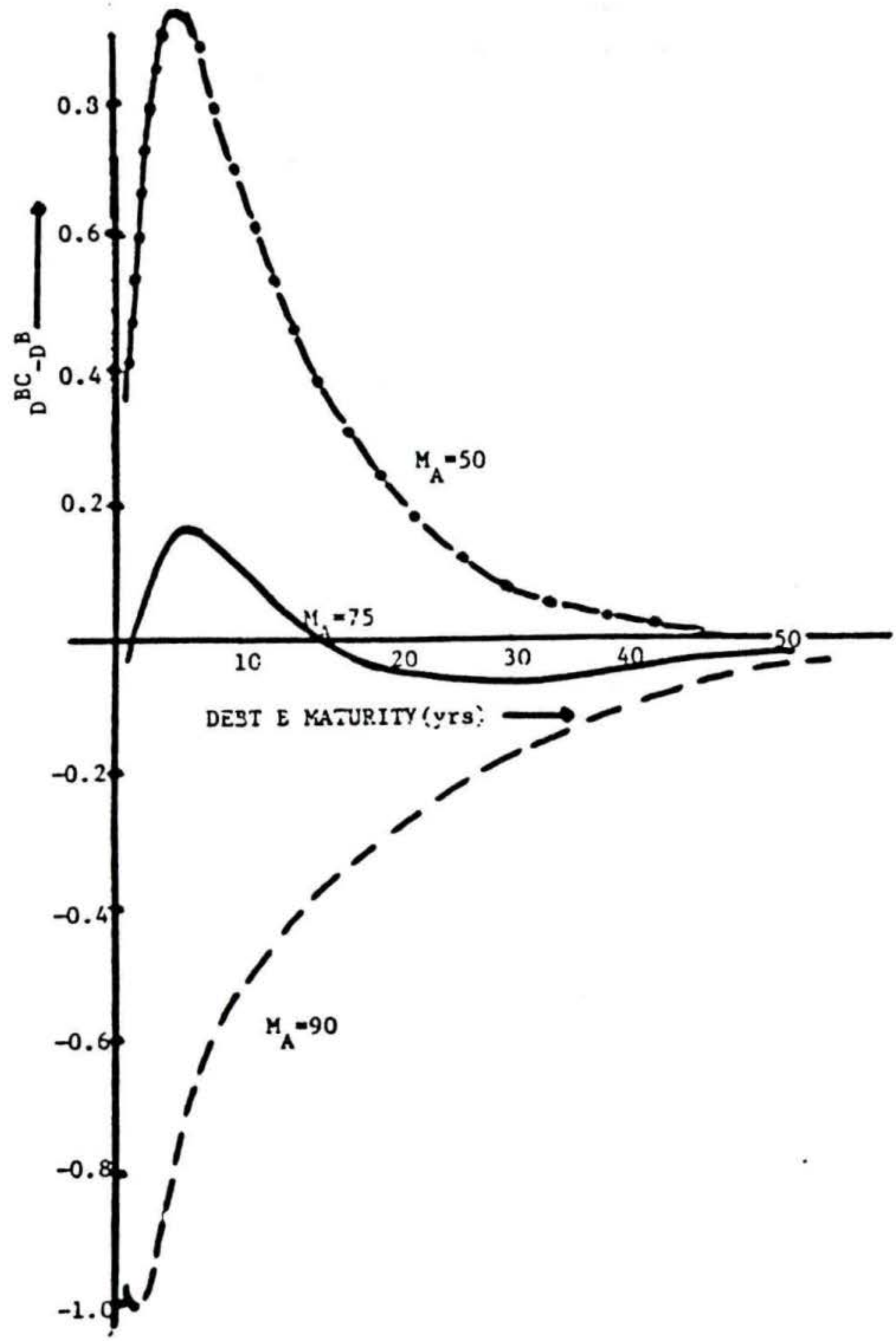
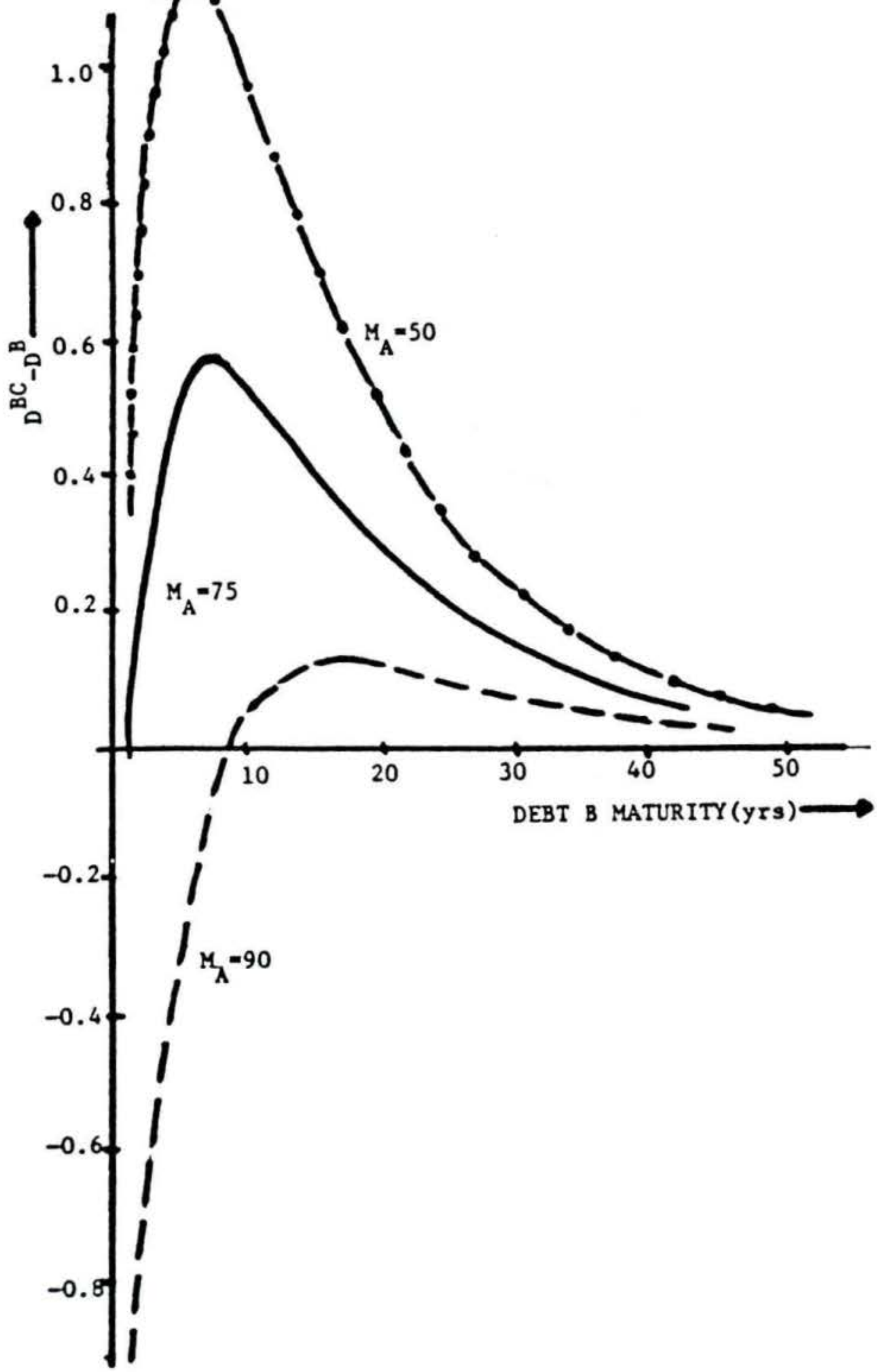


FIGURE 15: CHANGE IN VALUE OF DEBT B AS A RESULT OF THE MERGER
(20% DECREASE IN STANDARD DEVIATION)



in cases where the decrease in variance after retirement of debt A is extremely large.[25]

b) <u>The Effect of Coupon Payments on Debt</u>

Using techniques similar to those employed in the preceding sections, one can derive valuation equations under this additional assumption of coupon payments. These equations have a form similar to those derived in Geske (1979) and involve multivariate normal distribution functions. The difficulties involved in the evaluation of three or more integrals required us to take a slightly different approach in obtaining security values. The values of debt A, debt B, and stock are now obtained using the binomial approximation as developed in Cox, Ross and Rubinstein (1979). Although the binomial approximation reinterprets the problem of evaluating many integrals, it involves a large number of computations, with the latter increasing geometrically with increasing number of coupon payments on debt.[26] Thus, because of cost

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[25]For the simulations conducted here, the fall in the variance should be sufficient to make σ^2_{C2} equal to or slightly less than σ^2_B.

[26]The cost of evaluation of the post-merger values of debt A, debt B, and equity for one coupon payment on debt is approximately \$5.00. The evaluation cost for two coupon payments is approximately \$20.00

considerations, the simulations conducted here assume only one coupon payment on debt.

Simulations conducted here are with firm A and B values of \$100, debt A face value of \$75, debt B face value of \$50, \$75, and \$90, debt A maturity of one year, debt B maturity of two years, firm A variance of 0.09, firm B variance of 0.04, a correlation between firms A and B of 0.85, and a risk-free rate of 6%. The coupon rate on bond A takes on values of 0%, 8%, and 15%. Debt B coupon rate takes on values of 8% and 15%. The interest is assumed to be paid in 0.5 years for debt A and either 0.5 years or 1.5 years for debt B. The results of the simulations are presented in Table 9.

The introduction of coupon payments on debt does not in any way change the conclusions drawn in earlier sections.[27] The results of the simulations conducted under the conditions specified here are very similar to those obtained in earlier sections. In sixteen of the twenty-four cases for which security are presented in table 9, the holders of debt B experience a loss in

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[27]It is assumed that the firm goes bankrupt if the firm value is less than the coupon payment. If the firm value is greater than the interest payment, then the payment is made. This paper does not consider the effects of different suspension policies.

TABLE 9

Pre- and Post- Merger Security Values with Coupon Payments on Debt

$V^A = V^B = 100,\ M_A = 75,\ T_A = 1.0,\ T_B = 2.0, \sigma_A = 0.3, \sigma_B = 0.2,\ \rho = 0.85,\ r_f = 0.06$

T_A	r_B	t_A	t_B	M_B	D^A (\$)	$D^{AC} - D^A$ (\$)	D^B (\$)	$D^{BC} - D^B$ (\$)	S^A (\$)	S^B (\$)	$S^C - S^A - S^B$ (\$)
0	8%	-	0.5	50	69.11	1.52	48.21	1.62	30.89	51.79	-3.14
				75	69.11	1.52	71.10	-0.88	30.89	28.90	-0.64
				90	69.11	1.52	82.15	-1.07	30.89	17.85	-0.45
0	15%	-	0.5	50	69.11	1.52	51.60	3.20	30.89	48.40	-4.72
				75	69.11	1.52	75.57	-0.90	30.89	24.43	-0.62
				90	69.11	1.52	86.31	-1.07	30.89	13.69	-0.45
8%	8%	0.5	0.5	50	74.05	2.40	48.21	1.47	25.95	51.79	-4.17
				75	74.05	2.40	71.10	-1.51	25.95	28.90	-0.79
				90	74.05	2.40	82.15	-2.15	25.95	17.85	-0.25
8%	15%	0.5	0.5	50	74.05	2.40	51.60	2.99	25.95	48.40	-5.39
				79	74.05	2.40	75.57	-1.56	25.95	24.43	-0.84
				90	74.05	2.40	86.31	-2.33	25.95	13.69	-0.07
15%	8%	0.5	0.5	50	78.09	3.46	48.21	1.28	21.91	51.79	-4.74
				75	78.09	3.46	71.10	-2.18	21.91	28.90	-1.28
				90	78.09	3.45	82.15	-3.26	21.91	17.85	-0.21
15%	15%	0.5	0.5	50	78.09	3.45	51.60	2.75	21.91	48.40	-6.20
				75	78.09	3.45	75.57	-2.37	21.91	24.43	-1.08
				90	78.09	3.45	86.31	-3.61	21.91	13.69	+0.16
0%	8%		1.5	50	69.11	1.52	47.79	1.47	30.89	52.02	-2.99
				75	69.11	1.52	70.65	-0.71	30.89	29.35	-2.81
				90	69.11	1.52	81.53	-0.97	30.89	18.47	-0.55
0%	15%	-	1.5	50	69.11	1.52	51.16	2.88	30.89	48.84	-4.40
				75	69.11	1.52	74.67	-0.82	30.89	25.33	-0.70
				90	69.11	1.52	85.16	-0.93	30.89	14.84	-0.59

value as a result of the merger. The same parameter values with a no coupon payment assumption also yielded a loss in debt B value, although the magnitude of the loss was different with and without interest payments. The magnitude of the loss is an increasing function of debt B face value, and coupon rates on debt A and debt B. It is only in the eight cases where debt B face value is $50, that long-term debtholders experienced a gain in value.[28] The amount of gain is an increasing function of debt B coupon rate, and a decreasing function of both debt A coupon rate and time of interest payment. Debt A values always increase, with all or part of their gain resulting from a loss in value of debt B. Therefore, in a majority of the simulations there exists a wealth transfer from the holders of debt B to the holders of debt A. In one particular case, there also exists a transfer of wealth from debt B to stock. Thus, the four possible wealth transfers discussed in section 4 can occur even with coupon payments on debt.

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[28]This was also true with the no coupon payment assumption.

6. ON BOND COVENANTS AND METHODS OF COMPENSATION

Bond covenants are provisions that restrict the firm from engaging in certain activities after the bonds are sold. Restrictions against mergers commonly exist in debt contracts. To quote Smith and Warner (1979):

> Some indenture agreements contain a flat prohibition on mergers. Others permit the acquisition of firms provided that certain conditions are met...The merger can also be made contingent on there being no default on indenture provision after the transaction is completed.

The existence of these types of restrictions on merger activity are inconsistent with the result obtained by Galai and Masulis (1976) that the bondholders of the merged firm are always better off since the risk of bankruptcy always decreases. This inconsistency is a direct result of the fact that the conclusions drawn by Galai and Masulis (1976) are critically dependent on their assumptions. With a more general set of assumptions, it has bee show here that without the above restrictions the stockholders of firms can engage in merger activities that would cause the post-merger values of some bonds to be lower than their pre-merger values. The wide variety of situations under which bond values fall provides a rationale for the existence of bond covenants restricting merger activity.

Indenture agreements that contain a flat prohibition on mergers are in some sense "sub-optimal" from the point of view of bondholders, because there exist situations under which they would gain from the merger. Thus, we would expect some protective covenants that prevent mergers or require bondholders to be compensated under certain conditions. If the protective covenants do not ensure that the compensation is exactly equal to the loss in value, then wealth expropriation would still occur.

Given the existence of these protective covenants in debt contracts and the stockholders right of approval, one would not expect a merger, to take place unless the post-merger value of stock and debt is at least as great as the pre-merger value. In particular, in non-synergistic mergers, the approval would be obtained only if stock and debt values remain unchanged. In the remaining part of this section, we will discuss certain methods that can be used to compensate the stockholders and debtholders.

There are a number of ways in which bondholders could be compensated for losses and a few specific methods are now discussed. One method of compensation would be to increase interest payments on the debt that is adversely affected by the merger (for example, on debt B, in many of the cases discussed earlier). The effect of this action would be to increase the post-merger value of debt B, decrease the value of the stock, and decrease (or at best, leave unchanged) the value of debt A. A large enough increase in the interest payments could result in the post-merger value of debt A being less than its pre-merger value. Thus, such a policy would be restricted by some indentures agreement in the debt A contract. A second method of compensation would be to retire debt B at a certain pre-specified value (e.g., the pre-merger value). Holders of debt A would always prefer the retirement of debt B, as long as it is financed by the issue of subordinated debt and/or equity. A third possible alternative involved a simple swap. Debt B can be exchanged for new debt in the merged firm, such that the post-merger value of new debt is equal to the pre-merger value of debt B. The extent to which the firm engages in this activity would again be restricted by some indenture agreement in the bond A contract.

The above discussion has concentrated on compensating bondholders for losses. Given that the firm is required to engage in these types of bondholder compensation activities, the merger will result in a decrease in the in the market value of equity, i.e., the post-merger value of equity will be less than its pre-merger value. Thus, with perfect compensation agreements in debt contracts, stockholders would always oppose non-synergistic mergers, unless they are also compensated for their losses. One such method of compensation would be to give shareholders some debt in the merged firm. We will now consider one specific situation.

Assume that the post-merger values of both debt A and debt B are greater than their pre-merger values, i.e., $D^{AC} > D^{A}$ and $D^{BC} > D^{B}$. The values of both debts can be lowered by issuing more short and long-term debt. The post-merger value of debt A would now be given by

$$D^{AC} = \delta_1\{bV^C\{1 - N_1(k_1 + \sigma_C/T_A)] + (M_A + M_A^S)N_1(k_1)\} \qquad (8)$$

where M_A^S = face value of short-term debt issued to stock

$$k_1 = \frac{\ln(V^C/M_A + M^S) + (r_f - \tfrac{1}{2}\sigma^2)T_A}{\sigma_C/T_A}$$

$$\delta_1 = M_A/(M_A + M_A^S)$$

$$b = (M_A + M_A^S)/\ M_A + M_A^S + M_B e^{-r_f(T_B - T_A)} + M_B^S e^{-r_f(T_B - T_A)}$$

and M_B^S = face value of long-term debt issued to stock.

The post-merger value of debt B would be given by

$$D^{BC} = \delta_2\{V^C[1 - b + bN_1(k_1 + \sigma_C/T_A) - N_2(k_1 + \sigma_C/T_A, k_7; \rho)] - (M_A + M_A^S)e^{-r_f T_A}[N_1(k_1) - N_2(k_1, k_8; \rho)]$$

$$- (M_B + M_B^S)e^{-r_f T_B} N_2(k_1, k_9; \rho)\} \qquad (9)$$

where $\delta_2 = M_B/(M_B + M_B^S)$

$$k_7 = \frac{\ln(V_1^C/M_B + M_B) + (r_f + \tfrac{1}{2}\sigma^2)T_B}{\sigma_C/T_B}$$

$$k_8 = \frac{\ln(V^C/M_B + M^S) + (r_f + \tfrac{1}{2}\sigma^2)T_B - \sigma^2 T_A}{\sigma_C/T_B}$$

$$k_9 = k_7 - \sigma_C/T_B$$

$$\rho = /T_A/T_B$$

and $V_1^C = V^C - (M_A + M_A^S)e^{-r_f T_A}$.

The stockholders would be exactly compensated for their losses if M_A^S and M_B^S are chosen such that $D^{AC} = D^A$ and $D^{BC} = D^B$. Any situation can be converted to the one used in the above case. For example, if we have $D^{AC} > D^A$ and $D^{BC} < D^B$, then first compensate debt B, such that their post-merger value becomes greater than their pre-merger value, so that the condition assumed in the previous example holds.

Thus, with perfect compensation it is possible to nullify all wealth expropriation effects of the merger. The methods discussed above mainly involve changing the capital structure of the merged firm in a way such that the post-merger values of debt A, debt B and holdings of stockholders (debt and equity) are equal to their pre-merger values.

APPENDIX A

DERIVATION OF VALUATION EQUATIONS

In this appendix we show that the values of stock and long-term debt are given by equations (6) and (7) in Section V.

As shown in Section II, the value of the short-term debt is given by

$$D^{AC} = bV^C[1 - N_1(k_1 + \sigma_{C1}\sqrt{T_A})] + M_A e^{-r_f T_A} N_1(k_1) \tag{A.1}$$

$$= \frac{\ln(V^C/M_A) + (r_f - \tfrac{1}{2}\sigma^2)T_A}{\sigma_C\sqrt{T_A}}$$

After retirement of debt A, the stock becomes a call option on the value of the firm with an exercise price equal to the face value of debt B and a time to maturity of $(T_B - T_A)$ years. Thus, the value of the stock at time T_A is given by

$$S^C = V_T \; N_1(k_7 + \sigma_{C2}\sqrt{T_B - T_A}) - M_B e^{-r_f(T_B - T_A)} N_1(k_7) \tag{A.2}$$

where $$k_7 = \frac{\ln(V_T / M_B) + (r_f - \tfrac{1}{2}\sigma^2)(T_B - T_A)}{\sigma_{C2}/T_B - T_A}$$

and $V_T = V_T - M_A$.

Assuming that a riskless hedge can be created involving the stock, the current value of the stock is given by

$$S^C = e^{-r_f T_A} \; E\{S^C \mid V^C\}$$

or
$$S^C = e^{-r_{f}T_A}[\int_\infty V_T \; N_1(k_7 + \sigma_{C2}\sqrt{T_B - T_A} - M_B e^{-r_f(T_B - T_A)} N_1(k_7)] f(V_T \mid V^C) dV_T \quad (A.3)$$

where $f(V_T \mid V^C)$ is the density function of V_T given a current firm value of V_C.

Let
$$r = \ln(V_T / V^C)$$

$$r^* = \frac{-r + (r_f + \tfrac{1}{2}\sigma^2) T_A}{\sigma_{C1}\sqrt{T_A}}$$

$$r^{**} = \frac{-r + (r_f - \tfrac{1}{2}\sigma^2 (T_A}{\sigma_{C1}\sqrt{T_A}}$$

using the above defined variable changes and with some algebraic manipulations, equation (A.3), becomes[29]

$$S^C = V^C \int_\infty {}_1{}^{+}\sigma_{C1}\sqrt{T}_A \; N_1(k_7) f(r^*) dr^* -$$

$$e^{-r_f T_A} M_A \int_\infty 1 \; N_1(k_8) f(r^{**}) dr^{**}$$

$$-e^{-r_f T_B} M_B \int_\infty 1 N_1(k_9) f(r^{**}) dr^{**} \quad (A.4)$$

where

$$k_7'' = \frac{-\sigma_{C1}\sqrt{T_A}}{\sigma_{C2}\sqrt{T_B - T_A}} r^* + \frac{\ln(V^C/M_B) + (r_f + \tfrac{1}{2}\sigma^2) T_B + \tfrac{1}{2}(\sigma^2 - \sigma^2) T_A}{\sigma_{C2}\sqrt{T_B - T_A}}$$

$$k_8' = \frac{-\sigma_{C1}\sqrt{T_A}}{\sigma_{C2}\sqrt{T_B - T_A}} r^{**} + \frac{\ln(V_C/M_B) + (r_f + \tfrac{1}{2}\sigma^2) T_B - \tfrac{1}{2}(\sigma^2 + \sigma^2) T_A}{\sigma_{C2}\sqrt{T_B - T_A}}$$

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[29] The final forms for k_7, k_8, and k_9 are obtained with two approximation $\ln(V_T / V^C) = \ln(V_T - M_A \; e^{-r_f T_A})$. This approximation causes the stock to be under-valued by eq. (A.4).

$$k_9^* = \frac{-\sigma_{C1}\sqrt{T_A}}{\sigma_{C2}\sqrt{T_B - T_A}} r^{**} + \frac{\ln(V^C/M_B) + (r_f - \frac{1}{2}\sigma^2)T_B - \frac{1}{2}(\sigma^2 - \sigma^2)T_A}{\sigma_{C2}\sqrt{T_B - T_A}}$$

$f(r^*), f(r^{**})$ are standard normal density functions

and $\qquad V^C = V_C - M_A e^{-r_f T_A}$

Let $N_2(h_1, h_2, \rho)$ be the bivariate normal cumulative function with h_1 and h_2 as the upper limits of the integrals and as the correlation coefficient.

Then[30]

$$N_2(h_1, h_2, \rho) = \int_{\infty}^{1} N_1 \left[\frac{h_2 - \rho_x}{\sqrt{1-\rho^2}}\right] f(x)dx \qquad (A.5)$$

where $f(x)$ is the standard normal density function.

Using equation (A.5) and with some algebraic manipulations equation (A.4) becomes[31]

$$S^C = V^C N_2(k_1 + \sigma_{C1}\sqrt{T_A}, k_7; \rho) - M_A e^{-r_f T_A} N_2(k_1, k_8; \rho)$$
$$-M_B e^{-r_f T_B} N_2(k_1, k_9; \rho) \qquad (A.6)$$

where

$$k_1 = \frac{\ln(V^C/M_A) + (r_f - \frac{1}{2}\sigma_{C1}^2)T_A}{\sigma_{C1}\sqrt{T_A}}$$

$$k_7 = \frac{\ln(V_1^C/M_B) + (r_f + \frac{1}{2}\sigma_{C2}^2)T_B + \frac{1}{2}(\sigma_{C1}^2 - \sigma_{C2}^2)T_A}{\sqrt{T_B \sigma_{C2}^2 + T_A(\sigma_{C1}^2 - \sigma_{C2}^2)}}$$

$$k_8 = \frac{\ln(V_1^C/M_B) + (r_f + \frac{1}{2}\sigma_{C2}^2)T_B - \frac{1}{2}(\sigma_{C1}^2 + \sigma_{C1}^2)T_A}{}$$

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[30]See Abromowitz and Stegun (1964).

[31]The correlation coefficient has the property $0 \leq \rho \leq 1$ as long as $\sigma^2 \geq 0$, $T_A \geq 0$. and $T_A \leq T_B$.

$$\sqrt{T_B\sigma_{C2}^2 + T_A(\sigma_{C1}^2-\sigma_{C2}^2)}$$

$$k_9 = \frac{\ln(V_I^C/M_B) + (r_f-\tfrac{1}{2}\sigma_{C2}^2)T_B - \tfrac{1}{2}(\sigma_{C1}^2-\sigma_{C2}^2)T_A}{\sqrt{T_B\sigma_{C2}^2 + T_A(\sigma_{C1}^2-\sigma_{C2}^2)}}$$

and

$$\rho = \sigma_{C1}\ T_A/\sqrt{T_B\sigma_{C2}^2+T_A(\sigma_{C1}^2-\sigma_{C2}^2}$$

Equations (A.1) and (A.6) and the fact the sum of security values should equal firm value yields the following expression for long-term debt value:

$$\begin{aligned} D^{BC} = {} & V^C[1-b+bN_1(k_1+\sigma_{C1}\sqrt{T_A}) - N_2(k_1+\sigma_{C1}\sqrt{T_A},k_7;\rho)] \\ & - M_A e^{-r_f T_A}[N_1(k_1) - N_2(k_1,k_8;\rho)] \\ & + M_B e^{-r_f T_B}\ N_2(k_1,k_9;\rho). \end{aligned}$$

APPENDIX B

THE LEVERAGE EFFECT

Assume that $T_A = T_B + T$ and $\sigma_A = \sigma_B = \sigma$ and $\rho_{AB} = 1$ i.e., the maturity and variance effects are zero.

Then, the pre-merger value of Bond i, i = A,B, is given by

$$D^i = V^i[1-N_1(k_i+\sigma/T)] + M_i e^{-r_f T} N_1(k_i) \qquad (B.1)$$

where $k_i = \dfrac{\ln(V^i/M_i) + (r_f - \frac{1}{2}\sigma^2)T}{\sigma/T}$

The post-merger value of Bond i, i = A,B, is given by

$$D^{ic} = M_i \alpha_c[1-N_1(k_c+\sigma/T)] + M_i e^{-r_f T} N_1(k_c) \qquad (B.2)$$

where $\alpha_c = V^c/M_c$ = reciprocal of leverage ratio for firm C

$M_c = M_A + M_B$

$V^c = V^A + V^B$

$k_c = \dfrac{\ln\alpha_c + (r_f - 1/2\sigma^2)T}{\sigma/T}$

Therefore, the change in value of Bond i, i = A,B, is

$$\Delta D^i = D^{ic} - D^i = M_i\alpha_c[1-N_1(k_c+\sigma/T)] + M_i e^{-r_f T} N_1(k_c)$$
$$- M_i\alpha_i[1-N_1(k_i+\sigma/T)] - M_i e^{-r_f T} N_1(k_i) \qquad (B.3)$$

where $\alpha_i = V^i/M_i$

Equation (B.3) is equivalent to

$$\frac{\Delta D^i}{M_i} = \alpha_c[1-N_1(k_c+\sigma/T)] + e^{-r_f T} N_1(k_c)$$

$$- \alpha_i[1-N_1(k_i+\sigma/T)] - e^{-r_f T} N_1(k_i) \qquad (B.4)$$

Therefore, the change in value of Bond i as a fraction of its face value is equal to the change in value of a bond with a face value of \$1, when the firm value changes from α_i to α_c. Since the value of a bond is a non-decreasing function of firm value, and given equation (B.4), the change in value of Bond i is a decreasing function of the leverage ratio. Therefore,

$$\Delta D^i \text{ is } \begin{cases} < \; 0 & \text{if } \alpha_c < \alpha_i \\ = \; 0 & \alpha_c = \alpha_i \\ > \; 0 & \alpha_c > \alpha_i \end{cases}$$

The pre-merger value of stock i, i = A,B, is given by

$$S^i = V^i N_1(k_i+\sigma/T) - M_i e^{-r_f T} N_1(k_i) \qquad (B.5)$$

Equation (B.5) can also be written as

$$S^i/M_i = \alpha_i N_1(k_i+\sigma/T) - e^{-r_f T} N_1(k_i) \qquad (B.6)$$

Similarly, the equation for the post-merger value of the combined stock can be written as

$$S^c/M_c = \alpha_c N_1(k_c+\sigma/T) - e^{-r_f T} N_1(k_c) \qquad (B.7)$$

Therefore, given the face that stock prices are increasing functions of firm value, we have

$$S^A/M_A < S^c/M_c < S^B/M_B \text{ if } \alpha_A < \alpha_c < \alpha_B$$

or $$S^B/M_B < S^C/M_C < S^A/M_A \text{ if } \alpha_B < \alpha_c < \alpha_B$$

In addition, since stock prices are convex in firm value, we must have or $\quad W\ S^A/M_A + (1-W)S^B/M_B > S^c/M_C \qquad$ (B.8)

where W satisfies $\quad W\ V^A/M_A + (1-W)V^B/M_B = V^c/M_c \qquad$ (B.9)

Solving equation (B.9) for W, yields $W = M_A/M_C$.

Substitution of this value of W in (B.8) gives,

$$S^A + S_B > S^c$$

Therefore, the leverage effect is always negative for the combined common stock.

APPENDIX C

THE MATURITY EFFECT

Assume $\sigma_A = \sigma_B$, $\rho_{AB} = 1$, and $M_A/V^A = M_B/V^B$ i.e., the variance and leverage effects are zero. Further assume that the expiration date of Bond A is one "period" away, while that for Bond B is two "periods" away. We will now use the binomial process to show that the merger results in a non-negative change in Bond A value, an ambiguous change in Bond B value, and a non-negative change in total debt value and thus a non-positive change in total stock value.

Since the binomial process is being used to describe firm value movements, therefore at the end on any period, the value of the firm can either move up by a proportion u, or move down by proportion d (=1/u). The possible end of period values for firm A, firm B, and firm C are shown in Figure C.1.

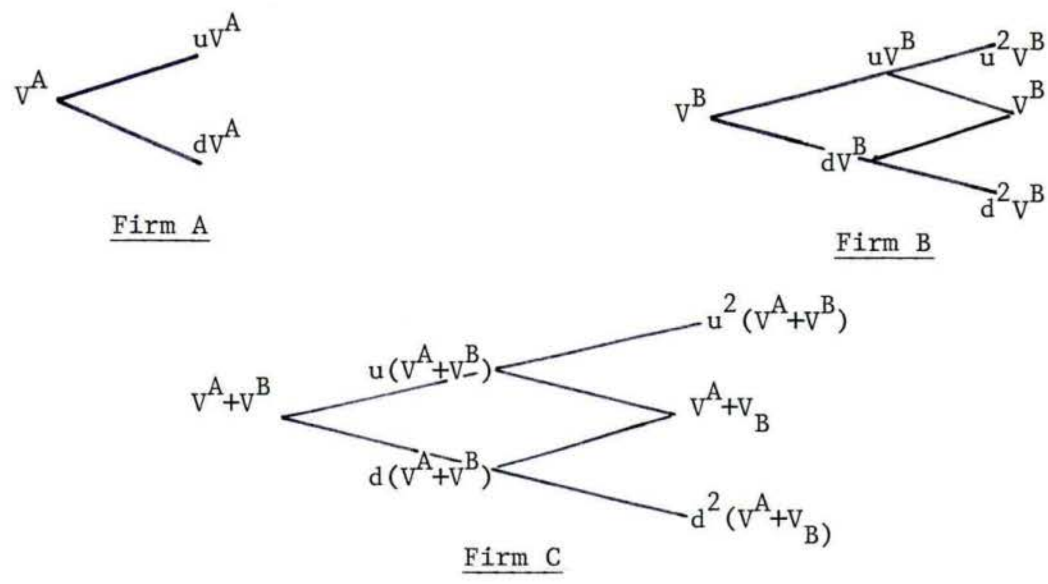

Figure C.1

Case 1: Assume $M_A < dV^A$ and $M_B < d^2V^B$

In this case the pre-merger values of Bond A and Bond B are given by

$$D^A = M_A/r \tag{C.1}$$

and $$D^B = M_B/r^2 \tag{C.2}$$

where r = one plus the risk-free rate/period.

The post merger payoffs, at the end of period 1, to Bond A is always M_A, since $d(V^A+V^B) > M_A$. Therefore, the post-merger value of Bond A is given by

$$D^{AC} = M_A/r \tag{C.3}$$

This implies that the change in value of Bond A as a result of the merger is zero. The post-merger payoffs, at the end of period 2, to Bond B is always M_B, since $d^2 (V_A+V_B) - dM_A > M_B$. Therefore, the post-merger value of Bond B is given by

$$D^{BC} = M_B/r^2 \tag{C.4}$$

This implies that the change in value of Bond B as a result of the merger is zero. Therefore, the total change in value of debt is zero.

Case 2: Assume $M_A < dV^A$ and $d^2V^B < M_B < dV_B$

In this case, the pre and post-merger values of Bond A are given by equations (C.1) and (C.3). Using the techniques developed in Cox, Ross, and Rubinstein (1979), it can be shown that the pre-merger value of Bond B is given by

$$D^{BC} = \frac{p(2-p)M_B}{r^2} + \frac{(1-p)^2 d^2 V^B}{r^2} \tag{C.5}$$

where $p = (r-d)/(u-d)$.

Similarily, the post-merger value of Bond B can be shown to be

$$D^{BC} = \frac{p(2-p)M_B}{r^2} + \frac{(1-p)^2 \mathrm{Min}[M_B, d^2(V^A+V^B)-dM_A]}{r^2} \tag{C.6}$$

Therefore, the change in value of debt B is

$$\Delta D^B = D^{BC} - D^B - D^B = \frac{(1-p)^2}{r^2} \{\mathrm{Min}[M_B, d^2(V^A+V^B)$$
$$-dM_A] - d^2V^B$$

$$\text{or} \quad \Delta D^B = \frac{(1-p)^2}{r^2} \quad \{\mathrm{Min}\ [M_B - d^2V^B, d^2V^A - dM_A]\} \tag{C.7}$$

ΔD^B is always greater than zero, since $M_B > d^2V^B$ and $d^2V^A > dM_A$. Therefore, Bond B gains as a result of the merger. Since there is no change in Bond A value, the total value of debt increases in this case.

Case 3: $dV^A < M_A < uV^A$ and $dV_B < M_B < V^B$

In this case, the pre-merger value of Bond a is given by

$$D^A = \frac{pM_A}{r} + \frac{(1-p)dV^A}{r} \tag{C.8}$$

The pre-merger value of Bond B is given by equation (C.5). To derive equations for post-merger debt value we have to consider two different situations:

a) $d(V^A+V^B) < M_A < u(V^A+V^B)$

In this situation it can be shown that the post-merger value of Bond a is given by the equation

$$D^{AC} = \frac{pM_A}{r} \quad \frac{(1-p)db(V^A+V^B)}{r} \tag{C.9}$$

where $b = M_A/(M_A+M_Br^{-1})$ = proportion of liquidated firm obtained by Bond A.

Therefore, the change in value of Bond A is given by

$$\Delta D^A = D^{AC} - D^A = \frac{(1-p)d}{r}[b(V^A+V^B)-V^A]$$

$$\text{or} \quad \Delta D^A = \frac{(1-p)dV^A}{r}\left[\frac{M_A + M_B}{M_A + M_Br^{-1}}\right] \tag{C.10}$$

Equation (C.10) implies that Bond A increases in value as a result of the merger.

The post-merger value of Bond B is given by

$$D^{BC} = \frac{p(2-p)M_B}{r^2} + \frac{(1-p)(1-b)d(V_A+V^B}{r} \tag{C.11}$$

Therefore, the change in value of Bond B is

$$\Delta D^B = D^{BC} - D^B = \frac{(1-p)d}{r}\left[(1-b)(V^A+V^B) - \frac{(1-p)}{r}dV^B\right]$$

$$\text{Or} \quad \Delta D^B = \frac{(1-p)dM_B}{r^2}\left[\frac{M_A + M_B}{M_A + M_B\, r^{-1}} - (1-p)d\right] > 0 \tag{C.12}$$

$$r^2 \qquad M_A + M_B r^{-1} \tag{C.12}$$

Equation (C.12) implies that Bond B increases in value in this case. Since both bonds increase in value in this case, the total value of debt must increase.

b) $M_A < d(V^A+V^B)$

In this situation, Bond A becomes riskless after the merger and, therefore, its value is given by equation (C.3). The change in value of Bond A is given by

$$\Delta D^A = \frac{(1-p)}{r}\,[M_A - dV^A] > 0 \tag{C.13}$$

The post-merger value of Bond B is given by the equation

$$D^{BC} = \frac{pM_B}{r^2} + \frac{p(1-p)}{r^2}\,\mathrm{Min}[M_B, V^A+V^B-uM_A] + \frac{(1-p)^2}{r^2}\,(d^2(V^A+V^B)-dM_A) \tag{C.14}$$

Therefore, the change in value of Bond B is

$$\Delta D^B = \frac{p(1-p)}{r^2}\,\{\mathrm{Min}[0, V^A+V^B-uM_A-M_B]\} + \frac{(1-p)^2}{r^2}\,d(dV^A-M_A) < 0 \tag{C.15}$$

Therefore, Bond B decreases in value in this case. The total change in debt value is $\Delta D = \Delta D^A + \Delta D^B$. In the case where the first term of equation (C.15) has value zero, the equation for ΔD is

$$\Delta D = \frac{1-p}{r}\,(M_A - dV^A)\left[1 - \frac{(1-p)d}{r}\right] > 0 \tag{C.16}$$

or the merger results in a decrease in the value of the combined common stock. When the first term of equation (C.15) is negative, the equation for ΔD is

$$\Delta D = \frac{1-p}{r}\,(M_A - dV^A)\left[1 - \frac{(1-p)d}{r}\right] + \frac{p(1-p)}{r^2}\; V^A + V^B$$

$$-uM_A - M_B$$

or $$\Delta D > \frac{1-p}{r}(M_A - dV^A)\ \{1 - \frac{(1-p)d}{r} - \frac{pu}{r}\} + 0 \qquad (C.17)$$

Since $V^B > M_B$ and $p = (r-d)/(u-d)$

or the merger results in a decrease in the value of the combined common stock.

Case 4: $dV^A < M_A < uV^A$ and $V^B < M_B < u^2V^B$

In this case, the pre-merger value of Bond A is given by equation (C.8). The pre-merger value of Bond B is given

by $$D^B = \frac{p^2}{r} M_B + \frac{2p(1-p)}{r^2} V_B + \frac{(1-p)^2 d^2 V^B}{r^2} \qquad (C.18)$$

As in Case 3, we have to consider two different situations to derive the post-merger values of Bond A and Bond B.

a) $d(V^A + V_B) < M_A < u(V^A + V_B)$.

In this situation, the post-merger value of Bond A is given by equation (C.9), and the change in Bond A value is given by equation (C.10). The post-merger value of Bond B is given by

$$D^{BC} = \frac{p^2}{r^2} M_B + \frac{p(1-p)}{r^2}\{\text{Min } M_B, V^A + V^B - dM^A\} + \qquad (C.19)$$

Therefore, the change in value of Bond B is

$$\Delta D^B = \quad \text{Min}\{M_B - V^B, V^A - dM_A\} +$$

$$-(1-p)d \qquad > 0 \qquad (C.20)$$

Equation (C.20) implies that Bond B increases in value in this case. Since both bonds increase in value, this situation will always result in a decrease in the combined common stock value.

b) $\quad M_A < d(V^A+V^B)$

In this situation, Bond A becomes riskless after the merger and, therefore, its value is given by equation (C.3). The change in value of Bond A is given by equation (C.13). The post-merger value of Bond B can be shown to be

$$D^{BC} = \frac{p^2}{r^2} M_B + \frac{p(1-p)}{r^2} \text{Min}\{M_B, V^A+V^B-dM_A\}$$
$$+ \frac{p(1-p)}{r^2}(V^A+V_B - uM_A) + \frac{(1-p)}{r^2}(d^2(V^A+V^B)-dM_A) \tag{C.21}$$

Therefore, the change in value of Bond B is given by

$$\Delta D^B + \frac{p(1-p)}{r^2}\{\text{Min } M_B-V^B, V^A-dM_A\} + \frac{(1-p)}{r^2}(dV_A - M_A) \quad 0 \tag{C.22}$$

Equation (C.22) implies that the change in Bond B value is ambiguous in this case. The total cahnge in value of debt is

$$\Delta D = D^A + D^B = \frac{p(1-p)}{r^2}\{\text{Min } M_B-V_B, V^A-dM_A\} \quad 0 \tag{C.23}$$

or the merger results in a decrease in the value of the combined common stock.

There are two more cases that can possibly occur. These cases are not considered in this appendix, because the firms would effectively have no equity when M_A is greater than uV^A and M_B is greater than u^2V^B. The four cases considered above are summarized in Table C.1.

TABLE C.1

THE MATURITY EFFECT

	BOND A	BOND B	COMMON STOCK
CASE 1: $M_A < dV^A$ $M_B < d^2V^B$	0	0	0
CASE 2: $M_A < dV^A$ $d^2V^B < M_B < dV^B$	0	>0	<0
CASE 3a: $dV^A < M_A < uV^A$ $dV^B < M_B < V^B$ $d(V^A + V^B) < M_A < u(V^A + V^B)$	>0	>0	<0
CASE 3b: $dV^A < M_A < uV^A$ $dV^B < M_B < V^B$ $M_A < d(V^A + V^B)$	>0	<0	<0
CASE 4a: $dV^A < M_A < uV^A$ $V^B < M_B < u^2V^B$ $d(V^A + V^B) < M_A < u(V^A + V^B)$	>0	>0	<0
CASE 4b: $dV^A < M_A < uV^A$ $V^B < M_B < u^2V^B$ $M_A < d(V^A + V^B)$	>0	$\gtrless 0$	<0

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Essay 2

H

VALUING CORPORATE SECURITIES: SOME EFFECTS OF STOCK REPURCHASE OFFERS

1. INTRODUCTION AND SUMMARY

A tender offer to repurchase common stock is a bid by a firm to acquire a portion of its outstanding equity at a price specified in the offer. The number of companies repurchasing their own stock has grown dramatically in the last years. In 1983, companies listed on the New York Stock Exchange have repurchased nearly $3 billion of their own stock. The overall effect has been a swing from a net corporate issuance of $12.9 billion in 1982 to a net reduction of $7 billion in 1983.

Recent empirical studies on repurchases report abnormal stock price increases after a repurchase announcement, but provide different explanations for the observed returns.[1] With

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[1]See Masulis (1980), Dann (1981), and Vermaelen (1981).

regard to the rationale for stock repurchases, the most frequently mentioned reasons are:[2]

(i) The Wealth Expropriation Hypothesis: It has been argued that since the repurchase transfers the ownership of some of the firm's assets to the stockholders, this transfer would result in an expropriation of wealth from the bondholders to the stockholders.

(ii) The Signalling Hypothesis: It has been argued that the announcement of the repurchase constitutes a signal from management about the firm's future prospects.

(iii) The Leverage Hypothesis: It has been argued that when the firm issues new debt to finance the repurchase, it engages in this activity to attain its 'optimal' capital structure. This change in capital structure would result in an increase in stock value. This hypothesis also predicts that debt values will remain unchanged.[3]

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[2]These hypotheses are not mutually exclusive. Masulis (1980) uses a combination of (i), (iii), and (iv) to explain his results, while Dann (1981), and Vermaelen (1981) state that their results can only be indicative of the predominant hypothesis.

[3]This statement follows from the fact that debt values are determined by the before-tax value of the firm, and leverage changes only alter the after-tax value. Therefore, tax gains from leverage will benefit stockholders only.

(iv) The Personal Taxation Hypothesis: Firms repurchase stock rather than paying stock dividends because this action enables the stockholders to benefit from the preferential tax treatment of repurchases relative to dividends. This hypothesis predicts that stock value will increase, and debt value will remain unchanged.

This paper demonstrates that the Wealth Expropriation, Leverage, and Personal Taxation hypotheses are not sufficient to explain the results obtained in recent studies. Thus, we focus on the Signalling hypothesis, and derive conditions under which different types of signals are consistent with the recent empirical results.

In Section 2, the assumptions used in this paper are presented. In Section 3, valuation equations for debt and equity are derived using the techniques developed by Black and Scholes (1973) for valuing securities as simple and compound options.[4] The results of simulations using these valuation equations are presented in Section 4. Contrary to previous work, it is shown that a repurchase can result in a wealth transfer from stockholders

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[4]These equations are derived assuming that the repurchase is not financed with a new debt issue. The Leverage and Personal Taxation Hypotheses are not explicitly modeled in these equations since we assume a world without taxes.

to bondholders.[5] Therefore, the expropriation effect would result in either a decrease in debt value or a decrease in stock value.[6] In either situation, the adversely affected securityholders would attempt to prevent the firm from engaging in the repurchase. Therefore, given a perfect and competitive market for protective covenants in debt contracts, and the voting rights of shareholders, the expropriation effect cannot provide a rationale for the existence of repurchases.

If the repurchase were a signal from management about the firm's future prospects, the announcement could result in a change in firm value, and (or) firm riskiness. For both securityholders to approve the repurchase, it is shown that the announcement must have a positive impact on firm value.

In addition, for 'reasonable' increases in firm value, the announcement must be accompanied by either one of the following changes in the variance of returns of the firm.[7]

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[5]See Masulis (1980), Dann (1981), and Vermaelen (1981).

[6]This follows from the fact that without information and debt financing, the value of the firm does not change at the announcement date of the repurchase offer. It must, however, be noted that the repurchase can be neutral in the sense that the wealth expropriation effect is zero.

[7]It is theoretically possible for all securityholders to approve the repurchase if the variance remains the same and the information effect on firm value is large.

(a) Variance decreases at the announcement date and then increases (or remains the same) at the expiration date of a successful offer.

(b) Variance increases at the announcement date and then decreases at the date of a successful offer.

Section 5 demonstrates that levered repurchases do not significantly alter the conclusion that the signal from management must affect both firm value and firm variance at the announcement date. In Section 6, the theoretical signalling effects of repurchases are compared with some of the empirical results obtained by Masulis (1980), Dann (1981), and Vermaelen (1981), and are found to be consistent with each other if the following hypotheses hold:[8]

(a) The repurchase announcement conveys a value increasing signal.

(b) The repurchase announcement conveys a risk decreasing signal.

(c) At the expiration date of the repurchase, the risk of the firm increases.

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[8]It must be pointed out that Dann (1981), and Vermaelen (1981) conclude that their results are consistent with the hypothesis that a repurchase announcement discloses favorable information about the firm's future prospects, but they fail to specify what kinds of signals are consistent.

(d) The information effect of the announcement on stock price is an increasing function of the repurchase premium, and the fraction of outstanding shares repurchased.

Finally, Section 7 develops testable hypotheses, and discusses methods for estimation of the parameters in the valuation equations.

2. THE ASSUMPTIONS

As stated earlier, the corporate securities considered her are valued using the option pricing framework. The valuation equations derived in later sections follow from the fact that stocks can be viewed as call options with the underlying securities being the assets of the firm. These equations are derived under the following set of assumptions:

A1. The firm has common stock and one debt issue outstanding.

A2. The outstanding debt issue is a pure discount bond with face value of M, and a maturity of T. years.

A3. There are no dividend payments on the stock.

A4. The distribution of changes in firm asset values at the end of any finite time interval is stationary log-normal.

A5. There exists a riskless asset paying a known constant rate of interest, r_f.

A6. Trading takes place continuously.

A7. Individuals can sell any security short and receive proceeds of the sale.

A8. Capital Markets are perfect.

A9. The firm announces an offer to repurchase a certain fraction of the outstanding shares, f_A, at a price P. The repurchase is executed at date T_E ($T_E < T$).

A10. New information relevant to the value of the firm and (or) the standard deviation of returns on the firm is associated with the repurchase offer.

A11. If the number of shares tendered by the stockholders is less than the number of shares sought by the firm, all shares tendered are repurchases. If the number of shares tendered exceed the number of shares sought, the firm repurchases an equal fraction from each tendering shareholder such that the total number of shares repurchases is equal to a fraction of f_A of the outstanding stock.

A12. The firm cannot change the terms of the repurchase offer.

A13. The firm liquidates some of its assets to finance the repurchase offer. Therefore, the variance of returns on the firm could change after the execution of the repurchase.

In Section 5, assumption A13 is replaced with the following assumption:

A14. The firm issues some debt to finance the repurchase offer. The new debt issue is a pure discount bond with a face value of M_N, and a maturity of T_N years. The new issue is subordinated to the existing debt.

The notations used in the paper is:

V_B = Value of firm before repurchase announcement.

σ_B = Standard deviation of returns of the firm before repurchase announcement.

M = Face value of outstanding debt.

T = Maturity date of debt (measured from date of repurchase announcement).

S^B = Value of stock before repurchase announcement.

D^B = Value of debt before repurchase announcement.

P = Repurchase premium (fraction of pre-announcement stock price per share).

T_E = Expiration date of repurchase offer (measured from date of repurchase announcement).

f_A = Fraction of outstanding shares firm wants to repurchase.

f_E = Fraction of outstanding shares firm actually repurchases.

V^A = Value of firm after announcement of repurchase offer.

= $V^B(1 + \delta_V)$.

δ_V = Information effect of repurchase announcement on firm value.

σ_{A1} = Standard deviation of returns on the firm after announcement of repurchase offer.

= $\sigma_B(1 + \delta_\sigma)$.

$\delta\sigma$ = Information effect of repurchase announcement on firm standard deviation.

σ_{A2} = Standard deviation of returns on firm after expiration of repurchase offer.

S^A = Value of stock after repurchase announcement.

D^A = Value of debt after repurchase announcement.

v = Value of the firm at which stock holders are indifferent to the repurchase.

$\underline{v}$ = Value of the firm below which the repurchase offer is suspended.

r_f = Risk-free rate per annum.

In Section 5 of the paper, the following additional notation is used:

M_N = Face value of new debt issue.

T_N = Maturity date of the new debt issue (measured from date of repurchase announcement.

D^B = Value of new debt before repurchase announcement.

3. THE VALUATION EQUATIONS

3.1 BEFORE REPURCHASE ANNOUNCEMENT

For the firm described in Section 2, the shareholders can be viewed as having an option to buy back the firm at an exercise price equal to the face value of the debt, at the maturity date of the latter. Thus, the value of the stock at the maturity date, T, is $Max(0, V_T - M)$, where V_T is the value of the firm at time, T, and M is the face value of the maturing debt. This implies that the stock is a call option on the value of the firm, and can be valued using the Black-Scholes option pricing model.[9] The value of the stock of the firm is:

$$S^B = V^B N(k_B + \sigma_B\sqrt{T}) - Me^{-r_f T} N(k_B) \tag{1}$$
$$= S(V^B, \sigma_B, r_f, M, T)$$

where $N(.)$ = univariate normal cumulative distribution function and

$$\text{and } k_B = \frac{\ln(V^B/M) + (r_f - \frac{1}{2}\sigma^2)T}{\sigma_B\sqrt{T}}$$

The current market value of debt is $V^B - S^B$. Thus,

$$D^B = V^B[1 - N(k_B + \sigma_B\sqrt{T})] + Me^{-r_f T} N(k_B). \tag{2}$$

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[9]See Black and Scholes (1973).

3.2 AFTER REPURCHASE ANNOUNCEMENT

At the expiration date of the repurchase offer, by arbitrage conditions, the repurchase is successful for those firm values where the value of the stock if the shareholders do not tender is less than the price specified in the firm's repurchase offer. Algebraically, the repurchase is successful is[10]

$$S(V^A_{T_E}, \sigma A1, r_f, M, T - T_E) \leq S^B(1 + p)$$

where $V^A_{T_E}$ = Value of the firm at date T_E.

Assume that $\bar{V}$ is that firm value that solves equation (3) as an equality. This implies that the repurchase is successful if $V^A_{T_E} < \bar{V}$. It is also assumed that the firm cancels the repurchase offer if $V^A_{T_E} < \underline{V}$, where $\underline{V} \geq f_A S^B(1 + p)$.[11] Therefore, the value of stock an instant before T_E is

$$S_A = \begin{cases} S(V^A_{T_E}, \sigma A1, r_f, M, T - T_E) & \text{if } V^A_{T_E} < \underline{V} \text{ or } V^A_{T_E} > \bar{V} \\ S(V^A - f_E S^B(1+p), \sigma A2, r_f, M, T-T_E) + f_E S^B(1+p) & \end{cases} \quad (4)$$

otherwise Cox and Ross (1975) have shown that if one can create a riskless hedge involving the security that one is interested in

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[10] S(.) has been defined in equation (1).

[11] This assumption guarantees that the firm will not repurchase if this activity results in bankruptcy at date T_E. In later sections, it will be assumed that $V^A = f_A S^B(1 + p)$. Under these conditions, the repurchase is successful if and only if the value of the firm at date T_E is between the value at which the firm cancels the repurchase and that at which the stockholders are indifferent between rendering and selling in the market, i.e. $\underline{V} < V^A < V$.

pricing, the current value of such a security can be obtained by discounting the expected value of the security at some future date by the risk-free rate.[12] Assuming that such a hedge can be created in this case, the current post-purchase announcement value of the stock is given by

$$S^A = e^{-r_f T_E} E\{S^A_{T_E} | V^A\} \tag{5}$$

Combining equations (4) and (5) and evaluating the integrals as in Geske (1979) yields:

$$S^A = V^A W - ME^{-4_F T} W_2 + f_A S^B (1 + p) e^{-r_f T_E} \tag{6}$$

Where $W_1 = N(_1 + {}_{\sigma A1}\sqrt{T_E}, K_1 + A1\sigma\sqrt{T}, \rho) + N({}^h w - {}_{\sigma A1}\sqrt{T_E}, k_{2\rho} 2)$

$$+ N(h_3 - {}_{\sigma A1}\sqrt{T_E}, k_1 + {}_{\sigma A1}\sqrt{T}, {}_{\rho 3}) - N(h_3 - {}_{\sigma A1}\sqrt{T}, k_{2\rho 2})$$

$$W_2 + N(h_1, k_1, {}_{\rho 1}) + N(h_2, k_3, {}_{\rho 2}) + (N(h_3, k_1, {}_{\rho 3}) - N(h_3, k_3, {}_{\rho 2})$$

$$W_3 + N(_3, k_4, {}_{\rho 2}) - N(h^2, k_3, \rho 2) + N(h_2) - N(h_3)$$

$$h_1 + \ln(V^A/V) + (r_f - \tfrac{1}{2}\sigma^2)T_E / \sigma A1\sqrt{T_E}$$

$$h_2 = -h_1$$

$$h3 = \ln(\underline{V}/V^A) - (r_f - \tfrac{1}{2}\sigma^2)T_E / \sigma A1\sqrt{T_E}$$

$$k_1 + \ln(V^A/M) + (r_f - \tfrac{1}{2}\sigma^2_{A1})T \; {}_{A1\sigma}\sqrt{T}$$

$$k_2 = \ln(V^A/M) + (r_f + \tfrac{1}{2}\sigma^2_{A2})T + \tfrac{1}{2}(\sigma^2_{A2} - \sigma^2_{A1})T_E \sqrt{\sigma^2_{A2}} T + (\sigma^2_{A1} - \sigma^2_{A2})T_E$$

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[12]The hedging argument results in an option price that does not depend directly on the structure of investor preferences. The preferences determing equilibrium parameter values, but given these parameter values, the solution obtained is preference free. Therefore, we can assume risk neutrality to obtain the equilibrium price of the option. See Cox and Ross (1975, 1976) for more details.

$$k_3 = k_2 - \sqrt{\sigma^2_{A2}T + (\sigma^2_{A1} - \sigma^2_{A2})T_E}$$

$$k_4 = \ln(V^A_1/M) + (r_f + \tfrac{1}{2}\sigma^2_{A2})T - \tfrac{1}{2}(\sigma^2_{A2} + \sigma^2_{A1})T_E/\sqrt{\sigma^2_{A2}T + (\sigma^2_{A1} - \sigma^2_{A2})T_E}$$

$$V^A_1 = V^A - f_E S^B(1 + p)e^{-r_f T_E}$$

$$\rho_1 = \sqrt{T_E/T}$$

$$\rho_2 = (\sigma_{A1}\sqrt{T_E})/(\sqrt{\sigma^2_{A2}T + (\sigma^2_{A1} - \sigma^2_{A2})T_E})$$

$$\rho_3 = -\rho_1$$

and $N(Z_1, Z_2, \rho)$ = bivariate normal distribution function with Z_1 and Z_2 as upper limits and ρ as the correlation coefficient.

Since the current values of the two outstanding securities should sum to the current value of the firm, the current value of debt is

$$D^A = V^A(1 - W_1) + Me^{-r_f T} W_2 - f_E S^B(1 + p)e^{-r_f T_E} W_3 \qquad (7)$$

4. PRE VS POST-ANNOUNCEMENT SECURITY VALUES: SOME COMPARISONS

In the framework considered here, any difference between the post-announcement and pre-announcement values of securities can be separated into three casual components. The first is the leverage effect, the second is the asset structure effect, and the third is the information effect. The leverage effect is caused by the change in the leverage ratio after a successful repurchase. The asset structure effect is the result of a change in the asset structure of the firm after a successful repurchase.[13] The

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[13]Although the leverage and asset structure changes take place at the expiration date of a successful repurchase offer, the post-announcement security values will reflect the expected effects.

information effect is caused by a change in the market's perception of (a) the value of the firm, and (b) the variance of returns on the firm that accompany the repurchase announcement.[14]

Since the three components do not necessarily act in the same direction, the change in value of any security is determined by the relative magnitude of each effect. Initially, in order to isolate these three effects, each one is considered separately while the others are forced to zero.

4.1 THE LEVERAGE EFFECT

The leverage effect is concerned with a post-repurchase change in bankruptcy risk caused by a change in the leverage ratio of the firm after the expiration of a successful repurchase offer.[15] The repurchase, when successful, increases the leverage ratio of the firm after date T_E. Therefore, the repurchase results in an increase in the leverage related risk for the bond which implies a decrease in the value of the debt

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[14]The changes in firm value, and variance due to the information effect take place at the announcement date.

[15]The leverage ration (ℓ) is defined as the ratio of the face value of debt to the market value of the firm i.e. $\ell = M/V$.

$(\Delta S\ell > 0, \Delta D\ell < 0)$.[16]

The magnitude of the leverage effect on stock and debt values would depend on the values of the various parameters in the valuation equation. The change in the leverage ratio ($\Delta\ell = \ell_B(\text{payout}/V\text{-payout})$) is an increasing function of the payout to the stockholders.[17] This relationship when coupled with the fact that $\partial\text{payout}/\partial f_E > 0$, and $\partial\text{payout}/\partial p > 0$, implies that the leverage effect on stock is an increasing function of the repurchase premium, and the fraction of outstanding shares repurchased.[18] Algebraically, $\partial\Delta S\ell/\partial f$, $\partial\Delta S_\ell/\partial p > 0$
The magnitude of the leverage effect would also depend on the pre-announcement debt to equity ratio of the firm. The effect of changes in the debt to equity ratio of the firm on change in stock value due to the leverage effect has two components, and can be written algebraically as:

$$\frac{d\Delta S\partial}{\partial(D^B/S^B)} = \left.\frac{\partial\Delta S\ell}{\partial(D^B/S^B)}\right|_{\substack{\text{Payout}\\=\text{Const.}}} + \frac{\partial\Delta S\ell}{\partial\text{payout}}\,\frac{\partial\text{ payout}}{\partial(D^B/S^B)}$$

If the firm is almost all equity (D^B/S^B large), the value of equity would increase by the amount of the payout while there would be no effect if $D^B/S^B \Rightarrow 0$. This implies that the first derivative

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[16]S ($\Delta D\ell$) is the leverage effect on stock (debt). Since the leverage effect causes no change in firm value at the announcement date of the repurchase, $\Delta S\ell + \Delta D\ell \Rightarrow 0$.

[17]This follows from the fact that $\partial\Delta\ell/\partial\text{payout} = \ell_B V/(V\text{-payout})^2 > 0$.

[18]The derivatives follow from payout = $f_E S^B(1+p)$.

on the RHS of the above equation is positive. But since the payout is a decreasing function of the debt to equity ratio, this implies that the second term on the RHS of the above equation is negative.[19] Therefore, the total effect of changes in the debt to equity ratio on changes in debt and equity values is ambiguous. Since the debt to equity ratio of the firm depends on the pre-announcement leverage ration, the pre-announcement variance, the maturity of debt, and the risk-free rate, changes in these parameters would have an ambiguous impact on the magnitude of the leverage effect. Algebraically,[20]

$$\frac{\sigma \Delta S\ell}{\partial(M/V^B)}, \quad \frac{\partial \Delta S\ell}{\partial \sigma_B}, \quad \frac{\partial \Delta S\ell}{\partial T}, \quad \frac{\partial \Delta S\ell}{\partial r_f} \gtrless 0$$

The magnitude of the leverage effect on stock is a decreasing function of the time to expiration of the repurchase offer i.e. $\partial \Delta S\ell / \partial T_E < 0$. The comparative statics for the leverage effect on debt values is exactly the opposite of those for stock.

4.2 THE ASSET STRUCTURE EFFECT

At date T_E, the firm liquidates some of its assets to finance the repurchase. The liquidation of these assets changes the asset

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[19] This assumes that p and f are fixed, and follows from the fact that S^B decreases as the debt to equity ratio increases (ceteris paribus).

[20] Based on simulations, these derivatives are unambiguous for reasonable parameter values. If $M/V^B<1$, and annual σ_B <0.6 we get $\partial \Delta S\ell / \partial(M/V^B) > 0$, $\partial \Delta S\ell / \partial \sigma_B > 0$, $\partial \Delta S\ell / \partial T > 0$, $\partial \Delta S\ell / o r_f < 0$.

structure of the firm, which in turn would change the variance of returns on the firm.[21] We assume that the variance of returns after a successful repurchase is given by

$\sigma_{A2}=(1+\Delta)\sigma_{A1}$

where σ_{A1} = Variance of returns before repurchase

To analyze the asset structure effect we consider two cases:

(i) Risk-increasing Asset Structure Change ($\Delta>0$): After a successful repurchase, the securities under consideration are part of a riskier firm. Therefore, the asset structure effect would be positive for common stock, and negative for debt ($\Delta S_{AS}>0, \Delta D_{AS}<0$).

The magnitude of the asset structure effect on stock value is an increasing function of the repurchase premium, and the fraction repurchased to the extent that these variables determine the size of the asset structure change i.e. $\partial\Delta S_{AS}/\partial p>0$, $\partial\Delta S_{AS}/\partial f>0$. In addition, since the asset structure change takes place at the expiration date of the repurchase offer, the magnitude of the asset structure change is a decreasing function of T_E i.e. $\partial\Delta S_{AS}/\partial T_E<0$. The effects of the four remaining parameters are now considered separately:

a) Leverage Ratio(M/V^B): If the firm has (almost) no debt in its capital structure, then increases in the firm variance at date T_E

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[21]The variance would remain the same if the firm liquidates an equal proportion of all its assets. In all other cases, the variance would either increase or decrease at date T_E.

will have (almost) no impact on the value of debt and equity. As the leverage ratio is increased from zero, increases n riskiness will cause a transfer of wealth from debt to equity. But if the firm is all debt, debt values will be invariant to changes in the firm's riskiness. Therefore, the effect of changes in pre-announcement leverage on the change in equity value is ambiguous, i.e. $\partial\Delta^{S}_{AS} / \partial(M/V^{B}) \gtrless 0$.[22]

b) Standard Deviation (σ_B): If the variance of returns on the firm is such that the debt is riskless before the announcement, then increases in variance that do not make the debt risky will have no effect on debt value. On the other hand, if the increase in variance makes the debt risky, then the asset structure change has a positive effect on stock value. This implies that in certain ranges of pre-announcement variance, the magnitude of the asset structure effect is an increasing function of σ_B. In the limit, if σ_B is large enough such that the pre-announcement debt value is approximately zero, the risk-increasing asset structure change will have negligible impact on debt and equity values. Therefore, the

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[22]For $M/V^B < 1$, and annual $\sigma_B < 0.6$, simulations result in a positive derivative.

effect of changes in the pre-announcement variance on the change in equity value is ambiguous i.e. $\partial \Delta S_{AS}/\partial \sigma_B \gtrless 0$.[23]

c) Risk-free Rate (r_f): The risk-free rate determines the magnitude of the asset structure effect through its effect on the pre-announcement debt to equity ratio of the firm. The debt to equity ratio is a decreasing function of the risk-free rate. Since the magnitude of the asset structure effect on equity is an increasing function of the debt to equity ratio, the change in equity value is a decreasing function of the risk-free rate, i.e. $\partial \Delta_{AS}/\partial r_f < 0$.

d) Maturity of Debt (T): If $T = T_E$, the value of debt is unaffected by changes in the variance at date T_E. As T is increased, changes in the variance caused by the asset structure change will result in a decrease in debt value. At the extreme, if T is large, the pre-announcement value of debt is almost zero, and changes in variance will have little effect on debt value. Therefore, changes in debt maturity have an ambiguous effect on the change in equity value i.e. $\partial \Delta_{AS}/\partial T > 0$.[24] The comparative statics for debt are exactly the opposite of those for equity.

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[23] For $M/V^B < 1$, and $\sigma_B < 0.6$, simulations result in a positive derivative.

[24] All the above results can be obtained by evaluating $\partial^2 S/\partial X \partial \sigma$ for $X = M/V^B$, σ_B, r_f, and T where S is valued as a call option using the Black-Scholes equation.

(ii) Risk-decreasing Asset Structure Change ($\Delta<0$): The results, in this case, are exactly the opposite of those for a risk-increasing asset structure change, i.e. $\Delta S_{AS}<0$, $\Delta_{AS}>0$.

4.3 THE INFORMATION EFFECT

As stated earlier, the leverage and asset structure effects are caused by changes in the firm's leverage ratio, and asset structure after a successful repurchase. If the probability of a successful repurchase is zero i.e. when $\underline{V}$ = or $V^A >>> V$, then these two effects would be zero. Even in this situation, the repurchase announcement may result in a change in security values if it conveys some information about the investment opportunities of the firm. The information effect on firm value is measured by δ_V where

$$\delta_V=(V^A-V^B)/V^B$$

V^A= Value of firm after announcement.

V^B= Value of the firm before announcement.

The information effect on firm variance is measured by $\sigma\sigma$ where

$$\delta_o=(\sigma_{A1} - \sigma_b)\ B$$

σ_{A1} = Standard deviation of returns after announcement.

σ_B = Standard deviation of returns before announcement.

To analyze the information effect we have to consider four cases:

(i) Value increasing Signal ($\delta_V>0$). Since stock and debt values are increasing functions of firm value, a value increasing signal would result in an information effect that is positive for both debt and equity ($\Delta S_V, \Delta D_V > 0$). The effects of the various parameters are now considered separately:

a) Leverage Ratio (M/V^B): If the leverage ratio of the firm is (almost) zero, increases in firm value will have (almost) no effect on debt, and equity value will increase by an amount (almost) equal to the increase in in firm value. On the other hand, if the leverage ratio of the firm is extremely large, increases in firm value will affect debt, and have almost no effect on equity. This implies that the information effect on equity is a decreasing function of the pre-announcement leverage ratio of the firm i.e. $\partial \Delta S_V / \partial (M/V^B) < 0$.

b) Standard Deviation (σ_B): If the variance is such taht the debt is riskless, then increases in firm value will have no effect on debt, and equity value will increase by an amount equal to the increase in firm value. If debt is risky, increases in firm value will be shared by both debt and equity. At the extreme, when variance is large, the debt to equity ratio of the firm is almost zero and increases in firm value will have almost no effect on debt. Therefore, changes in the pre-announcement variance has an ambiguous impact on the change in equity value i.e. $\partial \Delta S_V / \partial \sigma_B$.[25]

c) Risk-free Rate (r_f): The risk-free rate determines the the magnitude of the information effect through its effect on the debt to equity ratio of the firm which is a decreasing function of the risk-free rate. Since the magnitude of the firm value information effect on equity is a decreasing function of the debt-equity ratio, the change in equity value is an increasing function of r_f i.e. $\partial\ \partial S_V / \partial r_f > 0$.

d) Debt Maturity (T): If T=0, the pre-announcement value debt is approximately equal to its face value.[26] Since this is the maximum attainable value for debt, increases in firm value will have no effect on debt. As T is increased, the value of debt falls and (for 'reasonable' values of σ_B) will be below $Me^{-r_f T}$. This implies that increases in firm value will be shared by both debt and equity. In the limit, if T is large, the debt to equity ratio is almost zero, and increases in firm value will have little effect on debt. Therefore, changes in debt maturity has an

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[25]For reasonable values of the leverage ratio and pre-announcement variance, the derivative is negative eg. $M/V^B < 1$, and $\sigma_B < 0.6$.

[26]Assuming $M/V^B < 1$.

ambiguous impact on the change in the value of equity, i.e. $\partial \Delta S_V / \partial T > 0$.[27]

(ii) Value Decreasing Signal ($\delta_v <$)0: The results for this case are exactly the opposite of those for a value increasing signal.

(iii) Risk Increasing Signal ($\delta_o > 0$): Since stock (debt) values are increasing (decreasing) functions of firm risk, a risk increasing signal would result in an information effect that is positive (negative) for equity (debt) i.e. $\Delta S_s > 0$, $\Delta D_s > 0$. The comparative statics for this case are identical to those derived for a risk-increasing asset structure change, i.e.

$$\frac{\partial \Delta S_s}{\partial (M/V^B)} \quad \frac{\partial \Delta S_s}{\partial \sigma_B}, \quad \frac{\partial \Delta S_s}{\partial T} \gtrless 0, \frac{\partial \Delta S_s}{\partial r_f} < 0$$

$$\frac{\partial \Delta D_s}{\partial (M/V^B)} \quad \frac{\partial \Delta D_s}{\partial \sigma_B} \quad \frac{\partial \Delta D_s}{\partial T} \quad 0, \frac{\partial \Delta D_s}{\partial r_f} > 0$$

(iv) Risk Decreasing Signal ($\delta_o < 0$): The results in this case are exactly the opposite of those for a risk increasing signal. The information effect on stock value is unambiguously positive in the situation where the repurchase announcement results in an increase in both the firm value and variance. In the case where

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[27]All the above results can be obtained by evaluating $\partial^2 S/\partial X \partial V$ for $X = M/V^B$, σB, r_f, and T where S is valued as a call option.

magnitude of the decrease is an increasing function of the repurchase premium, the fraction of stock repurchased, and a decreasing function of the change in variance at date T_E.[28] Since the repurchase does not involve any payouts to debt, and we do not allow for revision of the terms of the repurchase offer, debt values will not change at the expiration date of the repurchase offer.

If the asset structure change is risk-decreasing ($\Delta \leq 0$), the post-repurchase price/share of stock will, in general, be lower than the pre-announcement price/share. If the asset structure change is risk-increasing ($\Delta > 0$), the post-repurchase price of stock could be higher than the pre-announcement price. The difference between pre-announcement and post-repurchase debt values would always be positive for a risk-increasing asset structure change ($\Delta > 0$), and is in general negative for a risk-decreasing asset structure change ($\Delta < 0$).[29]

Figures 1 and 2 portray time series of stock and debt values for zero, risk-increasing, and risk-decreasing asset structure

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[28]This follows from the fact that the change in stock price at date T_E is approximately $f_E(P-P_B) / (1-f_E)$ where f_E is the fraction of stock repurchased, P is the price in the tender offer, and P_B is the price before expiration of the tender offer.

[29]This statement does not hold when the risk-decreasing asset structure change is such that the leverage effect is still much larger than the asset structure effect.

changes. In both figures, V^B=\$100, M=\$75, r_f=10% per year, σ_B=0.3 per year, T=5 yrs T_E=0.1 yrs, p=25%, and f_A=f_E=20%.

In figure 1, the value of stock is a monotonically decreasing function of time, with discontinuities at the announcement and expiration dates of the repurchase offer.[30] The change in stock price at the announcement date varies from -0.98% for the risk-decreasing asset structure change to 5.47% for the risk-increasing asset structure change. The change in price at the repurchase expiration date is negative for all three cases, and varies from -6.91% for Δ=20% to-4.82% for Δ=20%. The difference between pre-announcement and post-repurchase stock price is positive for all situations portrayed in this figure, and varies from +8.69% for Δ=-20%, to +0.04% for Δ=20%.

In figure 2, the value of debt is a monotonically increasing function of time with a discontinuity at the offer announcement date. The discontinuity is negative for the zero, and positive asset structure changes, and positive for the negative asset structure change.[31] For a 20% increase in standard deviation after expiration of the repurchase offer, the value of

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[30] The decrease in stock price over time follows from the fact that the value of a call option is a decreasing function of time to maturity.

[31] The combined discontinuity may be positive for a small risk-decreasing asset structure change. In this example, if Δ is such that $-10<\Delta<0$, debt falls in value at the announcement date.

debt falls by 7.27% at the date of announcement. The corresponding change for a 20% decrease in standard deviation is 1.31%.

4.5 THE COMBINED EFFECT

If a "signalling" effect does exist, the value of equity increases unambiguously at the date of announcement, if the asset structure changes, and the firm variance signal are risk-increasing, and the firm value signal is value increasing ($\Delta \geq 0$, $\delta_V \geq 0$, $\delta_O \geq 0$). In all other cases, the combined effect on equity is ambiguous. Similarly, debt experiences an unambiguous decrease in value at the announcement date, if the asset structure change and the firm variance signal are both risk-increasing, and the firm value signal is value-decreasing ($\Delta \geq 0$, $\delta_V \leq 0$, $\delta_O \geq 0$). In all other situations, the combined effect on debt is ambiguous.

The behavior of stock and debt values at the expiration date of the repurchase offer in the "signalling" case is identical to that described earlier in the no "signalling" situation. Between the announcement and expiration dates, the stock price is a decreasing function of time when the firm value signal is value-decreasing, and (or) the asset structure change is risk-decreasing. If the asset structure change is risk-increasing and the firm value signal is value-increasing, the increase in firm value at the date of repurchase announcement increases the probability of an unsuccessful repurchase. This in turn, decreases the probability of the risk-increasing asset structure

FIGURE 1: A TIME SERIES PLOT OF STOCK PRICE

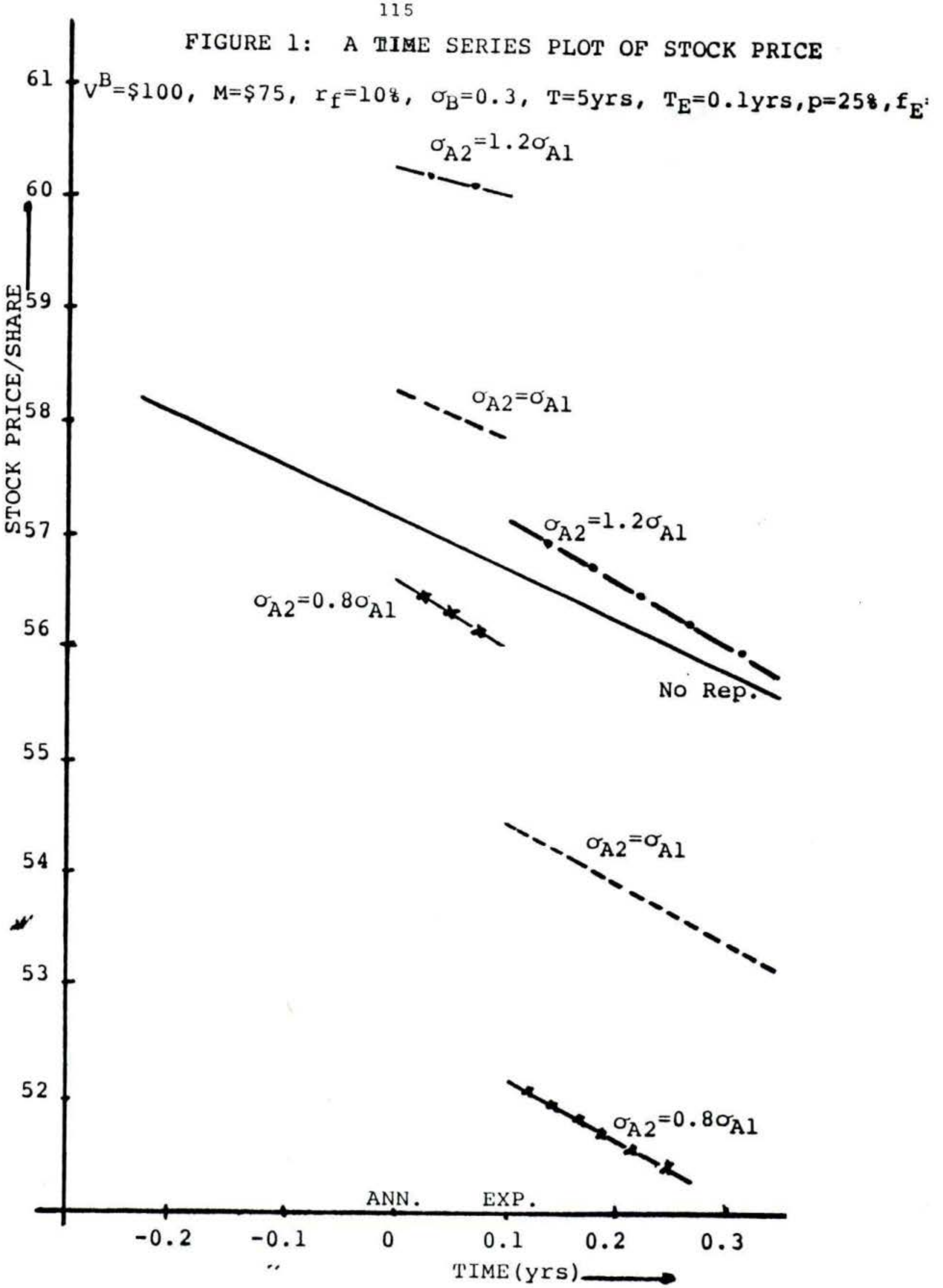


FIGURE 2: A TIME SERIES PLOT OF DEBT VALUE

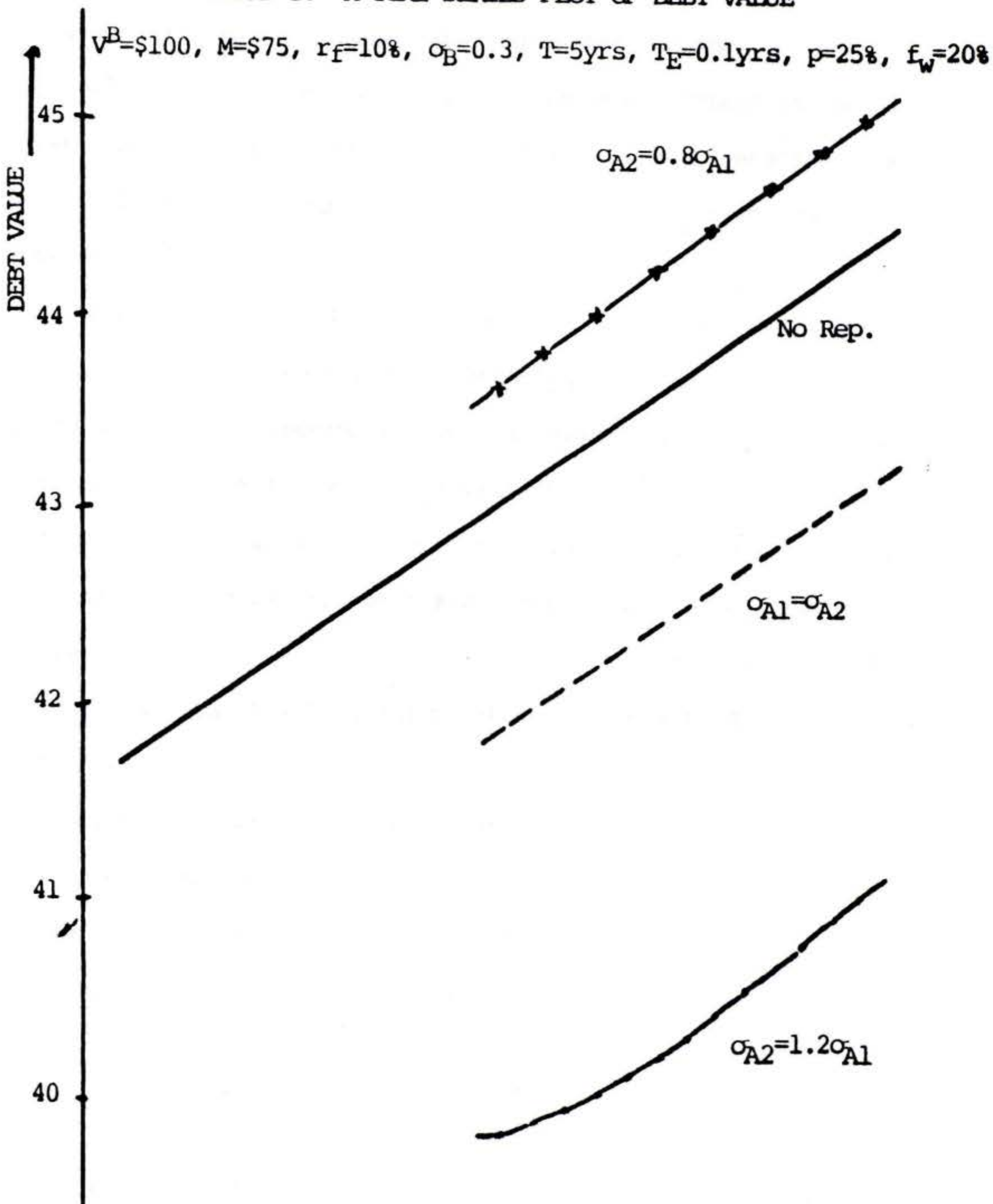
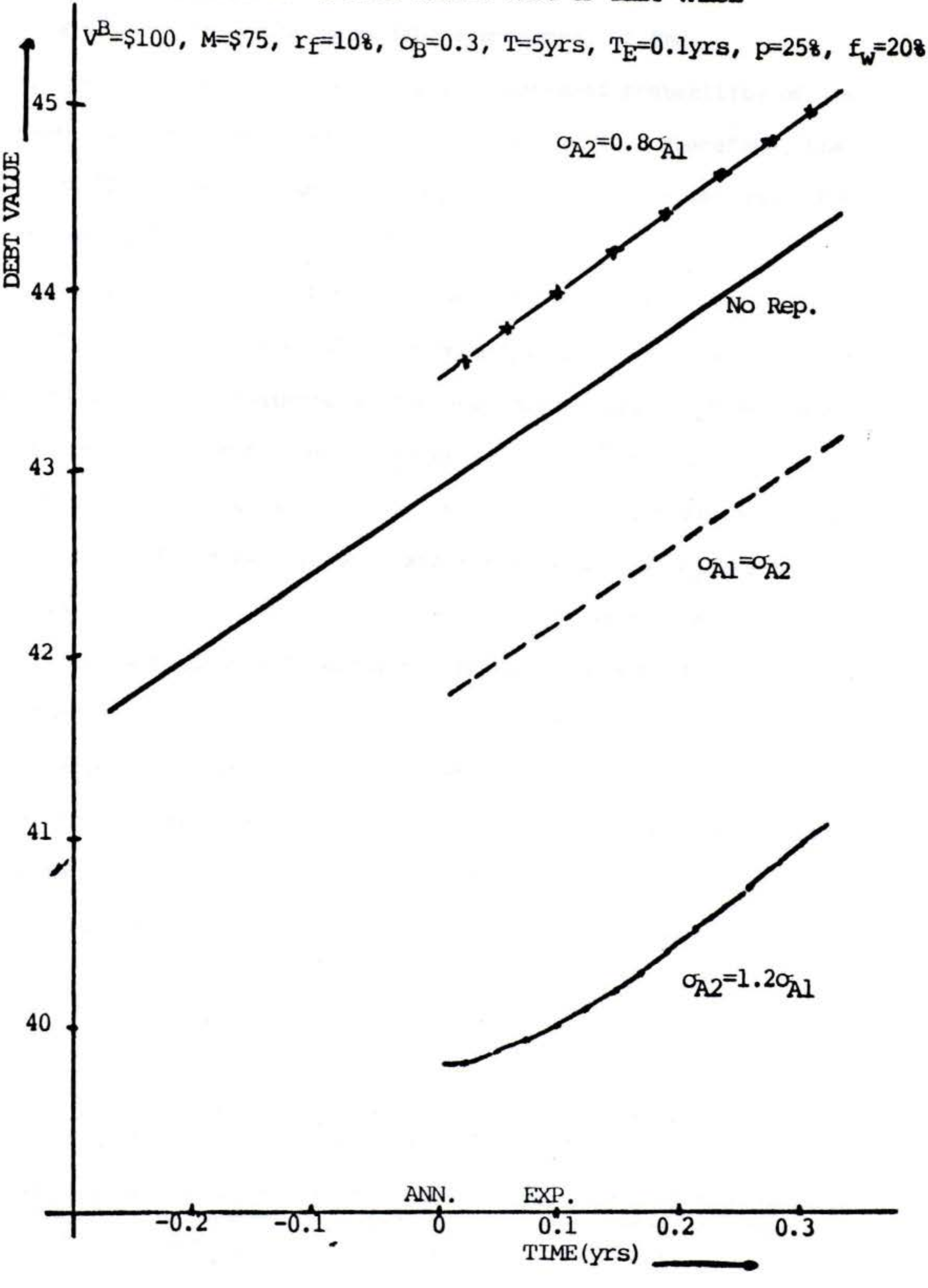


FIGURE 2: A TIME SERIES PLOT OF DEBT VALUE



change.[32] As the repurchase date approaches, if the repurchase becomes more certain, the increased probability of the asset structure change adds value to the equity. Therefore, the effect of changes in current time on stock price is ambiguous for this case.[33]

Figures 3, 4, 5, and 6 portray time series of stock and debt values for risk-increasing and decreasing asset structure changes, assuming that the repurchase involves "signalling". These figures use the same parameter values as those in Figures 1 and 2. In Figures 3 and 4 the firm value signal, and the firm variance signal are both -10%, while Figures 5 and 6 assume δ_V, and δ_σ are both 10%. The pattern remains essentially the same in these cases as in the situation without the information effect. The main difference is in the behavior of stock and debt values between announcement and expiration dates when both the asset structure change, is risk-increasing, and the firm value signal is value-increasing. Figure 5 indicates that when $\Delta>0$, and $\delta_V > 0$, the value of stock increases from \$69.50 to \$70.10 as current time passes, reducing

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[32]This follows from the fact that the asset structure change is contingent on the repurchase being successful.

[33]The sign would depend on the relative magnitudes of the positive effect of the increased probability of repurchases, and the negative effect of the decreasing time to maturity.

FIGURE 3: A TIME SERIES PLOT OF STOCK PRICE

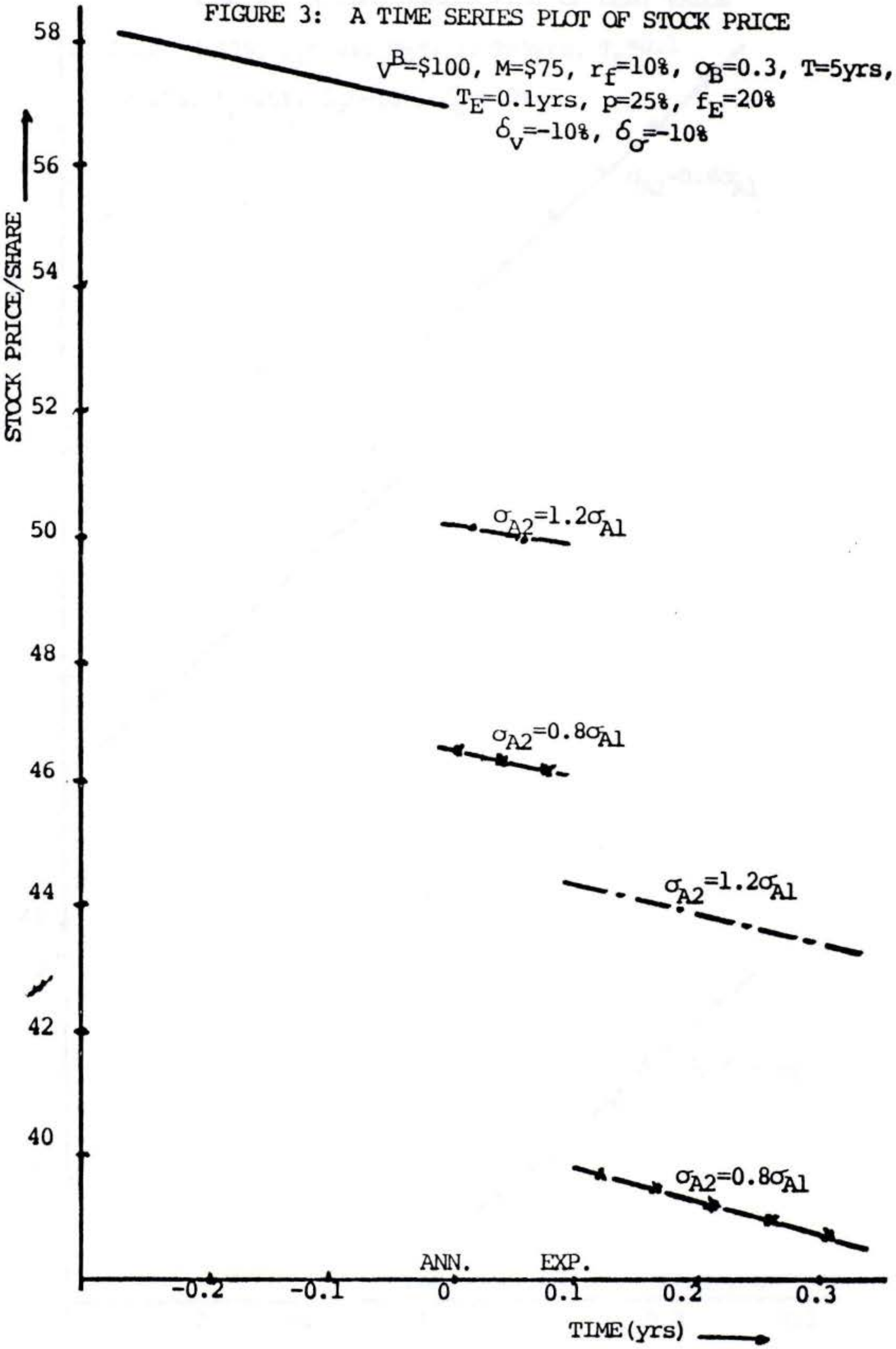


FIGURE 4: A TIME SERIES PLOT OF DEBT VALUE

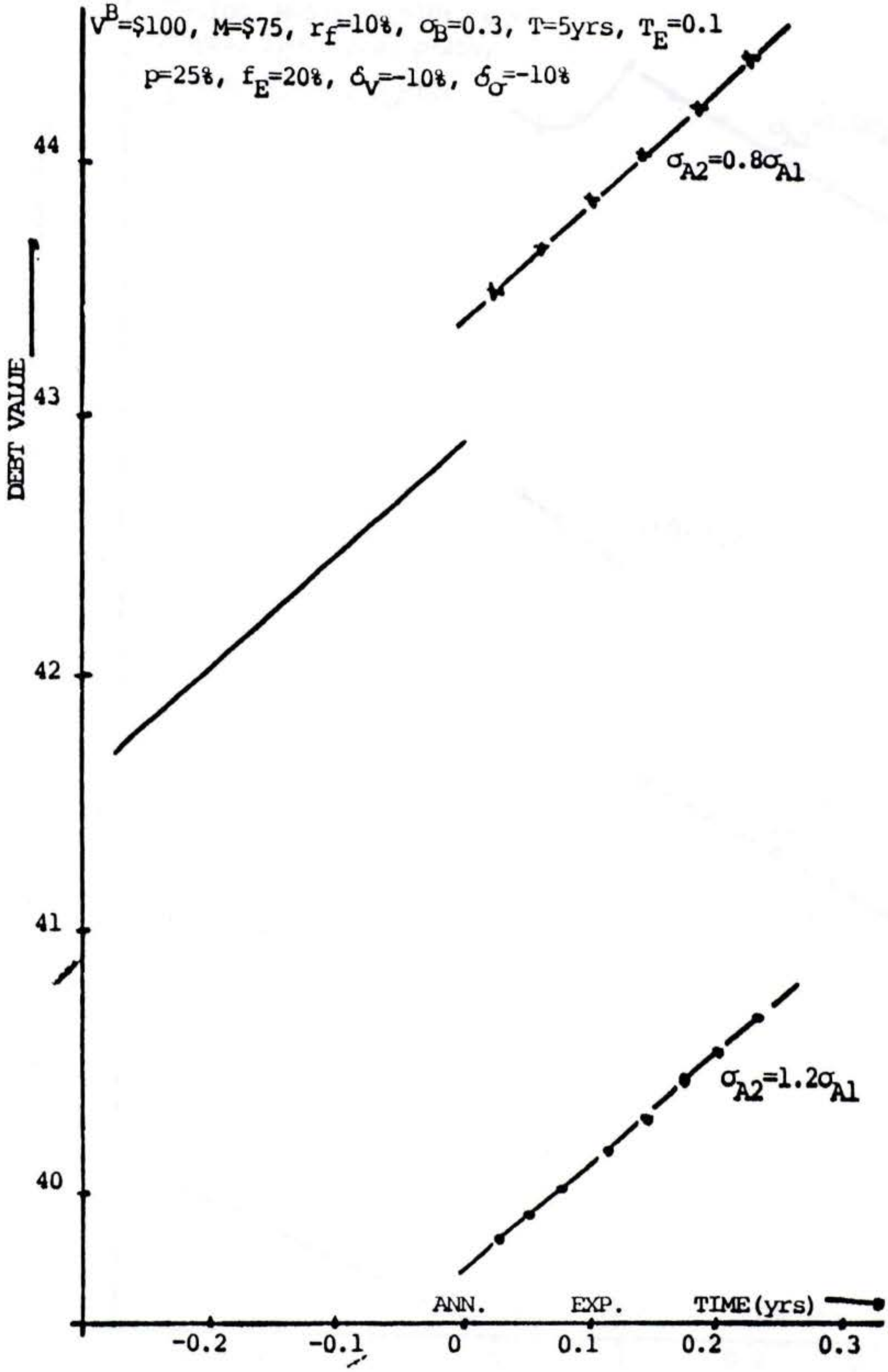


FIGURE 5: A TIME SERIES PLOT OF STOCK PRICE

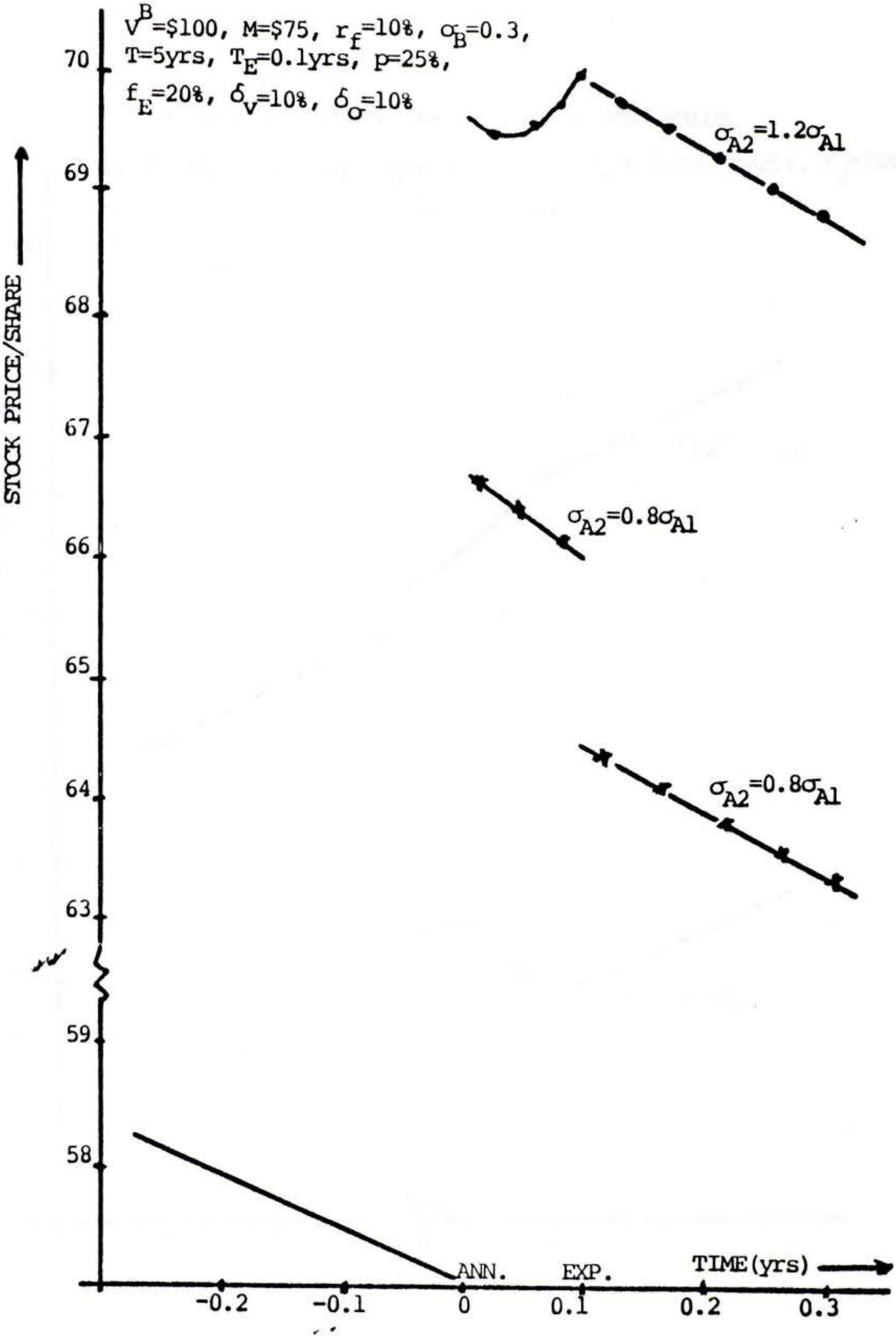
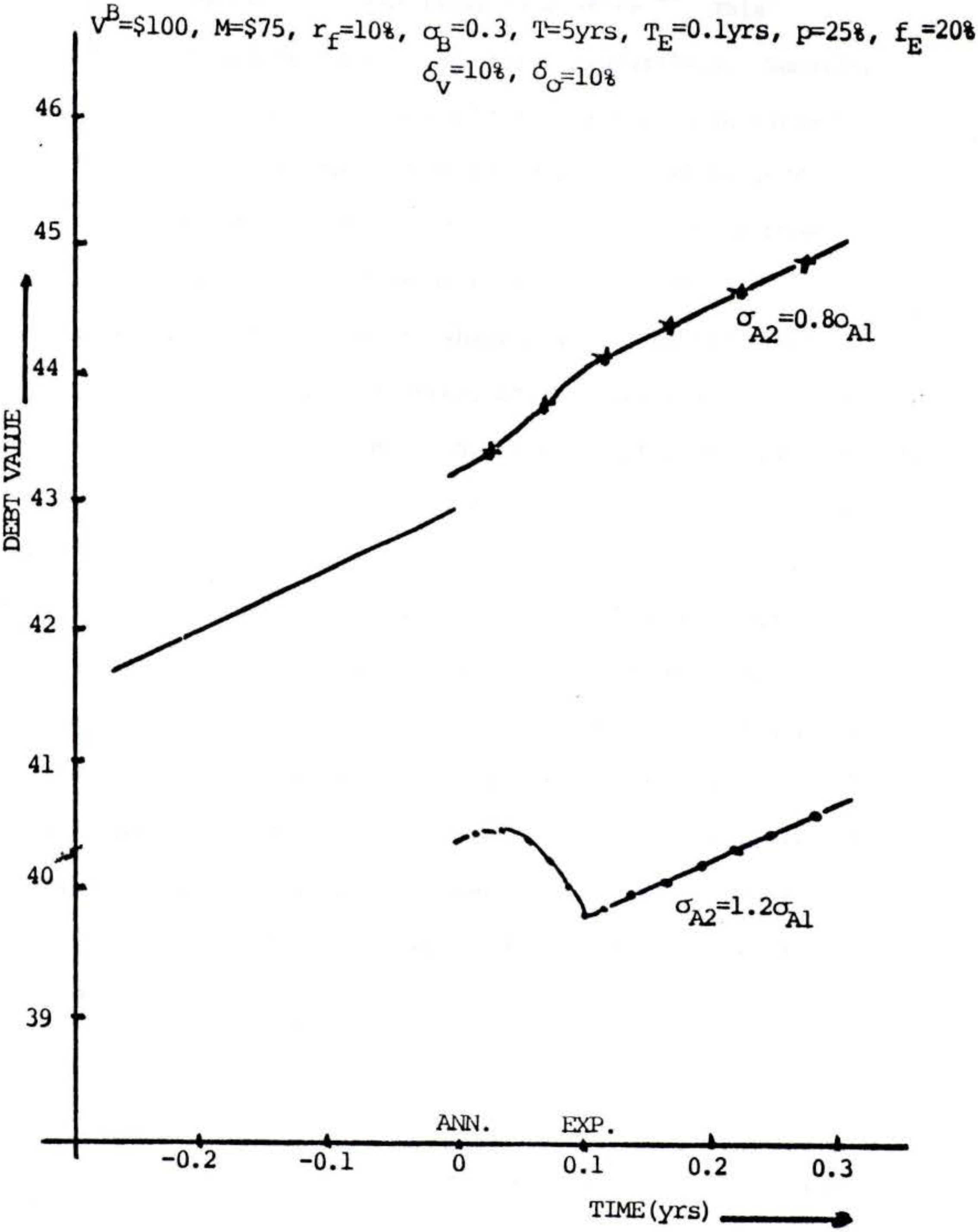


FIGURE 6: A TIME SERIES PLOT OF DEBT VALUE



The time to expiration of the repurchase offer.[34] This reflects the impending increase in firm's volatility. Numerical values for changes in stock and debt are presented in Table 1.

4.6 ON PROTECTIVE COVENANTS AND FEASIBLE INFORMATION SETS

Bond covenants are provisions that restrict the firm from engaging in certain activities after the bonds are sold. Restrictions against payouts to shareholders commonly exist in covenants. In competitive markets, bondholders would negotiate covenants which would prevent firms from engaging in repurchases if this activity results in a decrease in debt values. Therefore, the bondholders would prevent the firm from announcing the repurchase, unless the post-announcement debt value is as least as great as the pre-announcement debt value of debt. Further restriction on the stockholders point of view. The stockholders would not approve the repurchase unless the post-announcement stock price/share is at least equal to the pre-announcement price/share. Therefore, given the protective covenants in debt contracts, the stockholders right of approval, the repurchase would be approved only if

$$S^A \geq S^B \tag{8}$$

$$D^A \geq D^B \tag{9}$$

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[34]The opposite holds true for debt. The value of debt increases from $40.50 to $39.90.

TABLE 1

The Combined Effect on Stock and Debt Values

$V^B = \$100$, $M = \$75$, $r_f = 10\%$, $\sigma_B = 0.3$
$T = 5$yrs., $T_E = 0.1$yrs., $p = 25\%$, $f_E = 20\%$

INFORMATION EFFECT ON VALUE		-10%		10%	
INFORMATION EFFECT ON VARIANCE		-10%		10%	
ASSET STRUCTURE CHANGE		-20%	20%	-20%	20%
% CHANGE IN STOCK	AT ANNOUNCEMENT	-18.4	-11.9	16.9	21.9
	AT EXPIRATION	-13.6	-11.0	-2.1	-0.2
	PRE-ANN vs PRE-EXP	-30.1	-22.1	13.1	22.5
% CHANGE IN DEBT	AT ANNOUNCEMENT	1.1	-7.5	0.8	-5.8
	AT EXPIRATION	—	—	—	—
	PRE-ANN vs PRE-EXP	2.2	6.5	2.7	-7.3

If announcements do not convey any information, repurchases will be approved only if the leverage and asset structure effects are such that equations (8) and (9) hold simultaneously. Given the results derived earlier, that the leverage effect is always positive for stock and negative for debt, the repurchase would be approved only if the firm finances the repurchase with a risk-decreasing asset structure change. Even with a risk-decreasing asset structure change, the best the bondholders and stockholders can do in this situation is to realize no gains or losses from the repurchase announcement.[35] Another implication of his hypothesis is that the post-expiration price/share of equity would always be less than its pre-announcement price.

The purpose of this section is to derive feasible information sets (defined as (δ_V, δ_O) combinations) that leave all security holders at least as well off after the repurchase announcement as before. A further restriction can be placed on the feasible information set by considering the fact that the repurchase premium reflects the information contained in the repurchase announcement. Therefore, it is assumed that the information conveyed by the repurchase offer is such that the post-announcement price/share is

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[35]This result follows because of two restrictions. The first is that the wealth expropriation effects assume no change in firm value at the announcement date, and second that equations (8) and (9) must hold simultaneously.

at most equal to the repurchase offer price.[36] The third restriction on the feasible information set is [37]

$$S^A \leq S^B(1+p) \tag{10}$$

Equation (8), (9), and (10) in conjunction with the valuation equations derived in Section 3 would yield the feasible information set. Figures 7 and 8 depict these sets for risk-decreasing and increasing asset structure changes respectively. In both these figures, the parameter values are identical to those used in the earlier sections. Figure 7 assumes a 20% decrease in standard deviation at date T_E because of the asset structure change. In Figure 8, a 20% increase in standard deviation is assumed.

If the announcement of the repurchase conveys information on firm value and variance that falls along line AB, it would have no effect on debt values. Therefore, if the information conveyed lies either on the line AB or to the left of it, debtholders would not prevent the firm from announcing the repurchase. Similarly, the stockholders would not prevent the repurchase announcement, if they believe that it would convey information that lies either on or above line EA. The dashed line in both figures depicts those information combinations that result in equal pre-announcement and

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[36]This assumption has been found to be supported empirically. See Dann (1981).

[37]Equations (8) and (9) place lower bounds on the feasible information set, while equation (10) is the upper bound.

FIGURE 7: THE FEASIBLE INFORMATION SET FOR RISK DECREASING ASSET STRUCTURE CHANGES

V^B=\$100, M=\$75, r_f=10%, σ_E=0.3, T=5yrs, T_E=0.1yrs, p=25%, f_E=20%, Δ=-20

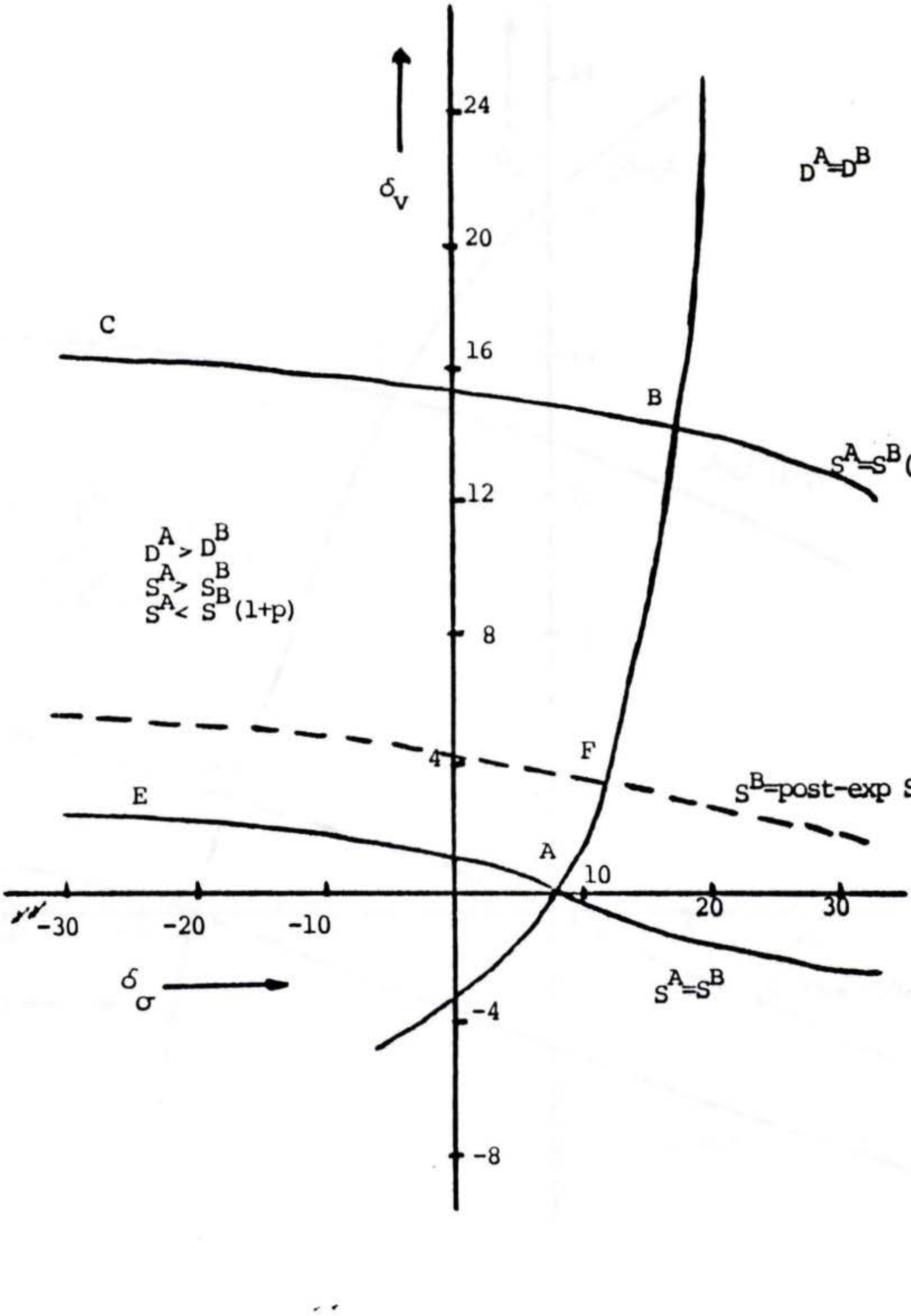
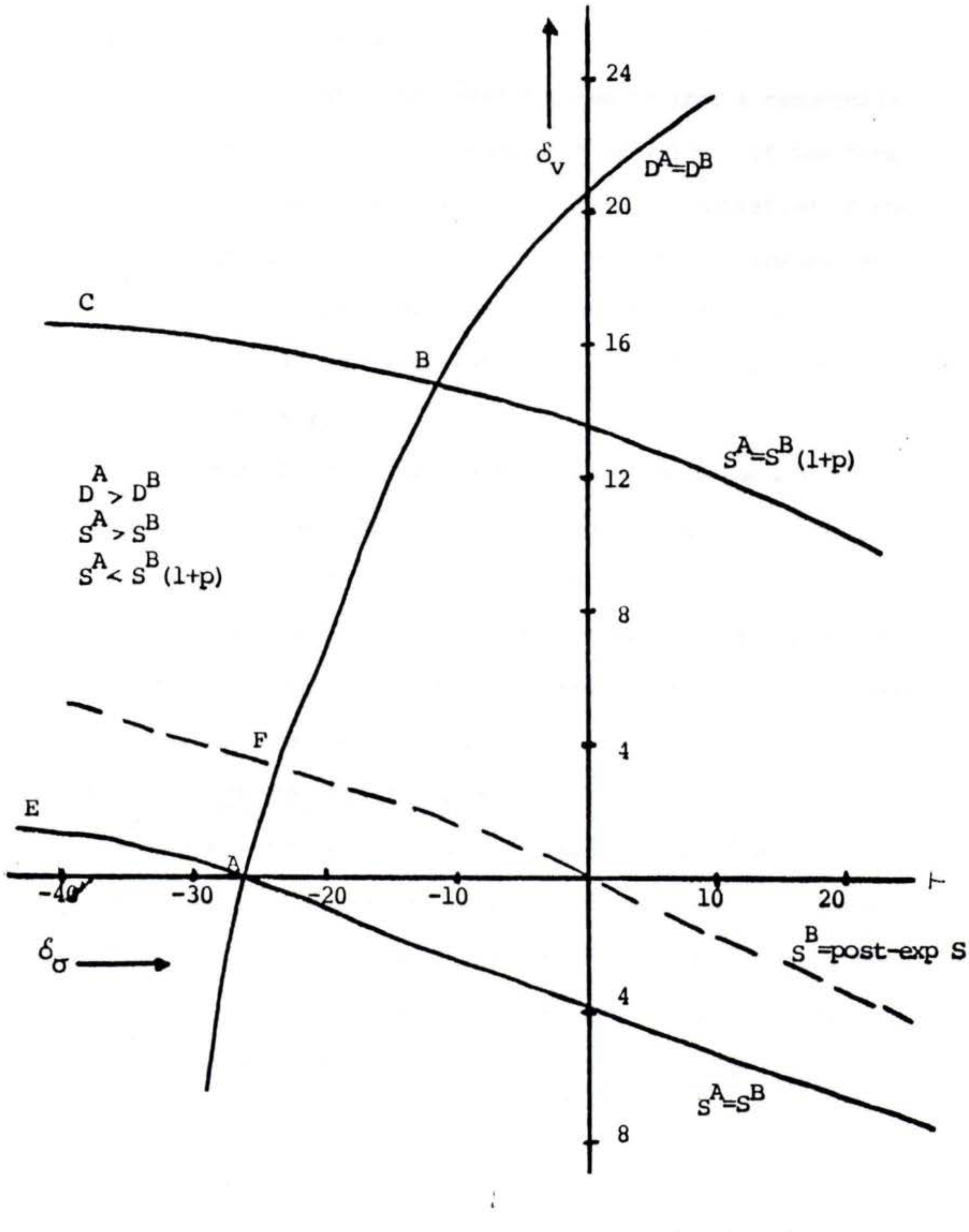


FIGURE 8: THE FEASIBLE INFORMATION SET FOR RISK INCREASING ASSET STRUCTURE CHANGES

V^B=\$100, M=\$75, r_f=10%, σ_B=0.3, T=5yrs, T_E=0.1yrs, p=25%, f_E=20%, Δ=20%



post-expiration stock prices.

The first implication of these figures is that a repurchase announcement cannot signal a decrease in firm value. If the firm value signal is value-decreasing, equation (8) is satisfied if the firm variance signal is risk-increasing. But if the firm variance signal is risk-increasing, equation (9) cannot hold. Therefore, if the repurchase conveys any information on firm value, it has to be a value increasing signal.

The figures also show that with a risk-increasing asset structure change at date T_E, the firm variance signal must be risk-decreasing for both equations (8), and (9) to hold. Similarly, if the firm variance signal is risk-increasing, then for all securityholders to approve the repurchase, the asset structure change at date T_E must be risk-decreasing.[38]

5. ON THE EFFECTS OF DEBT FINANCING

The results derived to this stage are based on the assumption that the firm liquidates some of its assets to finance the repurchase. Masulis (1980) found that 37% of the firms in his sample financed some portion of the repurchase with a new debt issue. The purpose of this section is to analyze the effects of debt financing on the results derived in the earlier sections.

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[38]If the asset structure change is risk-decreasing, and small in absolute value, then it is possible for all securityholders to approve the repurchase even if the firm variance signal is negative.

If the announcement of the new debt issue coincides with the announcement of the repurchase, the market will price the debt issue accordingly, and the repurchase will have no effect on new debt. The wealth transfer from old debt to equity will be smaller than that in the case where the firm uses no debt financing, and would depend on the fraction of the repurchase financed by the new issue.

If the new debt issue is completed prior to the announcement of the repurchase, and if the prospectus for the new issue states what the firm plans to do with the money, the repurchase will have no effect on new debt, and the wealth transfer from old debt to stock will be a decreasing function of the fraction of the repurchase financed by the new issue. If the firm's plans are not known at the time of the new issue, the effects of the repurchase announcement are very similar to those derived in section 4. The wealth expropriation effect is negative for both old and new debt, and positive for equity.[39] The magnitude of the wealth expropriation effect on equity is an increasing function of the

[39]This assumes a risk-increasing asset structure change.

fraction of the repurchase financed by the new issue.[40] In addition, since the new debt issue is junior to the old debt, the wealth expropriation effect on old debt is an increasing function of the fraction of the repurchase financed by the new issue.

The maturity of the new issue will also affect the magnitude of the wealth expropriation effect. Since it has been assumed that the new debt issue is junior to the old debt, the maturity of the new issue must be greater than (or equal to) the maturity of old debt i.e. $T_N > T$. Since the leverage ratio of the firm is an increasing function of T_N, the magnitude of the wealth expropriation effect on equity is an increasing function of T_N[41]. Since old debt is senior to new debt changes in T_N will have no effect on the magnitude of the wealth transfer from old debt to equity. Therefore the magnitude of the wealth transfer from new debt to equity is an increasing function of the fraction

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[40] This statement can be easily explained with an example. Assume that a firm valued at \$V has \$M of old debt. The firm plans to issue new debt with a market value of D_N. The face value of the new issue has to be at least $D_N e^{r_f T_N}$, where T is the maturity of the new debt issue. Therefore, the leverage ratio after the new issue is $(M+M_N) / (V+D_N) \geq (M+D_N e^{r_f T_N}) / (V+D_N) > M/V$, the leverage ratio before the debt issue. The statement then follows from the fact that the magnitude of the wealth expropriation on stock is an increasing function of the pre-announcement leverage ratio of the firm.

[41] The leverage ratio after the debt issue is $(M+M_N) / (V+D_N) \geq (M+D_N e^{r\ T_N}) / (V+D_N)$. The statement follows from the fact that the term to the right of the inequality is an increasing function of T_N.

of the repurchase financed by the new issue, and the maturity of the new issue.

Figure 9 depicts sets of (δ_V δ_O) combinations (for a risk-increasing asset structure change) that leave all debt-holders indifferent to the repurchase. The dashed line in the figure portrays the combinations assuming no debt financing, and V^B=\$100, M=\$60, r_f=10%, σ_B=0.3, T=5 yrs, T_E=0.1 yrs, p=25%, and f_A=f_E=20%. The solid lines portray combinations assuming that approximately 30% of the repurchase is financed with a new issue. The additional parameter values used to obtain these plots are M_N=\$37.50, T_N=8 yrs, and D=\$5.00.

As can be seen from the figure, the use of debt financing has a negligible impact on the feasible information set. Since the issuance of new debt reduces the wealth transfer from old debt to equity, the (δ_V,δ_O) combinations that leave old debt unaffected by the repurchase lie below those for the case where no new debt is issued. Similar results were obtained for 60% and 90% debt financing. Therefore, the financing of a repurchase with a prior issue of debt does not change any of the results derived in the earlier sections under the assumptions of no debt financing.[42]

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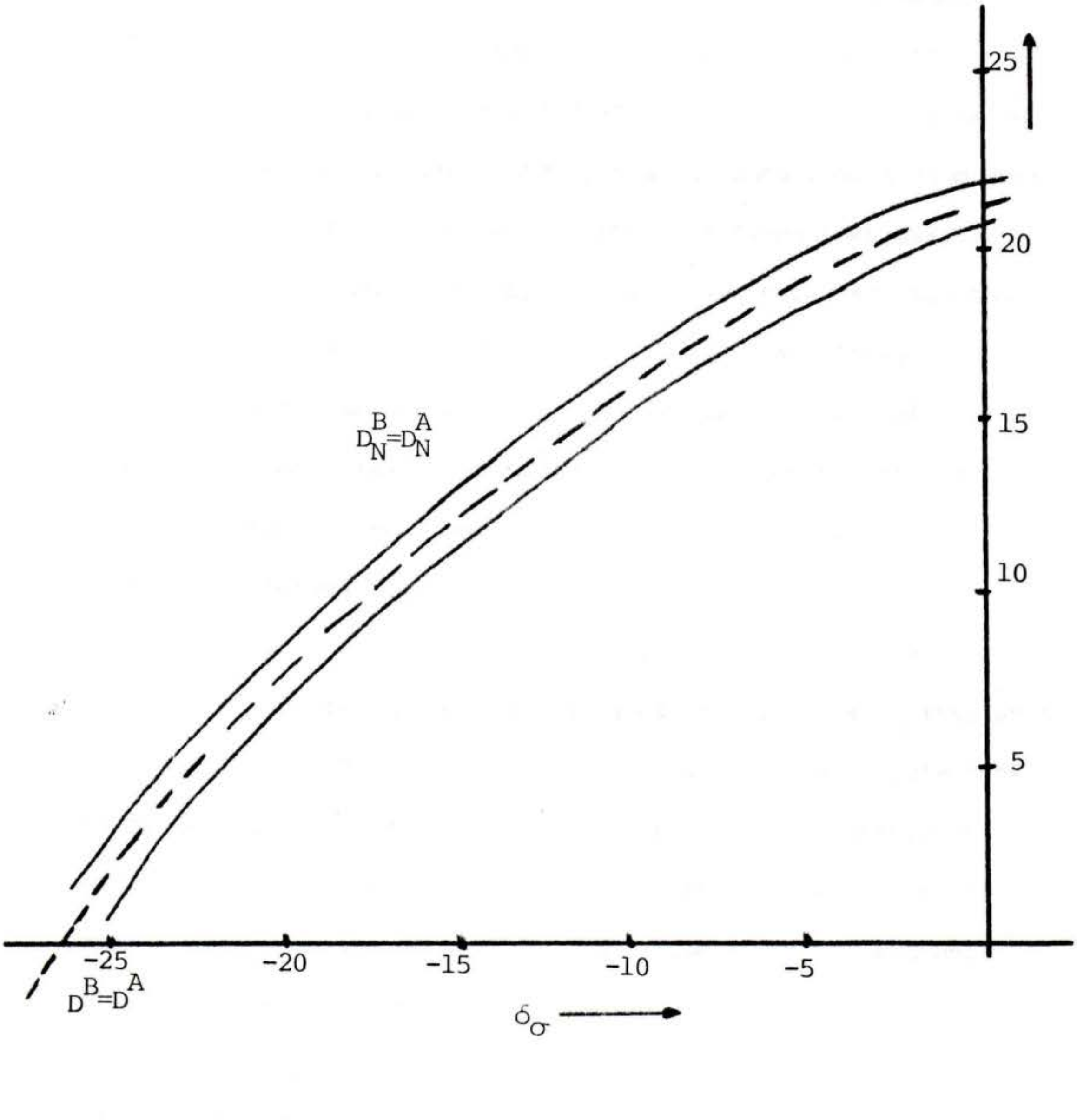
[42] In the analysis of the effects of new debt financing, we have not considered the effects of taxes. Even if taxes were included in the model presented here, they will not alter the conclusions drawn since tax gains from leverage only benefit shareholders.

FIGURE 9: THE EFFECT OF FINANCING WITH A NEW DEBT ISSUE



6. A COMPARISON WITH RECENT EMPIRICAL WORK

In this section the previous theory is compared with recent empirical evidence on repurchases obtained b Masulis (1980), Dann (1981), and Vermaelen (1981). Masulis (1980) found that the "announcement of tender offers are associated with a dramatic 17 percent two day return on common stock. Second,........the non-convertible debt and preferred stock either experience price declines or are unaffected." Masulis also found that announcement day returns on common stock were higher for firms that used more than 50% debt financing as compared to those that used less than 50% debt financing.[43] In addition, he found that stocks of firms that repurchased a greater than average fraction of outstanding shares have a much higher return than the stocks of firms that repurchased a smaller than average fraction of outstanding shares.

The results found by Dann (1981), and Vermaelen (1981) for stock price behavior around the announcement day is very similar to those obtained by Masulis. Dann found that non-convertible debt and preferred stock experienced a statistically insignificant change in value around the announcement date. In addition, Dan found a mean increase of 8.9% in firm value after the announcement of the repurchase offer.

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[43]The average premium for firms in the former category are higher than that in the latter.

The above results are consistent with the hypothesis that repurchases convey information about firm value, and variance. The increase in firm value found by Dann is consistent with the conclusion drawn in section 4 that a repurchase announcement cannot convey a value decreasing signal. The positive return on stock at the announcement date is consistent with information effects on firm value and variance being contained in the set ABCE in Figures 7 and 8. The result that returns on senior non-convertible securities are insignificantly different from zero would imply that the information conveyed by the repurchase offer lies somewhere along the line segment AB in the above mentioned figures. In addition, any information combination along AF would result in a post-expiration stock price that is lower than the pre-announcement stock price.[44] The opposite would hold for information sets along segment FB. Therefore, information sets along AB would be consistent with the announcement day impacts obtained by Masulis (1980), Dann (1981), and Vermaelen (1981).

A combination of the wealth transfer, leverage, and personal taxation hypotheses predict that stock values will increase and debt values will decrease at the announcement date. Therefore, they are not consistent with the results obtained by Dann

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[44]Masulis (1980) found this result for 33% of his sample.

(1981).[45] Although either one of the latter two hypotheses, or a combination of them predict stock and debt price changes that are consistent with the above results, they cannot be used as a rational exclusive of the wealth transfer hypothesis. We will now examine the consistency of the information hypothesis with results Masulis (1980) for sample divisions based on the fraction of the repurchase financed by debt, and the fraction of outstanding shares repurchased. In this discussion it will be assumed that the firm uses low variance assets (eg.cash) to repurchase its stock. This assumption implies that the asset structure change at the expiration date of the repurchase offer is risk-increasing.

As stated earlier, Masulis found that returns to common stock around the announcement day was an increasing function of the % of repurchase financed by a new debt issue. He also found that the firms in the greater-than 50% debt financing group had a larger average premium than that for the less-than-50% debt financing group. It was shown in Section 5 that the magnitude of the wealth transfer from debt to stock is an increasing function of the percentage debt financing in the repurchase (ceteris paribus). Therefore, the wealth transfer hypothesis is consistent with the result obtained by Masulis. In addition, the higher premium in the first group would affect the returns to stock in three ways.

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[45]They are consistent with the cases in which Masulis (1980) found a price decline in senior securities.

First, the higher premium would have a positive impact on returns through the leverage effect. Second, the higher premium implies a larger increase in risk due to the asset structure change (ceteris paribus), which in turn implies a higher return to stocks in the first category. Third, if the repurchase does not convey information on firm value and variance, and the signal is based partly on the dender offer premium, the higher premium would imply a larger information effect on stock prices. These four effects act in the same direction, and imply a higher return for stocks in the first category (i.e. the greater-than 50% debt financing group). Therefore, a combination of the wealth transfer, and the information hypothesis is consistent with this result. Similar arguments can be used to show that a combination of the wealth transfer, and the information hypothesis is consistent with the Masulis (1980) result that the returns to common stock are an increasing function of the fraction of outstanding shares repurchases.

In summary, the empirical results discussed above and the results derived in earlier sections are consistent with each other if the following hyposhtses hold:

(i) The repurchase offer signals an increase in firm value, and a decrease in the variance of returns on the firm at the date of announcement ($\delta_V > 0$, $\delta\sigma > 0$).

(ii) At the expiration date of a successful repurchase offer, the variance of returns on the firm increases ($\Delta>0$).

(iii) The information effect on stock price is an increasing function of the repurchasing premium (p), and the fraction of outstanding shares the firm wants to repurchase (f_A).

7. ON PARAMETER ESTIMATION AND TESTABLE HYPOTHESES

7.1 PARAMETER ESTIMATION

Any test of the model presented in this paper requires the estimation of the various parameters in the valuation equations. Some of the parameters are observable, while for others we would need to calculate implied estimates. Two estimation techniques are suggested, and the associated problems with each method are discussed.

The value of stock and debt before the announcement of the repurchase offer can be written as:[46]

$$S^B = f(V_B, \sigma_B, M, R, r_f) \qquad (11)$$
$$D^B = V^B - S^B$$

The value of stock and debt after announcement and before expiration of the repurchase offer can be written as:[47]

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[46] See Section 3, equation (1) for specification of f.

[47] See Section 3, equation (6) for specification of g.

$$S^A = g(V^A, \sigma_{A1}, \sigma_{A2}, M, T, r_f, p, f_E, T_E) \qquad (12)$$
$$D^A = V^A - S^A$$

The value of stock and debt after the expiration of the repurchase offer can be written as:

$$S^A = f(V_A, \sigma_{A2}, M, T, r_f) \qquad (13)$$

The parameters that are directly observable are S^B, S^A, D^B, D^A, $r_f p$, f_E, and T_E. The parameters that need to be estimated are firm value, firm standard deviation, debt face value, and debt maturity.[48] The estimation of these four parameters requires four equations. Since the model presented here provides a maximum of two equations at any one point in time, other theories will have to be used to generate the remaining two equations. Therefore, any test based on these estimates will be a joint test of the validity of both theories.

Duration is one method which could be used to calculate M and T at every point in time, and then equations (11), (12), and (13) yield implied values of $_B$, $_{A1}$ and $_{A2}$. A second technique used the price of options on the stock of the firm

[48] In cases where the firm has a very simple capital structure, M and T can be estimated using data sources such as annual reports or Compustat.

to calculate the implied values of $\sigma_B, \sigma_{A1}, \sigma_{A2}$, M and T.[49] Each technique is now discussed in greater detail:

(i) Estimation using Duration: Duration is a measure of the average maturity of a stream of payments. M and T are calculated using the expressions:

$$T = \sum_{1}^{N} tI^{e-it}/D \qquad (14)$$

and $M = De^{iT}$ (15)

where N = Maturity of the longest life liability

I_t = Total payment in time t to all securities (except equity)

D = Total current value of all securities in the firm's capital structure (except equity and current liabilities)

and i is that interest rate that solves $D = \Sigma I_t e^{-it}$ (16)

V^i can be obtained by using $V^i = S^i + D^i$, i=A,B, and σ_B, σ_{A1}, and σ_{A2} can be obtained by solving eqns. (11), (12), and (13).

The major problem with using duration is that there is no a priori reason to believe that the duration as calculated in eqn. (14) is the correct substitute for T. In addition specification of N, and I_t will require a number of simplifying assumptions.

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[49] Equations that relate call (and put) option prices to the above parameters are derived using techniques that are identical to those used in this paper. Therefore, tests using estimates that are obtained by this method would be preferable to those using estimates based on duration.

(ii) Estimation using Option Prices: Options on stocks can be valued using the compound option approach developed by Geske (1977). In that framework, the value of call (and put) options can be determined as functions of the various parameters in eqns. (11), (12), and (13). Therefore, the stock price, and the prices of three options on the stock can be used to obtain implied values of V^A, V^B, σ_B, σ_{A1}, and σ_{A2}. An alternate method is to use the stock price, the variance of returns on the stock, and the prices of two options on the stock to obtain implied parameter values.[50] The major advantage of this approach is that it gets around the problem of having to collect data on prices of long-term debt, preferred stock, and convertible debt.

7.2 TESTABLE HYPOTHESES

In this section we present testable implications of the signaling hyposthesis:

H1) Signal on Firm Value: It was argues in earlier sections of the paper that a repurchase announcement cannot convey a value decreasing signal. Therefore, one implication of the model is that there is an increase in firm value at the date of announcement of the repurchase, i.e. $V^A > V^V$ or $\sigma_V > 0$.

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[50] The variance of returns on the stock can be estimated using log relatives. The expression relating σ_s to σ_V is $\sigma_s = (\partial S/\partial V)(V/S)\sigma_V$.

H2) Signal on Firm Riskiness: The announcement of the re-purchase is accompanied by a decrease in the variance of returns on the firm, i.e.σ_{A1} or σ_B or $\delta\sigma$ >0.

H3) Asset Structure Change at Repurchase Expiration: If the firm uses low variance assets to finance the repurchase, the expiration of the repurchase offer will be accompanied by an increase in the variance of returns on the firm, i.e. $\sigma_{A2}>\sigma_{A1}$ or $\Delta > 0$.

H4) Effect of the Repurchase on 'Permanent' Risk: This hypothesis is concerned with the comparison of variances before announcement, and after expiration of the repurchase offer. In general, the two would be different, and the analysis in Section 4 indicates that for 'reasonable' increases in firm value, the repurchase results in a decrease in the 'permanent'risk of the firm, i.e. $\sigma_{A2} < {}_B$ or $(1 + \delta_\sigma)$ $(1 + \Delta) < 0$.

H5) Undersubscription and its effect on riskiness: It has been assumed that the information conveyed by the announcement of the repurchase depends on the repurchase premium (p), and the fraction of outstanding shares the firm wants to repurchase (f_A). This implies that δ_σ and Δ_{max} are determined by p, and f_A. For an under-scribed offer, the fraction of shares repurchases (f_E) will be less than F_A. This implies that the actual asset structure change (Δ) will be less than Δ_{max}. Therefore, σ_{A2}/σ_B will be an increasing function of f_E/f_A.

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