12-2022

Algebraic Reasoning and Conceptual Understanding: A Mixed Methods Comparison of In-Person and Virtual Classroom Strategies

Erneice J. Jackson

Algebraic Reasoning and Conceptual Understanding:

A Mixed Methods Comparison of In-Person and Virtual Classroom Strategies

by

Erneice J. Jackson

A Dissertation submitted to the Education Faculty of Lindenwood University

In partial fulfillment of the requirements for the

Degree of

Doctor of Education

School of Education

Algebraic Reasoning and Conceptual Understanding:

A Mixed Methods Comparison of In-Person and Virtual Classroom Strategies

by

Erneice J. Jackson

This dissertation has been approved in partial fulfillment of the requirements for the degree of

Doctor of Education

at Lindenwood University by the School of Education

Sherrie Wisdom ____________ 12/02/2022

Dr. Sherrie Wisdom, Dissertation Chair | Date

____________ 12/02/2022

Dr. Nevels Nevels, Committee Member | Date

____________ Ed.D. 12/02/2022

Dr. Monica Perry, Committee Member | Date

Declaration of Originality

I do hereby declare and attest to the fact that this is an original study based solely upon my own scholarly work here at Lindenwood University and that I have not submitted it for any other college or university course or degree here or elsewhere.

Full Legal Name: Erneice J. Jackson

Signature: ______________________ Date: __12/02/2022__

Acknowledgement

To accomplish a goal that one made in their teens is a dream that is becoming a reality. There are so many people to thank along this journey. To my dissertation committee, I would like to thank Dr. Sherrie Wisdom for guiding me through this process. You always reduced my stress level and always pointed me in the right direction. To committee member Dr. Nevels Navels, you have seen me grow from a 1st year teacher to a doctoral graduate. Thank you for being there every step of my professional journey. And, to committee member Dr. Monica Perry, thank you for your words, meetings, and encouragement, professionally and mentally.

I want to acknowledge my children, Brittany and Cassidy, for encouraging me not to give up when life had other plans; programs like Upward Bound, intended for first generation college students; school districts like Hazelwood School District, designed for developing my growth mindset about teaching mathematics; and universities like Lindenwood University. whose professors aimed at keeping me grounded throughout this process.

Abstract

Algebraic reasoning is the beginning school of thought to critical thinking. Employers are looking for this 21st century skill. The purpose of this research was to investigate equity in mathematics education using the NCTM Teaching and Learnings Beliefs Survey. Four area were studied: the number of years in education, the degree earned, the grade level taught in education, and the number of years in an educator's teaching position. A mixed methods inventory was used. Most results were not rejected in this study. Two statements in the survey warranted a discussion. Recommendations were made for further research.

Key Words: equity, COVID, student engagement, algebraic reasoning, critical thinking, professional development

.

Table of Contents

List of Tables

List of Figures

Chapter One: Introduction

Purpose of Dissertation

The purpose of this study is to investigate the equity in mathematics education through the instructional strategies used to increase students' conceptual understanding of algebraic reasoning in K – 12 classrooms. "Low algebraic reasoning abilities have been a long-standing issue in mathematics education. When a student develops algebraic reasoning, one must be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts" (Friel et al., 2001, p. 2). Students face considerable difficulties in moving from static arithmetic to dynamic algebraic situations, from concrete objects to formal symbols, and from specific to generalized thinking (PISA, 2003; Sfard & Linchevski 1994). Research findings indicated that a high percentage of students are unable to cope with algebraic letters as unknown or generalized numbers (Kuchemann 1981; Macgregor & Stacey 1997), find change and relationship properties (PISA 2003), and apply different algebraic representations (Kramarski 2004). In particular, PISA (2003) found that a high percentage of 15-year-old students have difficulties dealing with the complexity of problem solving in various procedural and real-life tasks, and in explaining their algebraic reasoning.

Compared to the 77 other education systems in the PISA (2018), the U.S. average mathematics literacy score was lower than the average. The U.S. average score was lower than the Organization for Economic Co-operation and Development (OECD) average score, and compared to the 36 other OECD members, the U.S. average in mathematics literacy was lower than the average in 24 education systems (PISA, 2018). Male 15-year-olds scored higher than their female peers and White and Asian students in the United States scored higher than the overall U.S. average in mathematics literacy, while Hispanic and Black students scored lower and students in U.S. public schools with the highest levels of poverty (75% or more of students eligible for Free or Reduced-Price Lunch) scored, on average, 50 points lower than the overall U.S. average in mathematics literacy (PISA, 2018).

According to the National Council of Teachers of Mathematics (2008), excellence in mathematics education rests on equity. Skovsmose and Valero (2001, 2002) stated that mathematics education generates selection, exclusion, and segregation of students along the lines of gender, race, language, and socioeconomic status. Most advanced math classes do not include students of color and are homogeneous. Past math classes have stuck to the procedural aspects of mathematics, leaving most students to try to memorize formulae and rules. Recent research has shown that mathematics education has taken a social turn (Lerman, 2000, 2006), resulting in a growing interest in exploring the social aspects of mathematics, as well as the socio-cultural, historical, and socio-political contexts that influence mathematics teaching and learning, in part in an effort to attend to issues of culture, race, and power in mathematics education (Atweh et al., 2001; Skovsmose & Valero, 2002; Valero & Zevenbergen, 2004).

Through this study, the researcher hopes to provide a foundation to improve teacher quality, increase student's conceptual understanding of algebraic reasoning and continue to promote critical thinking skills needed in the workforce.

Rationale

The National Assessment of Educational Progress (NAEP) is a congressionally mandated project administered by the National Center for Education Statistics (NCES) within the U.S. Department of Education. The NAEP mathematics assessment measures students' knowledge and skills in mathematics and their ability to solve problems in mathematical and real-world contexts (National Assessment of Educational Progress [NAEP], 2019). According to the NAEP report card (2019), 28 to 54% of fourth graders and 21 to 47% of eighth graders performed at or above NAEP Proficient in mathematics, and ranged from 28% to 54%. The average mathematics score of 150 for 12th grade students was not significantly different compared to 2015, the previous assessment year, nor was it significantly different compared to 2005 (NAEP, 2019). Students are losing reasoning skills as they advance to their higher grade levels.

Algebraic reasoning skills can affect a student's ability to enter into the workforce as an adult. Some of the most important skills needed in today's job market are: (1) critical thinking – minds that can evaluate, the ability to discern what information is trustworthy, viewing a problem from all angles and understanding how to analyze and evaluate the information before a decision is made; (2) problem solving - can approach a dilemma from various angles and find out-of-the-box solutions and (3) complex decision making – analyzing data to make intelligent decisions, the ability to take input from the data while considering how decisions can impact the broader community (Curtain, 2018; Marr, 2019; Maryville University, 2022). Algebraic reasoning supports these skills, once a child enters the educational system, and continues throughout their lives.

Distance learning (online learning, virtual learning) is an increasingly preferred option of educating K – 12 students, due to COVID-19. Virtual learning is defined as learning that can functionally and effectively occur in the absence of traditional classroom environments (Simonson & Schlosser, 2006). Students must be behaviorally, emotionally, and cognitively engaged in a virtual classroom (Fisher et al., 2021). Online learning requires teachers to have a basic understanding of using digital forms of learning and very often, teachers have a very basic understanding of technology or do not even have the necessary resources and tools to conduct online classes (Gautam, 2020). Teachers, especially math teachers, will need training on what tools to implement in an online platform.

Gender, income and socio-economic status, ethnicity, indigeneity, culture, language, and geographical location are just some of the factors that can contribute to inequitable opportunities and outcomes in education (Wood et al., 2011). For example, Maccoby and Jacklin's (1974) review of close to 1600 studies of gender differences concluded that boys were better in mathematics and physical sciences, whereas girls were better in reading and writing. In fact, mathematics was traditionally stereotyped as a male domain and societal influences tended to suggest that mathematical learning was not particularly appropriate for girls (Damarin, 1995; Fennema, 2000; Leder, 1992). Even today, current mathematic curriculum is still being written to favor the dominant white male by keeping this group in the enter and the marginalized groups on the outside (Grant, 2020). The existing research consistently demonstrates a positive correlation between students' socioeconomic status and their academic achievement levels from international large-scale assessments to school level assessments.

Participants

The participants in this research include 31 elementary and middle/high school math teachers on record from two school districts. These teachers were selected because every grade level teaches algebraic reasoning in some type of way.

The first research site is a school district located in a suburban community in the Midwest. The school district caters to over 11,000 students from pre-school to 12th grade. The district has seven primary schools (PreK-2), six elementary schools, (3-5), two sixth grade centers, two middle schools (7-8), two high schools, a STEAM middle school, and a STEAM high school. The second site is a school district in a city in the Midwest, which serves over 2000 students. The district consists of a preschool building, four neighborhood elementary schools, one middle school and one high school. The participants' responses were taken collectively, to protect participants from being identified.

Teachers will take several multiple-choice inventories and can add additional comments or reflections. Although no observations will take place, teachers will have the ability to describe students' actions and reactions, when completing inventories created by the researcher.

Limitations

Teacher Burnout Due to COVID

Due to the COVID-19 (coronavirus disease) pandemic, many school districts implemented alternative teaching approaches, including socially distanced classrooms, hybrid teaching, or 100% virtual instruction. Districts pushed teachers to learn new virtual instruction pedagogy and platforms and made teachers the first resource for parents using district instructional technology (Pressley, 2021). These new demands added to teachers' already full workloads, which even before COVID-19 affected teacher burnout and anxiety (Ferguson et al., 2012). COVID-19–related anxiety, anxiety about teaching demands, parent communication, and administrative support were some of the reasons why teachers left the classroom (Pressley, 2021). For those who stayed, participating in a study might not be a priority.

Small Sample Size

Teacher burnout and other factors can cause a small sample size. A sample that is smaller than necessary would have insufficient statistical power to answer the hypothesis representing the primary research question, and a statistically nonsignificant result could merely be because of inadequate sample size (Andrade, 2020). The researcher altered the inventories, due to the small sample size.

Definition of Terms

Algebraic Reasoning: 1) "A psychological process involved in solving problems that mathematicians can easily express using algebraic notation" (Carraher & Schliemann, 2007, p. 5); 2) "the use of any of a variety of representations that handle quantitative situations in a relational way" (Kieran, 1996, p. 4); or 3) to be able to reflect on, make sense of, and communicate about general numerical procedures (Kieran, 1992)

Behavioral Engagement: The quality of students' participation in the classroom and school community, evidenced by students' effort, persistence, participation, and compliance with school structures (Davis et al., 2012)

Critical Thinking: A careful way of thinking directed at a specific goal (Stanford Encyclopedia of Philosophy, n.d.). This definition will be expounded upon later in Chapter Two.

Conceptual, for this research study, is relating to or consisting of concepts (Merriam-Webster, n.d.a).

Cognitive Engagement: The quality of students' engagement, "whereas sheer effort refers to the quantity of their engagement in the class" (Pintrich, 2003, p. 105). The inclusion of cognitive engagement makes an important distinction between students' efforts to simply do the work and effort that is focused on understanding and mastery (Fredricks et al., 2004; Greene et al., 2004)

Digital competence: Digital competence is a combination of knowledge, skills and attitudes with regards to the use of technology to perform tasks, solve problems, communicate, manage information, collaborate, as well as to create and share content effectively, appropriately, securely, critically, creatively, independently and ethically (Skov, 2016, line 7).

Distance/Virtual learning: A method of study where teachers and students do not meet in-person, but use the Internet, email, or mail, etc., to have classes (Merriam-Webster, n.d.b)

"Doing school": Windschitl (2019) defines doing school as rote and shallow learning performances, which students and teachers give to each other to signify that they are accomplishing normative classroom tasks.

Inequality(ies): 1: the quality of being unequal or uneven: such as 1) social disparity or 2) disparity of distribution or opportunity (Merriam-Webster, n.d.c)

Learning: 1**)** The act or experience of one that learns; 2**)** knowledge or skill acquired by instruction or study; 3**)** modification of a behavioral tendency by experience such as exposure to conditioning (Merriam-Webster, n.d., n.p.)

Levels of Demand: Characteristics of mathematical tasks according to the NCTM. 1) Lowe-level demand tasks (memorization). Tasks reproduce previously learned facts, rules, formulas, definitions or committing them to memory, cannot be solved with a procedure, has no connection to concepts or meaning that underlie the facts, rules, formulas, or definitions. 2) Lower-level demands (procedures without connections). Tasks are algorithmic, require limited cognitive demand, have no connection to the concepts or meaning that underlie the procedure and focus on producing correct answers instead of understanding. 3) Higher-level demands (procedures with connections). Tasks use procedure for deeper understanding of concepts, uses broad procedures connected to ideas instead of narrow algorithms, has different representations and requires some degree of cognitive effort. 4) Higher-level demands (doing mathematics). Tasks require complex non-algorthmic thinking, requires students to explore and understand the mathematics, demands self-monitoring of one's cognitive process and requires considerable cognitive effort and may involve some level of anxiety because the solution path is not clear (as cited by Leinwand et al., 2014).

Mathematical Task: A mathematical task has been defined as a single problem or a set of problems that focuses student attention on a mathematical idea (Stein et al., 1996). Mathematical tasks can be designed with the purpose of being "interesting to the students, incorporate a rationale for them to engage, provide some challenges, reduce the risk of failure, and for which success provides the motivation for further engagement" (Sullivan et al., 2013, p. 38).

Relational/Emotional Engagement: Skinner and Belmont (1993) defined emotional engagement as students' feelings of interest, happiness, anxiety, and anger during achievement-related activities. Sciarra and Seirup (2008) defined emotional engagement as the extent to which students feel a sense of belonging "and the degree to which they care about their school" (p. 218).

Understanding: Merriam-Webster (n.d.) defines understanding as: 1) a mental grasp 2) the power of comprehension especially the capacity to apprehend general relations of particulars; 3) the power to make experience intelligible by applying concepts and categories.

Research Questions

RQ1: Does the number of years in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

RQ2: Does the degree earned in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

RQ3: Does the grade level taught in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

RQ4: Does the number of years in an educator's teaching position affect the results of the NCTM's Teaching and Learning Beliefs Survey?

Summary

This study was designed to examine the instructional strategies used in classrooms that promote conceptual understanding of algebraic reasoning. Examining each grade level on how they utilize students' algebraic reasoning skills can give insight on where the problem begins and where to fix it. Students in the United States are losing algebraic reasoning skills as they enter higher grades. The workforce needs critical thinkers and algebraic reasoning is the beginning.

COVID changed how the world facilitated education and the increase of virtual learning changed overnight. Most public-school districts lost attendance and engagement, due to virtual learning. Other factors that were out of districts' control (parental income, homes, meals) added to an already full plate for teachers.

When the United States finally turned to some form of "normalcy," teacher burnout increased dramatically. Teachers were slow to participate and many of the inventory questions were changed from essay type to multiple choice. In this study, teachers were asked a series of multiple-choice questions describing algebraic reasoning in the in-person and virtual classrooms. The study included 18 elementary teachers, five middle school teachers, and eight high school teachers. Data were collected over the course of one school semester.

Chapter Two: Review of Literature

Algebraic Reasoning

The word algebra derives from the book “Al-Kitab al-Jabr wal-Muqabala,” meaning “The Compendious Book on Calculation by Completion and Balancing,” written by the Persian mathematician Muhammad ibn Musa al-Khwarizmı (approx. 780-850 CE). This was the first book dealing systematically with solving linear and quadratic equations, based on earlier work by Greek and Indian mathematicians (Lempp, 2008). Algebra follows the study of arithmetic, whereas arithmetic deals with numbers and operations, algebra generalizes this from computing with “concrete” numbers to reasoning with “unknown” numbers (“variables”, usually denoted by letters) using equations, functions, etc. (Lempp, 2008). Algebraic reasoning is also a process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways (Kaput, 1998; 1999). Driscoll (1999, p. 2) affirms that critical to algebraic thinking is the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules.

Algebra has always been around. According to Lempp (2008), most of the attention in algebra centered around solving equations in one variable until the Renaissance. How to solve quadratic equations by completing the square was already known to the Babylonians. The Italian mathematicians Scipione del Ferro and Niccol`o Fontana Tartaglia independently, and the Italian mathematician Lodovico Ferrari, gave the first general solution to the cubic and quartic equations, respectively. The French mathematician Francois Viete is credited with the first attempt at giving the modern notation for algebra we use today; before him, very cumbersome notation was used. The 18th and 19th century then saw the birth of modern algebra in the other sense mentioned above, which also led to much more general techniques for solving equations (Lempp, 2008).

In 1923, Thorndike et. al (1923) published The Psychology of Algebra, in which he applied his "bond" theory to the learning of algebra. He is credited with bringing a systematic approach to research in the learning of algebra, including a careful analysis of the nature of algebraic tasks (as cited in Wagner & Parker, 1993). From 1930 to 1945, research in education declined, the nation focused on issues of survival surrounding the Great Depression and World War II. It was in the 1960s that mathematics educators, with academic backgrounds in higher mathematics and teaching experience in secondary schools, began to shift the focus of research toward conceptual understanding.

Moses (1997) stated that algebra is a way of thinking, a method of seeing and expressing relationships, and a way of generalizing patterns in everyday activities.

In the past, algebra and its concepts were left for middle and high school students. This method has been unsuccessful, in terms of student achievement (U.S. Department of Education & NCES. 1998a, 1998b, 1998c). It is now widely accepted that preparing elementary students for the increasingly complex mathematics of the new century will require a different type of school experience; specifically, one that cultivates habits of mind that attend to the deeper underlying structure of mathematics (Kaput, 1999; Romberg & Kaput, 1999). Research has led to the fact that algebraic reasoning can simultaneously emerge from and enhance elementary school mathematics (NCTM, 2000). Lempp (2008) states that the integration of algebraic reasoning into primary grades offers an alternative that builds the conceptual development of deeper and more complex mathematics into students' experiences from the beginning.

Common Core Mathematical Standards

The purpose of the Common Core Mathematical Standards was to build a mathematics curriculum that is more focused, coherent, clear and specific (Common Core States Standards Initiative, n.d.). The Standards are designed, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas (see Table 1).

Table 2 displays an overview of the high school standards by Common Core.

Table 1

Overview of Common Core Standards by Domain K – 8

K	1st	2nd	3rd	4th	5th	6th	7th	8th
Counting & Cardinality • Know number names and the count sequence. • Count to tell the number of objects. • Compare numbers. Operations & Algebraic Thinking • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. Number & Operations in Base Ten • Work with numbers 11–19 to gain foundations for place value.	Operations & Algebraic Thinking • Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Work with addition and subtraction equations. Number & Operations in Base Ten • Extend the counting sequence. • Understand place value. • Use place	Operations & Algebraic Thinking • Represent and solve problems involving addition and subtraction. • Add and subtract within 20. • Work with equal groups of objects to gain relationship for multiplication. Number & Operations in Base Ten • Understand place value. • Use place value understanding and properties of operations to add and subtract. Measurement & Data • Measure and estimate lengths in standard units. • Relate addition and subtraction to length. • Work with time and money.	Operations & Algebraic Thinking • Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship between multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations, and identify and explain patterns in arithmetic. Number & Operations in Base Ten • Use place value	Operations & Algebraic Thinking • Use the four operations with whole numbers to solve problems. • Gain familiarity with factors and multiples. • Generate and analyze patterns. Number & Operations in Base Ten • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic.	Operational & Algebraic Thinking • Write and interpret numerical expressions. • Analyze patterns and relationship. Number & Operations in Base Ten • Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths. Number & Operations – Fractions • Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions. Measurement & Data • Convert like measurement units within a given measurement system.	Ratios & Proportional Relationships • Understand ratio concepts and use ratio reasoning to solve problems. The Number System • Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Compute fluently with multi-digit numbers and find common factors and multiples. • Apply and extend previous understandings of numbers to the system of rational numbers. Expressions & Equations • Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationship	Ratios & Proportional Relationships • Analyze proportional relationship and use them to solve real-world and mathematical problems The Number System • Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Expressions & Equations • Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations. Geometry • Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure,	The Number System • Know that there are numbers that are not rational, and relationship them by rational numbers. Expressions & Equations • Work with radicals and integer exponents. • Understand the relationship between proportional relationship, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations Functions • Define, evaluate, and compare functions. • Use functions to model relationship between quantities. Geometry • Understand congruence and similarity using physical models, transparencies, or geometry software.

Measurement & Data • Describe and compare measurable attributes. • Classify objects and count the number of objects in categories. Geometry • Identify and describe shapes. • Analyze, compare, create, and compose shapes	value understanding and properties of operations to add and subtract. Measurement & Data • Measure lengths indirectly and by iterating length units. • Tell and write time. • Represent and interpret data. Geometry • Reason with shapes and their attributes	• Represent and interpret data. Geometry • Reason with shapes and their attributes.	understanding and properties of operations to perform multi-digit arithmetic. Numbers and Operations – Fractions • Develop understanding of fractions as numbers. Measurement & Data • Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. • Represent and interpret data. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	Number & Operations – Fractions • Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimal fractions. Measurement & Data • Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Represent and interpret data.	• Represent and interpret data. • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. Geometry • Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.	between dependent and independent variables. Geometry • Solve real-world and mathematical problems involving area, surface area, and volume. Statistics & Probability • Develop understanding of statistical variability. • Summarize and describe relationship.	area, surface area, and volume. Statistics & Probability • Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models.	• Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. Statistics & Probability • Investigate patterns of association in bivariate data.

	• Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures Geometry • Reason with shapes and their attributes	• Geometric measurement: understand concepts of angle and measure angles. Geometry • Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	

Source. Common Core State Standards (n.d., n.p.).

Table 2

Overview of High School Common Core Standards

Number & Quantity	Algebra	Functions	Geometry	Statistics & Probability
The Real Number System • Extend the properties of exponents to rational exponents • Use properties of rational and irrational numbers. Quantities • Reason quantitatively and use units to solve problems The Complex Number System • Perform arithmetic operations with complex numbers • Represent complex numbers and their operations on the complex plane • Use complex numbers in polynomial identities and equations Vector and Matrix Quantities • Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications	Seeing Structure in Expressions • Interpret the structure of expressions • Write expressions in equivalent forms to solve problems Arithmetic with Polynomials and Rational Expressions • Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions Creating Equations • Create equations that describe numbers or relationships Reasoning with Equations and Inequalities • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and	Interpreting Functions • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations Building Functions • Build a function that models a relationship between two quantities • Build new functions from existing functions Linear, Quadratic, and Exponential Models • Construct and compare linear, quadratic, and exponential models and solve problems • Interpret expressions for functions in terms of the situation they model Trigonometric Functions • Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions	Congruence • Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions Similarity, Right Triangles, and Trigonometry • Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Apply trigonometry to general triangles Circles • Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles Expressing Geometric Properties with Equations • Translate between the geometric description and the equation for a conic section • Use coordinates to prove simple geometric theorems algebraically	Interpreting Categorical and Quantitative Data • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models Making Inferences and Justifying Conclusions • Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiments and observational studies Conditional Probability and the Rules of Probability • Understand independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model Using Probability to Make Decisions • Calculate expected values and use them to solve problems • Use probability to evaluate outcomes of decisions

	inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically	• Prove and apply trigonometric identities	Geometric Measurement and Dimension • Explain volume formulas and use them to solve problems • Visualize relationships between two-dimensional and three-dimensional objects Modeling with Geometry • Apply geometric concepts in modeling situations

Source: Common Core State Standards (n.d., n.p.)

Common Core also included Standards for Mathematical Practices. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students (Common Core States Standards Initiative, n.d.). The Mathematical Practices are (1) Make sense of problems and preserve in solving them, (2) Reason abstractly and quantitatively, (3) Construct viable arguments and critique the reasoning of others, (4) Model with mathematics, (5) Use appropriate tools strategically, (6) Attend to precision, (7) Look for and make use of structure, and (8) Look for and express regularity in repeated reasoning. The content standards set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics (Common Core States Standards Initiative, n.d.). Common Core initiative attempted to apply algebraic reasoning throughout the K – 12 curriculums.

Missouri Learning Standards

The Missouri Learning Standards define the knowledge and skills students need in each grade level and course for success in college, other post-secondary training, and careers. These expectations are aligned to the Show-Me Standards, which define what all Missouri high school graduates should know and be able to do (MODESE, 2022c). In 2014, Missouri legislators passed House Bill 1490, mandating the development of the Missouri Learning Expectations (MODESE, 2022a). In April of 2016, these Missouri Learning Expectations were adopted by the State Board of Education. Groups of Missouri educators from across the state collaborated to create the documents necessary to support the implementation of these expectations.

One of the documents developed is the item specification document, which includes all Missouri grade level/course expectations arranged by domains/strands. It defines what could be measured on a variety of assessments. The document serves as the foundation of the assessment development process and was created by Missouri educators to provide classroom teachers a more descriptive version of the mathematics of the mathematics Grade– and Course-Level Expectations (GLESs and CLEs) (MODESE, 2022a).

Elementary

Relationships and Algebraic Reasoning is a strand in the Missouri Learning Standards (MODESE, 2022b). It covers K-12 learning as follows, in Table 3.

Table 3

Algebraic Reasoning

Grade Level	Algebraic Reasoning Standard
Kindergarten	Understand addition as putting together or adding to, and understand subtraction as taking apart or taking from
1st Grade	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20
2nd Grade	Add and subtract within 20 Develop foundations for multiplication and division
3rd Grade	Represent and solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply and divide within 100 Use the four operations to solve word problems Identify and explain arithmetic patterns
4th Grade	Use the four operations with whole numbers to solve problems Work with factors and multiplies Generate and analyze patterns
5th Grade	Represent and analyze patterns and relationships Write and interpret numerical expressions Use the four operations to represent and solve problems

(MODESE, 2022b)

Middle

Once a student becomes a middle school student, algebraic reasoning is interwoven throughout the standards. Students are to apply knowledge from previous years and utilize it in Ratios and Proportions, Expressions, Equations and Inequalities, Geometry, and Data and Statistics.

High

The Algebra (1 and 2) course number one goal is to develop, understand and model reasoning. The main focus at a workshop held in Hamburg, Germany was how algebra provided a good basis for a variety of types of reasoning: not only deductive reasoning, but also analogical, inductive, and qualitative reasoning (Gust et al., 2003). Gust et al. (2003) wrote that Algebras are important because of two aspects: first, many of the formal mathematical structures we are dealing with in AI and cognitive science, even in the case of logical approaches, are algebras. Second, algebras are a classical tool to represent analogical reasoning.

Expressing Geometry Properties with Equations is a standard within the Missouri Learning Standards (MODESE, 2022b). Students should be able to translate between the geometric description and the equations for a conic section and use coordinates to prove geometric theorems algebraically (MODESE, 2022b). Just like in middle school, the algebraic reasoning is spread throughout the course. Algebraic reasoning carries over into post high school and adulthood transformed into critical thinking skills.

Critical Thinking

Psychologist, Robert Sternberg (1985), defined critical thinking broadly as “the mental processes, strategies, and representations people use to solve problems, make decisions, and learn new concepts" (p. 49). Education professor Michael Scriven and philosopher Richard Paul (2008) defined it as "the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action" (para 2). Furthermore, it is based on universal intellectual values that transcend subject matter divisions: clarity, accuracy, precision, consistency, relevance, sound evidence, good reasons, depth, breadth, and fairness. According to Scriven and Paul (2008), critical thinking is incorporated in a family of interwoven modes of thinking, among them: scientific thinking, mathematical thinking, historical thinking, anthropological thinking, economic thinking, moral thinking, and philosophical thinking.

Linda Elder (as cited in Scriven & Paul, 2008) conceptualizes critical thinking in this way: Critical thinking is self-guided, self-disciplined thinking which attempts to reason at the highest level of quality in a fair-minded way. People who think critically consistently attempt to live rationally, reasonably, empathically (para. 7). They are keenly aware of the inherently flawed nature of human thinking when left unchecked. They strive to diminish the power of their egocentric and sociocentric tendencies. They use the intellectual tools that critical thinking offers – concepts and principles that enable them to analyze, assess, and improve thinking. They work diligently to develop the intellectual virtues of intellectual integrity, intellectual humility, intellectual civility, intellectual empathy, intellectual sense of justice and confidence in reason. They realize that no matter how skilled they are as thinkers, they can always improve their reasoning abilities and they will at times fall prey to mistakes in reasoning, human irrationality, prejudices, biases, distortions, uncritically accepted social rules and taboos, self-interest, and vested interest. They strive to improve the world in whatever ways they can and contribute to a more rational, civilized society. At the same time, they recognize the complexities often inherent in doing so. They avoid thinking simplistically about complicated issues and strive to appropriately consider the rights and needs of relevant others. They recognize the complexities in developing as thinkers, and commit themselves to life-long practice toward self-improvement. They embody the Socratic principle: The unexamined life is not worth living, because they realize that many unexamined lives together result in an uncritical, unjust, dangerous world.

Critical Thinking and Education

Kaput (2000) discussed the key to algebra reform is by integrating algebra in K-12 curriculum, therefore solving three major problems: (1) It opens curricular space for 21st century mathematics desperately needed at the secondary level, (2) It adds a new level of coherence, depth, and power to school mathematics as a habit of mind and as curriculum, and (3) It eliminates the late, abrupt, isolated and superficial high school algebra courses. He also stated that a strands approach that begins early fits well with an inclusive, big-idea strands-oriented approach to the curriculum and democratizes access to powerful ideas (Kaput, 2000). With the introduction of computers and Common Core, mathematics looks different than it did in the past.

In his paper, *Transforming Algebra from an Engine of Inequity to an Engine of Mathematical Power By "Algebrafying" the K-12 Curriculum,* Kaput (2000) discusses the five interrelated forms of reasoning within algebraic reasoning:

1. (Kernel) Algebra as Generalizing and Formalizing Patterns & Constraints, especially, but not exclusively, Algebra as Generalized Arithmetic Reasoning and Algebra as Generalized Quantitative Reasoning.
2. (Kernel) Algebra as Syntactically-Guided Manipulation of Formalisms
3. (Topic-strand) Algebra as the Study of Structures and Systems Abstracted from Computations and Relations.
4. (Topic-strand) Algebra as the Study of Functions, Relations, and Joint Variation.
5. (Language aspect) Algebra as a Cluster of (a) Modeling and (b) Phenomena-Controlling Languages. (p. 4)

The first two includes all of the others, the next two are topic strands of the curriculum and the last reflects algebra as a web of languages. Strand numbers one and two underlie all the others, with number one being based both inside and outside of mathematics and number two done in conjunction with one. Math activities involve generalizations and formalizing in one way or another. The activities a student does in number two are typical in algebra courses and can occur as a result of prior formalizations of situations. These types of manipulations yield general patterns and structures, which is number three form of algebraic reasoning (Kaput, 2000, p. 4). In order to use or communicate generalizations, one needs languages in which to express them, which leads to number 5, which in turn permeates all the others. While number 3 is a school mathematics topic strand occurring nowadays mainly at the advanced levels, it is also an important growing domain of mathematics in its own right - abstract algebra. Topic strand number 4, functions, is more a school mathematics domain, and lives in the world of mathematics more as a general-purpose conceptual tool rather than a branch of mathematics. And the fact that numbers 3 and 4 lie on the opposite sides of deep understanding in mathematics, both appear in algebra. Algebraic reasoning appears throughout all of mathematics and the right application will allow students critical think early in life (p. 2).

Developing critical thinkers has always been the focus of education (De Bono, 1987). Learners' effective thinking can be ensured through ways, such as asking questions requiring higher-order thinking, using writing activities, and applying several strategies (Marzano, 1993). Therefore, curricula for thinking skills, based on a theoretical framework and expected to be developed both within and out of the courses, not only to facilitate learners to transfer effective thinking skills into the other parts of their life, but also to have a positive impact on their academic achievement (Hu et. al, 2011). With society and technology changing so fast, critical thinking is considered and, is of the key skills needed to adapt to the global climate (Hughes, 2014; Yang & Chung, 2009). Studies are conducted for developing teaching or developing critical thinking skills in different levels of formal education (Jackson, 1986; Pena & Almaguer, 2012; Yang & Chung, 2009). The report published by the American Philosophical Association proposes that developing critical thinking skills and dispositions is needed to be regarded as an objective for K-12 curriculum (Facione, 1990). McCall (2011) notes that it is considerably important to provide learning environments, where different perspectives are welcomed and respected, and different opinions are encouraged. Studies indicate that the most effective method in developing critical thinking skills is to teach them directly, as well as providing opportunities to learners for practicing them in other courses (Abrami et. al, 2014; Heijltjes et al., 2014).

According to Pykett (2004), raising more democratic and better citizens in society is a result of providing critical thinking skills in schools. In an educational environment, teachers should ask the students to generate solutions for problems they have encountered, instead of discussing preplanned topics in textbooks; students should discuss their own ideas and opinions about the content that was covered, continually forming their own categorizations about the related content (Paul, 1990). Critical thinking is a form of reasoning in which an individual improves his/her thinking potential through analyses of the problems, issues, and content, along with evaluation and reconstructing processes (Paul & Elder, 2006). Logical thinking is a skill, which is seen during both the preoperational and concrete operational periods of Piaget's cognitive development (Senemoğlu, 2004,). This skill is explained as an individual's problem solving by means of different cognitive operations or reaching principles and codes by abstraction (Korkmaz, 2002). This process requires acquiring all of the ideas, facts and results and putting them in order in a chain (Logical Thinking, 2010). It is one of the sub-stages of problem solving (Howe & Jones, 1993). Logical thinking is "a skill of showing behaviors like using numbers effectively, generating scientific solutions to problems, identifying relations among concepts, classifying, generalizing, expressing in a mathematical formula, calculation, hypothesis, testing and drawing an analogy" (Bümen, 2010, p. 7). Moreover, scientists, mathematicians, accountants, engineers, computer programmers, statisticians, and others are examples of individuals with strong logical intelligence (Demirel, 2009). Research indicates a positive correlation between logical thinking skills and academic success (Johnson & Lawson, 1998). Logical thinking skills are one of the highest predictors of success, as stated by Tobin and Capie (1981). Moreover, it has a significant effect upon self-efficacy and academic success (Lawson et al., 2006).

Post High School

From previous sections, the researcher revealed that algebraic reasoning can begin in the early grades to grow linearly. But for most students, critical thinking classes are taught post high school. One strategy to increase critical thinking is collaborative learning. It is described as "the grouping and pairing of students for the purpose of achieving an academic goal" is beneficial in its promotion of a positive interdependence among students and in its contribution to their oral communication and social interaction skills (Gokhale, 1995, p. 22). The implementation of a collaborative learning environment in college classrooms more closely resembles what graduates should expect to encounter in workplaces (McCormick et al., 2015). Particularly in higher education, research has shown that using problem solving and group learning opportunities increased student involvement in the classroom (Bowen, 2000; Mahalingam et al., 2008). Critical thinking skills are better developed when students are given opportunities to think for themselves as opposed to being guided along step by step (Effects of Computer-Assisted Instruction, n.d.). This active exchange, clarification, and evaluation of ideas within a group setting not only encourages student involvement, it also fosters critical thinking (Gokhale, 1995).

Critical Thinking and Careers

Companies across the United States say it is becoming increasingly difficult to find applicants who can communicate clearly, take initiative, problem-solve and get along with coworkers (Davidson, 2016). Companies have automated or outsourced many routine tasks, and the jobs that remain often require workers to take on broader responsibilities that demand critical thinking, empathy or other abilities that computers cannot easily simulate (Davidson, 2016). Ninety-nine and two tenths' percent of employers surveyed considered critical thinking as an essential skill (25 In-Demand Jobs, 2022). Table 4 contains sampling of jobs that require critical thinking listed in the U.S. Bureau of Labor Statistics (2022) Occupational Outlook Handbook (n.p.).

Table 4

Jobs That Require Critical Thinking

Occupation	Jobs projected through 2030	Entry Level Education Requires	2021 Median Pay	Critical Skills needed
Nurse practitioners	393,300	Master's	$123,780	Working in variety of healthcare settings; provides full range of health care needs
Home Health Aides	4,600,600	High school Diploma	$29,430	Monitor health conditions of people with chronic illness or disabilities & assist them with daily activities
Statisticians	59,800	Master's	$95,280	Analyze data & use computational techniques to solve problems
Logisticians	247,400	Bachelor's	$77,030	Analyze, coordinate, & suggest improvements in an organization's supply chain
Tour/Travel Guides	56,800	High School Diploma	$29,780	Plan, organize, & arrange tailored vacations plans & sightseeing tours

Coaches/Scouts	313,800	Bachelor’s	$38,970	Evaluate & teach amateur or pro-athletes to succeed & improve on past performance
Actuaries	34,500	Bachelor’s	$105,900	Use math & statistics to analyze & economic costs
Mental Health Counselors	402,600	Bachelor’s	$48,520	Diagnose substance abuse behavioral disorders & mental health problems & counsel patients
Athletic Trainers	37,000	Bachelor’s	$498,420	Prevent, diagnose & treat muscle & bone injuries & illness
Software Developers	2,257,400	Bachelor’s	$110,140	Identify problems with software applications and report/correct defects
Phlebotomist	158,400	Post-Secondary nondegree	$37,800	Draw blood from patients with attention to detail & empathy towards patients who may be uncomfortable
Broadcast Technicians	168,300	Associates	$49,050	Set up, operate, maintain, & troubleshoot equipment for media programs
Market Research Analysts	904,500	Bachelor’s	$63,920	Study market conditions & examine potential sales & service opportunities & upgrades
Preschool Teachers	556,000	Associates/Bachelor’s	$30,210	Attend to the needs of younger

				children prior to them entering kindergarten
Social Service Assistants	487,100	High School Diploma	$37,610	Provide clients with tailored services to assist people in therapy & rehabilitation settings
Financial Managers	799,900	Bachelor's	$131,710	Create detailed financial reports & plan for the organization's long-term financial goals
Audiologists	15,800	Doctoral/Professional Degree	$78,950	Diagnose, manage, & treat patients experiencing hearing & balance problems
Veterinarians	101,300	Doctoral/Professional Degree	$100,370	Diagnose, treat, & provide care for animals
Management Analysts	1,032,000	Bachelor's	$93,000	Recommend ways for an organization to improve its operation & efficiency
Education Administrators	56,900	Bachelor's	$90,560	Manage, administer, & prepare budgets & education syllabi in a variety of educational settings
Postsecondary Teachers	1,433,600	Master's/PhD	$79,640	Prepare class syllabi & lesson plans with assessment methods to test student learning
Aircraft Mechanics	168,700	FAA tech training/on-the-job training	$65,550	Troubleshoot, repair, & preform scheduled maintenance on aircraft engines

				& supporting equipment
Computer & information Systems Manager	534,700	Bachelor's	$159,010	Plan, coordinate, & oversee IT related activities in a variety of organizations
Construction Managers	499,400	Bachelor's	$98,890	Coordinate, plan, budget, & oversee construction projects from inception to completion
Dietitians	73,000	Bachelor's	$61,650	Plan & implement food service & nutritional programs in a variety of settings

(25 In Demand Jobs, 2022; U.S. Bureau of Labor Statistics, 2022).

No matter what career is chosen, critical thinking is embedded in them.

Correlation Between Algebraic Reasoning and Creative Thinking

Creative thinking is one of the skills acknowledged as a 21st century skill (Larson & Miller, 2011; Ravitz, et al., 2012; Voogt & Roblin, 2012). Creative thinking incorporates aspects of fluency, flexibility, originality, and elaboration (Torrance, 1962). Research on creative thinking indicates that factors, such as flexible use of time and place, providing appropriate materials, working outside the school building, game-based learning approaches, interaction with teachers and learners, opportunities for peer interaction and awareness of learner needs promote learners' creative thinking skills (Davies, et al., 2013). Further research demonstrates that creative thinking skills can be developed via various methods and techniques, such as brainstorming, SCAMPER, analogies, and collaborative group work (Eragamreddy, 2013; Gregory, et. al, 2013).

Qualifications of Math Teachers

In previous years, if a person wanted to become a teacher, they would attend a teacher program at a college or university, take a qualifying exam and receive professional development throughout their career. A highly qualified math teacher must have state certification in a certain subject area and have completed a teacher program. States are afforded discretion in how teachers become highly qualified (Boyd et al., 2010). The United States has struggled to recruit and retain effective math teachers and this problem is more acute in schools serving high poverty student populations (Boyd et al., 2006; Boyd et al., 2008; Hanushek et al., 2004). School districts have employed a variety of strategies, including paying a one-time signing bonus or a subject-area bonus (Boyd, et. al, 2010) and allowing experienced teachers to keep their years when they change districts. Another strategy, alternative-route certification, was created to expand the pool of math teachers. For example, the New York City Teaching Fellows Program provided nearly 12,000 new teachers to New York City schools from 2003 to 2008 (Boyd, et. al, 2010). A teacher residency program called Math for America has been directing substantial effort to the recruitment and preparation of highly qualified math candidates (Boyd, et. al, 2010).

Professional Development

Almost every teacher participates in professional development. Research articles shout about the success of a particular method or program that appears practically monthly, and practitioner magazines burst with accounts of the phenomenal improvements in teacher knowledge and skills that result (Hill, 2009). But teachers have little use of their learning experiences. In 1999-2000, the National Center for Education

Statistics (NCES) data showed that just over half of respondents to an NCES survey reported spending a day or less in professional development over the past year; only a small minority reported attending four or more days within the past year (2001). This generally low rate of participation closely matches many states' re-licensure requirements, typically 15 days over a five-year period (NASDTEC 2004), suggesting that most teachers do the bare minimum required under law. When queried about the impact of the past three years of professional development experiences, less than a quarter, on average, reported that professional development affected their instruction (Horizon Research, 2002). Professional development reaching regular teachers through district contracts, regional conferences, and similar means can be quite poor and despite a number of high-quality programs and sessions, others covered math only superficially, contained mathematical ambiguities and errors, or provided inaccurate information about student learning (Hill, 2009).

One cause could be the variability in the capacity of providers. Some mathematics professional development providers stated that providing professional development for math teachers is not their only responsibility, and when given a math assessment, scored below the 50th percentile of the teacher sample (Hill, 2009). Another issue is the problem of transfer. In many cases, the activities were imported into classrooms without the mathematics they were meant to represent; in others, the math was present but distorted (Hill, 2008). Too much professional development can actually decrease instructional coherence. District officials have more than once expressed frustration because professional development advice and supplemental materials undermine district-adopted curriculum and instructional approaches (Hill, 2009).

Recognizing that teacher education is a business in the United States is the first step to repairing professional development. Estimates place professional development spending at between 1% and 6% of district expenditures (Hertert, 1997; Killeen et al., 2002; Odden et al., 2002; Miles, 2003). The National Science Foundation and U.S. Department of Education Math-Science Partnerships spent nearly $1.2 billion (NSF, 2007) on mathematics and science learning for preservice and in service teachers between the years 2002 and 2007. These numbers do not account for professional development paid for by the teachers. Economists often examine markets from four key perspectives: supply, demand, information, and efficiency which are all useful in delineating the challenges facing efforts to reform continuing teacher education (Hill, 2009).

Supply means all available professional development opportunities, analogous to the amount of oil on the world market or the number of widgets produced by manufacturers (Hill, 2009). This leads to products that are low-quality, offering teachers only quick fixes and, in some worst cases, misinformation (Hill, 2009). Demand means the average consumer’s desire for professional development and related programs (Hill, 2009). These opportunity costs, coupled with a misguided formal incentive structure, mean that demand for high-quality professional development is typically weak (Hill, 2009). Information about product quality is the glue that holds markets together and it allows consumers to make wise buying decisions and also informs suppliers about a product’s sales potential and price (Hill, 2009). Teachers gamble on whether professional development program A or B will improve their ability to connect with students, deliver content, and enhance learning (Hill, 2009). Efficiency asks whether teachers have access to the professional learning they need (Hill, 2009). Most districts pick what they think their teachers need without asking.

Gersten et al. (2014) conducted a study to answer the question: What does casual research say are effective math professional development interventions for K-12 teachers aimed at improving student achievement? Out of the 643 professional development approaches studied, only five were determined to meet the What Works Clearinghouse standards: two had statistically significant positive effects, such as intensive math content courses accompanied by follow-up workshops (McMeeking et al., 2012), and lesson study focused on linear measurement model of fractions (Perry & Lewis, 2011). One had limited effects, such as cognitively guided instruction (Carpenter et al., 1989; Jacobs et al., 2007), and two had no discernible effect, such as America's Choice (Garet, et al, 2010, 2011) and Pearson Achievement Solutions (Garet, et al, 2010, 2011).

COVID

Severe acute respiratory syndrome coronavirus 2 (Corona Virus Disease 19, as defined by the World Health Organization in February 2020) was discovered during the recent epidemic of pneumonia in January 2020 (Zhou, Yang, Wang, et. al, 2020; Wu et. al, 2020). Since then, the virus has spread all over the world, and as of 20 May 2020, it has infected 4,806,299 people, and caused 318,599 deaths (World Health Organization, 2020).

Being a respiratory disease, COVID-19 was spread through the air, which meant schools, colleges, universities, restaurants, and other social places were closed. Daily essentials, such as groceries, gas, and other industries decreased during the pandemic. Man is a social animal and social relations and the social interactions are integral to human civilization; but, due to the rapid pandemic spread of the virus and the increase of social distancing measures, this web of relationships was severely impacted (Singh & Singh, 2020). If there is absence of such deep meaningful connections it leads to stressful states of anxiety both in body and in mind such as loneliness, anxiety drives, depression, panic states, mental disorders, health hazards, and many other issues (Singh & Singh, 2020).

Effects on Education

The Institute of Educational Sciences conducted the 2020-2021 National Teacher and Principal Survey [IES-NTPS] (2022). The sample population included 9,920 public and public charter school principals, 3,000 private school principals, 68,300 public school teachers and 8,000 private school teachers. Topics that were covered in the survey included changes to instruction, real-time interactions, support and resources, computer distribution and internet access. The results of the survey stated that:

- 77% of public school moved to online distance learning formats while private schools who applied the option reported 73%.
- 83% of public-school teachers reported that all or some of their classes normally taught in person moved to an online distance-learning format.
- 63% of private school teachers, during the pandemic, used scheduled real-time lessons that allowed students to ask questions through a video or an audio call compared to 47% of public-school teachers.
- 61% of private school teachers had real time interactions with over 75% of their students where public school teachers had 32% of real-time interactions with their students

- Private school principals, 78%, somewhat or strongly agreed that they had the support and resources they needed to be effective at a higher rate than public school principals, 74%.
- Before COID, 23% of public-school principals reported that the schools assigned a computer or digital device that each student could take home at a higher rate than private school principals, 14%
- During COVID, 45% of public-school principals reported assigning computers or digital devices to all students than 20% of private school principals
- During COVID in spring of 2020, 58% of private school principals reported that all students in their school had home internet access verses 4% of public-school principals
- 61% of public-school principals sent home hotspots and other devices to students at home compared to 9% of private school principals.
- 52% of public-school principals in the city and 49% of suburban schools worked with internet providers to help students access the internet at home
- 47% of public-school principals in towns and 46% of rural schools offered spaces where students could access free WiFi (IES-NTPS, 2022)

A study comparing Indiana children in grades three through eight who switched from in-person to virtual learning, experienced large, negative effects in math and ELA (Fitzpatrick et al., 2020). Sal Khan reported that distance learning approaches did not work for younger students (as cited in Freedberg, 2020). The Illinois Department of Education recommended that primary school children have a maximum of 60 to 120 minutes per day in remote learning, representing a fraction of a regular school day (Illinois State Board of Education, 2020).

The pandemic mostly had an impact on low-income students of color. Lower income students are less likely to have access to high quality remote learning or to a conducive learning environment, such as a quiet space with minimal distractions, devices they do not need to share, high-speed internet, and parental academic supervision (Auxier & Anderson, 2020). Data from Curriculum Associates, creators of the i-Ready digital-instruction and assessment software, suggest that only 60% of low-income students are regularly logging into online instruction. Engagement rates are also lagging behind in school serving predominantly Black and Hispanic students, only 60 to 70% are logging in regularly (Hancock et al, 2020).

COVID 19 could increase high school drop-out rates. Hancock et al., (2020) suggest that the virus is disrupting many of the supports that can help vulnerable kids stay in school: academic engagement and achievement, strong relationships with caring adults, and supportive home environments. In normal circumstances, students who miss more than 10 days of school are 36% more likely to drop out (Utah Education Policy Center, 2012). An additional two percent to nine percent of high-school students could drop out as a result of the coronavirus and associated school closures—232,000 ninth-to-11th graders to 1.1 million (Hancock et al., 2020).

Social and emotional trauma intensified COVID-19. Increasing social isolation, increasing anxiety about jobs lost by parents and loved one becoming ill or even passing away to due virus placed a strain on students (Hancock et al., 2020). After school activities, sports, school dances and graduations reduced academic motivation, hurt academic performance and general levels of engagement (Lessard & Schacter, 2020).

Hancock et al., (2020) estimated that the average K–12 student in the United States could lose $61,000 to $82,000 in lifetime earnings (in constant 2020 dollars), or the equivalent of a year of full-time work, solely as a result of COVID-19–related learning losses. White students would earn $1,348 a year less (a 1.6% reduction) over a 40-year working life; the figure is $2,186 a year (a 3.3% reduction) for Black students and $1,809 (3.0%) for Hispanic ones (Hancock et al., 2020). This translates into an estimated impact of $110 billion annual earnings across the entire current K–12 cohort.

Virtual Learning

Even though virtual learning became the unconventional form of education during COVID –19, virtual learning is not new in education. Back in the 1700s, it was known as conventional learning. Students would receive instruction via the mail system and responded with assignments or questions to the instructor (Florida National University, n.d.). In 1858, the university of London became the first college to offer distance learning degrees; and in 1888, the International Correspondence Schools provided training for immigrant coal miners by sending out textbooks with the use of in-person salesman (Florida National University, n.d.). In 1922, Pennsylvania State College became the first college to “broadcast courses across radio networks” and about a decade later, the University of Iowa followed suit, becoming the “first university to employ television as a learning tool” (Florida National University. n.d.). Debter (2014) stated that in 1956, Chicago public television station WTTW, in partnership with the local Board of Education, televised college courses for credit; over 15,000 students enroll in five years. In 1984, National Technological University established the first accredited virtual university with financial support from companies like IBM, Motorola, and HP (Debter, 2014). In 1989, the University of Phoenix became the first institution to launch a fully online college institution that offered both bachelors' and masters' degrees and it was predicted that by 2006, 89% of four-year public colleges in the United States offer classes online, along with 60% of private institutions (Debter, 2014). COVID –19 has made distance learning the new norm.

Relevancy to Mathematics

According to the Center for Digital Dannelse (2016), digital competence is a combination of knowledge, skills and attitudes with regards to the use of technology to perform tasks, solve problems, communicate, manage information, collaborate, as well as to create and share content effectively, appropriately, securely, critically, creatively, independently and ethnically. Because of COVID-19 and the pandemic, the increase use of computers and the internet does not suggest that digital competence has increases. Research has shown that large amounts of computer, mobile and internet use only contribute to digital skills at the operational level (Skov, 2016).

Digital competence is divided into three domains: (1) Instrumental skills for using digital tools and media, (2) Knowledge, theories and principles related to technology, and (3) attitudes towards strategies use, openness, critical understanding, creativity, accountability and independence which are referred to the learning domains (Skov, 2016). Many areas of life are influenced by digital competence including critical understanding, employment, career, social inequality, and education.

Critical understanding when accessing online content can affect people's decisions and activities. In other words, it is crucial that people understand the internet as a resource where the validity of information is not necessarily verified (Skov, 2016). Many schools and education programs have banned the use of Wikipedia as a source, because they believe that students do not possess the skills for critical and responsible use (Skov, 2016). IT skills have become a main focus of employment, because of the need for IT-competent professionals in all sectors and for almost all types of tasks. A study showed that 58% believed that digital technologies had helped them find a good job (Van Deursen, 2010). A study found that about 50% of employers used social media to investigate job candidates, and 35% of them found content that caused them not to hire the candidate (Careerbuilder, 2009). For example, inappropriate photographs, attitudes, consumption of alcohol, drugs, or slander of colleagues. The economic, social, health, cultural, and societal benefits of good digital skills are more accessible to those who already have these benefits and less accessible to the neediest, such as low-skilled, unemployed, or elderly without social support (Van Deursen, 2010). Digital tools also provide a new dimension to lifelong learning. They provide a means of developing innovative learning methods and teaching with student-centered approaches, as well as connecting schools in an organized collaboration (Skov, 2016).

Teacher competency

The International Society for Technology in Education has created standards for teachers to deepen their practice, promote collaboration with peers, challenge them to rethink traditional approaches and prepare student to drive their own learning (ISTE, n.d.).

Table 5

ISTE Educator Standards

ISTE Educator Standards		
Learner	Educators continually improve their practice by learning from and with others and exploring proven and promising practices that leverage technology to improve student learning.	Set professional learning goas to explore and apply pedagogical approaches made possible by technology and reflect on their effectiveness Pursue professional interests by creating and actively participating in local and global learning networks Stay current with research that supports improved student learning outcomes, including findings from the learning sciences
Leader	Educators seek out opportunities for leadership to support student empowerment and success and to improve teaching and learning	Shape, advance and accelerate a shared vision for empowered learning with technology by engaging with education stakeholders Advocate for equitable access to educational technology, digital content and learning opportunities to meet the diverse needs of all students Model for colleagues the identification, exploration, evaluation, curation and adoption of new digital resources and tools for learning
Citizen	Educators inspire students to positively contribute to and responsibly participate in the digital world.	Create experiences for learners to make positive, socially responsible contributions and exhibit empathetic behavior online that build relationships and community Establish a learning culture that promotes curiosity and critical examination of online resources and fosters digital literacy and media fluency Mentor students in safe, legal and ethical practices with digital tools and the protection of intellectual rights and property

		Model and promote management personal data and digital identity and protect student data privacy
Collaborator	Educators dedicate time to collaborate with both colleagues and students to improve practice, discover and share resources and ideas, and solve problems.	Dedicate planning time to collaborate with colleagues to create authentic learning experiences that leverage technology Collaborate and co-learn with students to discover and use new digital resources and diagnose and troubleshoot technology issues Use collaborative tools to expand students' authentic, real-world learning experiences by engaging virtually with experts, teams and students, locally and globally Demonstrate cultural competency when communicating with students, parents and colleagues and interact with them as co-collaborators in student learning
Designer	Educators design authentic, learner-driven activities and environments that recognize and accommodate learner variability.	Use technology to create, adapt and personalize learning experiences that foster independent learning and accommodate learner differences and needs Design authentic learning activities that align with content area standards and use digital tools and resources to maximize active, deep learning Explore and apply instructional design principles to create innovative digital learning environments that engage and support learning
Facilitator	Educators facilitate learning with technology to support student achievement of the ISTE Standards for Students.	Foster a culture where student take ownership of their learning goals and outcomes in both independent and group setting Manage the use of technology and student learning strategies in digital platforms, virtual

		environments, hands-on makerspaces or in the field Create learning opportunities that challenge students to use a design process and computational thinking to innovate and solve problems Model and nurture creativity and creative expression to communicate ideas, knowledge or connections
Analyst	Educators understand and use data to drive their instruction and support students in achieving their learning goals.	Provide alternative ways for students to demonstrate competency and reflect on their learning using technology Use technology to design and implement a variety of formative and summative assessments that accommodate learner needs, provide timely feedback to students and inform instruction Use assessment to guide progress and communicate with students, parents and education stakeholders to build student self-direction

Issues With Math Work

Shulman (1986) proposed three categories of content knowledge: (1) subject matter content knowledge, (2) pedagogical content knowledge and (3) curricular knowledge. Shulman's (1986) model has influenced other frameworks. Technological Pedagogical Content Knowledge (Mishra & Koehler, 2006) attempts to identify the nature of knowledge required by teachers for technology integration in their teaching, while addressing the complex, multifaceted and situated nature of teacher knowledge (Koehler, 2012). The teacher is the key person in integrating technology into classrooms (Emprin, 2010). Jones (2004) stated that there is a great deal of literature evidence to suggest that effective training is crucial if teachers are to implement digital competency effectively in their teaching. Teachers need to develop new knowledge and skills for designing relevant technology-mediated tasks, monitoring student work and assessing student learning using technology (Spiteri & Rundgren, 2020). The gap between teachers' needs and the teacher education PD content has been identified as one of the main reasons for the unsuccess of professional developments (Emprin, 2010). One reason could be the mismatch between teachers' needs and the PD with which they are provided, and may be that the PD targets a change in teachers' knowledge and perhaps even skills yet fails to address attitudes (Inprasitha et al., 2021).

Best Practices

In order to achieve equitable teaching in mathematics, equity education as a whole should be the main goal (Brenner, 1998; Bonner, 2009; Bowman et al., 2022; Gutstein et al., 1997; Matthews, 2003; Nasir, 2002; Osisioma et al., 2008; Tate, 1995). Boaler and Staples (2008) conducted a longitudinal study comparing how equitable teaching impacted students' math achievement in three high schools. With the pretest scores being much lower than the comparison schools, students in this school outperformed the others in years two and three on post-test measures of math achievement. The researchers concluded that because the focus school held high expectations for students, presented all students with a common, rigorous curriculum to support their learning; offered learning supports to struggling students; and enacted a high level of challenge in classroom tasks, inequalities in teaching practices were reduced, thus increasing students' math achievement levels (Boaler & Staples, 2008).

According to Kaput (2000), reasoning and communicating in arithmetic and reasoning and communicating in situations is the beginning of elementary students' conceptual understanding algebra. He claims, for example, that a student who is generalizing patterns in sequences of numbers or working with objects and relations is conceived as mathematical. However, if a student is making comparisons of difference in prices between cashews and peanuts, the generalizing is from the situation rather than the mathematics. This same student can later use algebra inequalities, to model the same situation. Early introductions to algebra allows students to continuously make connections throughout their math classes.

Bender (2017) wrote 20 Strategies for Increasing Student Engagement and categorized them into four sections: instructional organization, technology strategies, collaborative instruction, and personal responsibility. Differentiated Instruction, the Flipped classroom, Project-Based learning, and makerspace are discussed within the instructional organization section, Augmented reality, games and simulations, virtual field trips, coding and robotics, individualized computer-driven instruction, storyboarding for comprehension, and animation are explained in the flipped classroom section, blogging, social networking, class Wikis, peer tutoring, and role-playing are considered in the collaborative instruction section, and mindfulness, reward and response, growth mindset strategies, goal-setting and self-monitoring are examined in the personal responsibility section of the book. All of these strategies are designed to bring the best out of today's students.

Equity in Mathematics Education

An equitable classroom is defined many ways. Snyder et al. (2019) stated that it is the driving force behind ensuring that all students, everywhere, receive rigorous, rich educational experiences that are designed to meet their specific learning needs. The National School Board Association affirmed that it is the intentional allocation of resources, instruction, and opportunities according to need, requiring that discriminatory practices, prejudices, and beliefs be identified and eradicated when looking at best practices and resources (NSBA, 2020). Geneva Gay (1998) confirmed that a focus on equitable outputs should lead to the development and selection of the inputs, or materials and practices used in the classrooms. In other words, the real focus of equity is not the sameness of content for all students, but equivalency of effect potential, quality status, and significance of learning opportunities. The National Council of Teachers of Mathematics (2020) added that acknowledging and addressing factors that contribute to differential outcomes among groups of students is critical to ensuring that all students routinely have opportunities to experience high quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful. Equity ensures that all students are learning from rigorous materials and where teachers are supporting students through material that creates a positive learning environment.

Gay (1998) introduced culturally responsive teaching, which focuses on teacher practice and ways to make learning more relevant and effective for all students. She (2010) promotes certain elements to help guide teachers in culturally responsive teaching: (1) being socially and academically empowering, (2) setting high expectations for all students, (3) engaging in multidimensional knowledge building, contributions and perspectives, (4) validating all students' cultures through diverse instructional strategies and materials, (5) being socially, emotionally, and politically comprehensive in educating the whole child, (6) using student's strengths to drive instruction and (7) being thoughtful and critical about how educational practices and ideals may form barriers to student success. Research has discovered several components that support classroom equity and echo the tenets of culturally responsive and sustaining practices (Aronson & Laughter, 2016; Gay, 2010; Krasnoff, 2016; Ladson-Billings, 2006; Morrison et al., 2008; New York State Education Department [NYSED], 2019; Saphier, 2017; Snyder et al., 2019; Waddell, 2014).

Summary

Algebra is a concept that has been around for many years, dating back to 700 CE. Italians, Greeks, and even the Romans used algebra in some form. Algebra became more systematic during the Great Depression and World War II. Algebra has always been a concept that was left for middle and high school teachers. But many are concluding that algebra should begin in the earlier grades.

Common Core Mathematical Standards was an initiative designed for mathematics to become for focused for students to understand and clearer for teachers to demonstrate. The standards also have built in practices to ensure connection between the standards and the practices. The state of Missouri has adopted their own version of the Common Core Mathematical Standards called the Missouri Learning Standards (MODESE, 2022b). These standards included algebraic reasoning from kindergarten until high school.

Critical thinking is a way to solve problems, make decisions and learn new concepts by using mental processes and strategies. People who utilize this way of thinking become more self-guided and self-disciplined, which is the one 21st century skill that employers are looking for. Every career requires critical thinking skills, from home health aides to statisticians.

In order to aid employers, teachers need to be highly qualified. Teacher educational programs are using many efforts from alternative certifications to signing bonuses to get teachers into the classrooms. Once teachers are there, the professional developments have to meet the needs of the students. Most professional development programs are new approaches that do not last a year.

COVID-19 caused a big rip in the educational system. Many people lost their lives from March 2019 until May 2020. This respiratory disease spread through the air. People were quarantined in their homes, which left a lot of students without adequate teachers, school materials, computers, and internet. This increased an old, but new style of learning, virtual learning.

Virtual Learning is as old as the 1700s. Students would send work through the mail and wait for teacher responses. Today, apps have been developed, such as Canvas, as a way for all course work to be taught through the Internet. Teachers' and students' digital competencies increased over the COVID-19 pandemic. Math teachers had to develop digital ways for students to engage in their work.

Thus, equity in education is a very important conversation that needs to happen. All students should receive a rigorous rich educational experience. Teachers have to be intentional about their lessons and the school districts have to be intentional about the professional development plans.

Chapter Three: Methodology

Purpose

The purpose of this study was to search out the instructional strategies that will lead students to the conceptual understanding of algebraic reasoning in the in-person and virtual classrooms. Critical thinking is one of the most important skills needed in the 21st century and algebraic reasoning contributes to that skill. The researcher is hoping that through this research, teachers from elementary to high school will gain understanding that algebraic reasoning is a continuum that carries into adulthood, through critical thinking and reasoning. This mixed method research may add to the research in future curriculum writing and best practices in mathematics.

Research Design

Algebraic Reasoning Correlation With Critical Thinking

Kaput (2008) specified that there are two core aspects of algebraic thinking: (i) making generalizations and expressing those generalizations in increasingly, conventional symbol systems, and (ii) reasoning with symbolic forms, including the syntactically guided manipulations of those symbolic forms. In the case of the first aspect, generalizations are produced, justified and expressed in various ways. The second aspect refers to the association of meanings to symbols and to the treatment of symbols independently of their meaning. Kaput (2008) asserted that these two aspects of algebraic thinking denote reasoning processes that are considered to flow through varying degrees throughout three strands of algebraic activity: (i) generalized arithmetic, (ii) functional thinking, and (iii) the application of modeling languages for describing generalizations. English and Sharry (1996) showed that analogical reasoning constitutes an essential mechanism when students resolve algebraic tasks. Specifically, they describe analogical reasoning as the mental source for extracting commonalities between relations and constructing mental representations for expressing generalizations.

Radford (2008) developed a definition of the process of generalizing a pattern which unfolds the involvement of various forms of reasoning: Generalizing a pattern algebraically rests on the capability of grasping a commonality noticed on some particulars (say p1, p2, p3…pk); extending or generalizing this commonality to all subsequent terms (pk + 1, pk + 2, pk + 3, …), and being able to use the commonality to provide a direct expression of any term of the sequence. (p. 84)

This process first involves the identification of differences and similarities between the parts of the sequence – described as analogical reasoning by English and Sharry (1996). Then the commonality founded is generalized through predicting a plausible generalization. This stage is considered by Rivera and Becker (2007) as abductive in nature, since it is abductive reasoning that boosts conjecturing and adopting a hypothesis that is considered testable. Finally, the tested commonality becomes the basis for inducing the generalized concept of the sequence. Here, the role of inductive reasoning is considered as pivotal (Ellis, 2007). Chimoni and Pitta-Pantazi (2015) examined the relationship between specific reasoning processes and an individuals' algebraic thinking abilities with students from grade four through grade seven. The results showed that there was a significant correlation between algebraic thinking and deductive reasoning ad reasoning by analogy. Introducing algebraic reasoning early will increase a student's critical thinking and reasoning skills.

Instructional Strategies

A research-based instructional strategy is any teaching approach supported by a statistical analysis of data from the learning environment (Apostolou et al., 2014). The use of research-based, high-impact teaching innovations increases the probability of strong student outcomes (Bolt-Lee, 2021). Instructional strategies can motivate students and help them focus attention, organize information for understanding and remembering, monitor and assess learning (Alberta Government, 2002). The participants in the study will describe the strategies they used in the classroom.

Foundational Skills of Teachers

According to Ball (2003), three actions are needed for teachers to improve students' learning. First, teaching mathematics entails a respect for the integrity of the discipline. Procedures are reasoned, and the efficiency and meaningfulness of those procedures are deeply intertwined. Second, knowledge of mathematics for teaching entails more than knowing it for oneself. Knowing mathematics sufficiently for teaching requires being able to unpack ideas and make them accessible as they are first encountered by the learner, not only in their finished form. Third, and closely related to the first two qualities, mathematical knowledge for teaching must be reasoned. Teachers have to know why procedures work, that certain properties are true, that particular relationships exist, and on what bases Ball (2003). Teachers use this knowledge to describe how their background knowledge played a role in conceptual understanding.

Teachers' Mindsets About Teaching Mathematics

Teachers will take the Teaching and Learning Beliefs Survey created by the NCTM (2014a). A teacher's mindset can influence their pedagogical decisions, how small groups are created, how feedback is given (Rattan et al., 2012), and other factors.

Teachers exhibiting a fixed mindset may constrict children's aspirations and shape their future academic goals and identities (Cvencek et al., 2014).

Student Engagement For In-person Classrooms

Researcher for years have been investigating student engagement. Zepke and Leach (2010) defined engagement as a students' cognitive investment in, active participation with, and emotional commitment to learning particular content. Student engagement has a direct correlation with student learning (Zilvinskis, Masseria & Pike, 2017). Participants will determine how their students responded to their activities using Schlechty's Levels of Engagement.

Schlechty's Levels of Engagement was developed by Phil Schlechty to transform learning experiences of students. Once given a task, a student may respond in one or more ways within the task: authentic engagement, ritual engagement, passive compliance, retreatism and rebellion (Schlechty, 2001).

- Authentic engagement. The task, activity, or work the student is assigned or encouraged to undertake is associated with a result or outcome that has clear meaning and relatively immediate value to the student—for example, reading a book on a topic of personal interest to the student or to get access to information that the student needs to solve a problem of real interest to him or her.
- Ritual engagement. The immediate end of the assigned work has little or no inherent meaning or direct value to the student, but the student associates it with extrinsic outcomes and results that are of value—for example, reading a book in order to pass a test or to earn grades needed to be accepted at college.

- Passive compliance. The student is willing to expend whatever effort is needed to avoid negative consequences, although he or she sees little meaning in the tasks assigned or the consequences of doing those tasks.
- Retreatism. The student is disengaged from the tasks, expends no energy in attempting to comply with the demands of the tasks, but does not act in ways that disrupt others and does not try to substitute other activities for the assigned task.
- Rebellion. The student summarily refuses to do the task assigned, acts in ways that disrupt others, or attempts to substitute tasks and activities to which he or she is committed in lieu of those assigned or supported by the school and by the teacher.

The participants measured their students' levels of engagement through an inventory for the classrooms that were in-person.

Student Engagement for Virtual Classrooms

COVID-19 fast-forwarded the implementation of the virtual classroom. Because of more school closures during this research study, participants assigned virtual work during the study. Teachers measured students' levels of engagement of engagement in the virtual classroom setting. The levels of engagement were behavioral, cognitive and relational/emotional engagement. Behavioral engagement is the quality of students' participation in the classroom and school community; students' effort, persistence, participation, and compliance with school structures (Davis et al., 2012). Cognitive Engagement refers to the quality of students' engagement whereas sheer effort refers to the quantity of their engagement in the class" (Pintrich, 2003). The inclusion of cognitive engagement makes an important distinction between students' efforts to simply do the work and effort that is focused on understanding and mastery (Fredricks et al., 2004; Greene et al., 2004). Skinner and Belmont (1993) defined emotional engagement as students' feelings of interest, happiness, anxiety, and anger during achievement-related activities. In contrast, Sciarra and Seirup (2008) defined emotional engagement as the extent to which students feel a sense of belonging "and the degree to which they care about their school" (p. 218). The purpose was to gage if a student was engaged during the virtual classroom.

Tasks

The task chosen by the teacher can guide the conceptual understanding of a student and is one of the most important decisions a teacher can make (Lappan & Briars, 1995; Smith & Stein, 2011). The task can influence how a student makes sense of the mathematics. Van de Walle (2003) finds that these factors in the task can promote learning: 1. What is problematic must be the mathematics? 2. Tasks must be accessible to students. 3. Tasks must require justifications and explanations for answers or methods. Worthwhile tasks encompass eight characteristics: uses significant mathematics for the grade level, is rich, is problem solving in nature, is authentic and interesting, is equitable, is active, connects to the Process standards and has a high cognitive demand (Van de Walle, 2003).

Research Questions and Null Hypothesis

RQ1: Does the number of years in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$1: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the number of years in education.

RQ2: Does the degree earned in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$2: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the degree earned in education.

RQ3: Does the grade level taught in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$3: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the grade level taught in education.

RQ4: Does the number of years in an educator's teaching position affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$4: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the number of years an educator's teaching position.

Hypotheses will be analyzed by applying Analysis of Variance to collected data.

Population and Sample

The total population included two school districts which totaled 230 teachers. The sample was twenty-five teacher participants. The teachers were on record as an elementary teacher, a middle school math teacher or a high school math teacher.

Instrumentation and Data Collection

Teachers' Inventory

Teachers completed an inventory asking about years' experience, highest degree earned, grades taught and their beliefs. Teachers' beliefs about teaching and learning were collected using the NCTM's (2014) Teaching and Learning Beliefs Survey. The survey consisted of 12 questions regarding aspects of both teaching and learning and aligned with what NCTM called productive and unproductive thinking (2014b).

Table 6

Beliefs About Teaching and Learning Mathematics

Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar

	contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Source: NCTM (2014b)

The beliefs that teachers have about teaching mathematics can impact students' learning. Respondents selected from a four-point Likert scale to indicate their levels of disagreement or agreement with each statement. The mean score for both productive and unproductive beliefs was computed, with high scores indicating strong alignment to productive or unproductive beliefs and low scores indicating little or no alignment.

Teachers characterized the data entry's level of demand. Stein and his colleagues developed a taxonomy of mathematical tasks, based on the kind and level of thinking required to solve them (Stein et al., 1996; Stein & Smith, 1998). Participants decided if their tasks had lower-level demands (memorization and procedures with connections) or higher-level demands (procedures with connections and doing mathematics). Teachers selected the students' levels of engagement during the data entry. Schlechty (2011) developed a list of possible indicators of engagement and reactions used to characterize student responses to the work presented to them.

Then teachers determined the students' virtual engagement level. Behavioral engagement is the quality of students' participation in the classroom and school community; students' effort, persistence, participation, and compliance with school structures (Davis et al., 2012). Cognitive Engagement refers to the quality of students' engagement whereas sheer effort refers to the quantity of their engagement in the class

(Pintrich, 2003). The inclusion of cognitive engagement makes an important distinction between students' efforts to simply do the work and effort that is focused on understanding and mastery (Fredricks et al., 2004; Greene et al., 2004). Skinner and Belmont (1993) defined emotional engagement as students' feelings of interest, happiness, anxiety, and anger during achievement-related activities. In contrast, Sciarra and Seirup (2008) defined emotional engagement as "the extent to which students feel a sense of belonging "and the degree to which they care about their school" (p. 218).

In this study, the researcher changed the number of originally planned inventories. Due to low enrollment and teacher burnout (Ferguson, et al, 2021), one teacher inventory was created and student samples were excluded from the study. The researcher decided to conduct an analysis of the NCTM's (2014) Teaching and Learning Beliefs Survey.

Equity within Participating Districts

The following tables show the students enrolled in AP classes.

Table 7

Students Enrolled in AP Classes: District # 1

Students enrolled in at least 1 AP Course District #1	Female/%	Male/%
American Indian/Alaska Native	0/0%	0/0%
Asian	2/25%	6/75%
Black	185/62.9%	109/37.1%
Hawaiian or Pacific Islander	0/0%	0/0%
Hispanic	9/64.3%	5/35.7%

continued

Table 7. Continued

White	26/60.5%	17/39.5
Multi-Race	15/68.2%	7/31.8
Limited English Proficient	0/0%	1/100%

Table 8

Students Enrolled in AP Classes: District # 2

Students enrolled in at least 1 AP Course District #2	Female/%	Male/%
American Indian/Alaska Native	0/0%	0/0%
Asian	0/0%	3/100%
Black	50/69.4%	22/30.6%
Hawaiian or Pacific Islander	0/0%	0/0%
Hispanic	3/100%	0/%
White	14/41.2%	20/58.8%
Multi-Race	5/83.3%	1/16.7%
Limited English Proficient	2/66.7%	1/33.3%

Table 9 shows the number of students enrolled in one AP course within the state:

Table 9

Students Enrolled in AP Classes: State

Students enrolled in at least 1 AP Course within the State	Female/%	Male/%
American Indian/Alaska Native	69/67%	34/33%
Asian	982/51.9%	909/48.1%
Black	2695/62.4%	1621/37.6%
Hawaiian or Pacific Islander	33/57.9%	24/42.1%
Hispanic	1158/57.4%	858/42.6%
White	15,214/56%	11965/44%
Multi-Race	697/59.8%	468/40.2%
Limited English Proficient	131/49.4%	134/50.6%

Ethical Considerations

Considerations were taken to preserve the identity of the participants. The researcher removed any identifiers for the participants, such as name and personal contact, etc. Students' work was not needed for this study design.

Limitations

A limitation for the study was COVID 19. With teacher burnout increasing, the teachers may not want to participate. Fifty five percent of public-school teachers, administrators and other staff said they were planning to leave the field sooner than they had planned, because of the crushing additional stresses brought on by the pandemic (Edelman, 2022).

Summary

To measure the instructional strategies used by teachers, the researcher applied a mixed methods design to determine how teachers are helping students to develop conceptual understanding of algebraic reasoning in grades kindergarten through high school. Instructional strategies, foundational skills of the teacher, mindset of the teachers, student engagement, educational tasks, and virtual classroom are the variables included in the study.

A multiple-choice inventory was created for data collection, which included the NCTM's (2014) Teaching and Learning Beliefs Survey, using *Qualtrics*. The inventory addressed professional development and teachers only identified what grade level they taught. The inventory was emailed to the teachers and all other identifying information was not included and nor necessary to the study. The inventory took five to fifteen minutes to complete, depending on the comments made by the participants.

Chapter Three outlined the research plan, tools, and strategies for collecting data. Chapter Four describes the results of the data collection, based on the hypotheses.

Chapter Four: Results and Findings

This study took place at two suburban school districts in the Midwest and was designed to examine the instructional strategies used to increase students' conceptual understanding of algebraic reasoning in grades K-12 in-person and virtual classrooms. Twenty-five educators were a part of the study, which included elementary teachers, middle school and high school math teachers.

Collected data were analyzed by applying Analysis of Variance. The research questions, with the null hypotheses that were analyzed in this study were:

RQ1: Does the number of years in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

H1: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the number of years in education.

1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.

Table 10

Hypothesis 1, Statement 1; n = 25

Groups	*Groups*	*Sum*	*Mean*	*Variance*
Group 1	0-5 yrs.	7	2.33	1.33
Group 2	6-11 yrs.	20	2.00	0.44
Group 3	12-17 yrs.	20	2.50	1.14
Group 4	18+ yrs.	8	2.00	0.07

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.33	3	0.44	0.56	0.647	3.072
Within Groups	16.67	21	0.79			

The null hypothesis is not rejected for this statement (F = 0.56; F-crit = 3.072).

2. The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.

Table 11

Hypothesis 1, Statement 2; n = 25

Groups	*Groups*	*Sum*	*Mean*	*Variance*
Group 1	0-5 yrs.	10	3.33	1.33
Group 2	6-11 yrs.	23	2.30	1.12
Group 3	12-17 yrs.	14	1.75	0.21
Group 4	18+ yrs.	9	2.25	0.25

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	5.54	3	1.85	2.58	0.080	3.072
Within Groups	15.02	21	0.72			

The null hypothesis is not rejected for this statement (F = 0.080; F-crit = 3.072).

3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

Table 12

Hypothesis 1, Statement 3; n = 25

Groups	*Groups*	*Sum*	*Mean*	*Variance*
Group 1	0-5 yrs.	11	3.67	0.33
Group 2	6-11 yrs.	38	3.80	0.18
Group 3	12-17 yrs.	27	3.38	1.41
Group 4	18+ yrs.	15	3.75	0.25

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.87	3	0.29	0.47	0.705	3.072
Within Groups	12.89	21	0.61			

The null hypothesis is not rejected for this statement (F = 0.47; F-crit = 3.072).

4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

Table 13

Hypothesis 1, Statement 4; n = 25

Groups	*Groups*	*Sum*	*Mean*	*Variance*
Group 1	0-5 yrs.	12	4.00	0.00
Group 2	6-11 yrs.	38	3.80	0.18
Group 3	12-17 yrs.	30	3.75	0.21
Group 4	18+ yrs.	14	3.50	0.33

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.46	3	0.15	0.79	0.515	3.072
Within Groups	4.10	21	0.20			

The null hypothesis is not rejected for this statement (F = 0.79; F-crit = 3.072).

5. Mathematics learning should focus on developing understanding of concepts and procedures through problems solving, reasoning, and discourse.

Table 14

Hypothesis 1, Statement 5; n = 25

Groups	*Groups*	*Sum*	*Mean*	*Variance*
Group 1	0-5 yrs.	12	4.00	0.00
Group 2	6-11 yrs.	37	3.70	0.23
Group 3	12-17 yrs.	29	3.63	0.27
Group 4	18+ yrs.	11	2.75	0.25

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	3.44	3	1.15	5.09	0.008	3.072
Within Groups	4.72	21	0.23			

The null hypothesis is rejected for this statement (F = 5.09; F-crit = 3.072). Mathematics teachers who have taught zero to five years and mathematics teachers who have taught 18 years or more agree that through problem solving, discourse and reasoning, mathematics is learned best. This could be due to pre-service programs, professional development offered, and on the job experiences.

6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

Table 15

Hypothesis 1, Statement 6; n = 25

Groups	***Groups***	***Sum***	***Mean***	***Variance***
Group 1	0-5 yrs.	10	3.33	1.33
Group 2	6-11 yrs.	22	2.20	0.62
Group 3	12-17 yrs.	17	2.13	0.41
Group 4	18+ yrs.	9	2.25	0.92

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	3.55	3	1.18	1.79	0.180	3.072
Within Groups	13.89	21	0.66			

The null hypothesis is not rejected for this statement (F = 1.79; F-crit = 3.072).

7. Students can learn to apply mathematics only after they have mastered the basic skills.

Table 16

Hypothesis 1, Statement 7; n = 25

Groups	***Groups***	***Sum***	***Mean***	***Variance***
Group 1	0-5 yrs.	9	3.00	1.00
Group 2	6-11 yrs.	24	2.40	0.71
Group 3	12-17 yrs.	16	2.00	0.57
Group 4	18+ yrs.	10	2.50	0.33

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	2.36	3	0.79	1.23	0.323	3.072
Within Groups	13.40	21	0.64			

The null hypothesis is not rejected for this statement (F = 1.23; F-crit = 3.072).

8. Students can learn mathematics through exploring and solving contextual and mathematical problems.

Table 17

Hypothesis 1, Statement 8; n = 25

Groups	***Groups***	***Sum***	***Mean***	***Variance***
Group 1	0-5 yrs.	11	3.67	0.33
Group 2	6-11 yrs.	37	3.70	0.23
Group 3	12-17 yrs.	28	3.50	0.57
Group 4	18+ yrs.	13	3.25	0.25

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.64	3	0.21	0.60	0.623	3.072
Within Groups	7.52	21	0.36			

The null hypothesis is not rejected for this statement (F = 0.60; F-crit = 3.072).

9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Table 18

Hypothesis 1, Statement 9; n = 25

Groups	***Groups***	***Sum***	***Mean***	***Variance***
Group 1	0-5 yrs.	11	3.67	0.33
Group 2	6-11 yrs.	40	4.00	0.00
Group 3	12-17 yrs.	29	3.63	0.55
Group 4	18+ yrs.	15	3.75	0.25

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.71	3	0.24	0.94	0.440	3.072
Within Groups	5.29	21	0.25			

The null hypothesis is not rejected for this statement (F = 0.94; F-crit = 3.072).

10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.

Table 19

Hypothesis 1, Statement 10; n = 25

Groups	***Groups***	***Sum***	***Mean***	***Variance***
Group 1	0-5 yrs.	8	2.67	2.33
Group 2	6-11 yrs.	18	1.80	0.62
Group 3	12-17 yrs.	13	1.63	0.55
Group 4	18+ yrs.	9	2.25	0.25

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	2.95	3	0.98	1.39	0.275	3.072
Within Groups	14.89	21	0.71			

The null hypothesis is not rejected for this statement (F = 1.39; F-crit = 3.072).

11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

Table 20

Hypothesis 1, Statement 11; n = 25

Groups	***Groups***	***Sum***	***Mean***	***Variance***
Group 1	0-5 yrs.	11	3.67	0.33
Group 2	6-11 yrs.	39	3.90	0.10
Group 3	12-17 yrs.	27	3.38	0.55
Group 4	18+ yrs.	15	3.75	0.25

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.25	3	0.42	1.41	0.267	3.072
Within Groups	6.19	21	0.29			

The null hypothesis is not rejected for this statement (F = 1.41; F-crit = 3.072).

12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Table 21

Hypothesis 1, Statement 12; n = 25

Groups	*Groups*	*Sum*	*Mean*	*Variance*
Group 1	0-5 yrs.	5	1.67	0.33
Group 2	6-11 yrs.	18	1.80	0.40
Group 3	12-17 yrs.	13	1.63	0.55
Group 4	18+ yrs.	9	2.25	0.25

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.11	3	0.37	0.87	0.471	3.072
Within Groups	8.89	21	0.42			

The null hypothesis is not rejected for this statement (F = 0.87; F-crit = 3.072).

RQ2: Does the degree earned in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$2: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the degree earned in education.

1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.

Table 22

Hypothesis 2, Statement 1; n = 25

Groups	*Degree*	*Sum*	*Mean*	*Variance*
Group 1	BS	11	2.20	0.70
Group 2	MS	39	2.17	0.62
Group 3	Doctorate	5	2.50	4.50

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.20	2	0.10	0.12	0.884	3.443
Within Groups	17.80	22	0.81			

The null hypothesis is not rejected for this statement (F = 0.12; F-crit = 3.443).

2. The role of the teacher is to tell students exactly what definitions, formulas and rules they should know and demonstrate how to use this information to solve mathematics problems.

Table 23

Hypothesis 2, Statement 2; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	13	2.60	1.80
Group 2	MS	39	2.17	0.74
Group 3	Doctorate	4	2.00	0.00

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.86	2	0.43	0.48	0.625	3.443
Within Groups	19.70	22	0.90			

The null hypothesis is not rejected for this statement (F = 0.48; F-crit = 3.443).

3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

Table 24

Hypothesis 2, Statement 3; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	18	3.60	0.30
Group 2	MS	68	3.78	0.30
Group 3	Doctorate	5	2.50	4.50

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	2.95	2	1.47	3.00	0.070	3.443
Within Groups	10.81	22	0.49			

The null hypothesis is not rejected for this statement (F = 3.00; F-crit = 3.443).

4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

Table 25

Hypothesis 2, Statement 4; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	19	3.80	0.20
Group 2	MS	69	3.83	0.15
Group 3	Doctorate	6	3.00	0.00

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.26	2	0.63	4.20	0.029	3.443
Within Groups	3.30	22	0.15			

The null hypothesis is rejected for this statement (F = 4.20; F-crit = 3.443). The teachers with Doctoral degrees have a little higher belief that the teachers with Masters' and Bachelors' degrees. Doctoral degrees require more time and research into scholarly articles, journals and books than the other two degrees. Bachelor degreed teachers is the first step of becoming a teacher and required more process and procedures to enter into the classroom.

5. Mathematics learning should focus on developing understanding of concepts and procedures through problems solving, reasoning, and discourse.

Table 26

Hypothesis 2, Statement 5; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	19	3.80	0.20
Group 2	MS	64	3.56	0.38
Group 3	Doctorate	6	3.00	0.00

Continued

Table 26. Continued

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.92	2	0.46	1.39	0.270	3.443
Within Groups	7.24	22	0.33			

The null hypothesis is not rejected for this statement (F = 1.39; F-crit = 3.443).

6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

Table 27

Hypothesis 2, Statement 6; n = 25

Groups	*Degree*	*Sum*	*Mean*	*Variance*
Group 1	BS	13	2.60	1.30
Group 2	MS	40	2.22	0.65
Group 3	Doctorate	5	2.50	0.50

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.63	2	0.31	0.41	0.668	3.443
Within Groups	16.81	22	0.76			

The null hypothesis is not rejected for this statement (F = 0.41; F-crit = 3.443).

7. Students can learn to apply mathematics only after they have mastered the basic skills.

Table 28

Hypothesis 2, Statement 7; n = 25

Groups	*Degree*	*Sum*	*Mean*	*Variance*
Group 1	BS	13	2.60	1.80
Group 2	MS	43	2.39	0.37
Group 3	Doctorate	3	1.50	0.50

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.78	2	0.89	1.40	0.267	3.443
Within Groups	13.98	22	0.64			

The null hypothesis is not rejected for this statement (F = 0.267; F-crit = 3.443).

8. Students can learn mathematics through exploring and solving contextual and mathematical problems.

Table 29

Hypothesis 2, Statement 8; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	18	3.60	0.30
Group 2	MS	65	3.61	0.37
Group 3	Doctorate	6	3.00	0.00

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.68	2	0.34	1.00	0.383	3.443
Within Groups	7.48	22	0.34			

The null hypothesis is not rejected for this statement (F = 1.00; F-crit = 3.443).

9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Table 30

Hypothesis 2, Statement 9; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	19	3.80	0.20
Group 2	MS	70	3.89	0.10
Group 3	Doctorate	6	3.00	2.00

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.42	2	0.71	3.42	0.051	3.443
Within Groups	4.58	22	0.21			

The null hypothesis is not rejected for this statement (F = 3.42; F-crit = 3.443).

10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.

Table 31

Hypothesis 2, Statement 10; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	11	2.20	1.70
Group 2	MS	32	1.78	0.54
Group 3	Doctorate	5	2.50	0.50

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.43	2	0.71	0.96	0.399	3.443
Within Groups	16.41	22	0.75			

The null hypothesis is not rejected for this statement (F = 0.90; F-crit = 3.443).

11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

Table 32

Hypothesis 2, Statement 11; n = 25

Groups	***Degree***	***Sum***	***Mean***	***Variance***
Group 1	BS	19	3.80	0.20
Group 2	MS	67	3.72	0.21
Group 3	Doctorate	6	3.00	2.00

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.03	2	0.51	1.77	0.195	3.443
Within Groups	6.41	22	0.29			

The null hypothesis is not rejected for this statement (F = 1.77; F-crit = 3.443).

12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Table 33

Hypothesis 2, Statement 12; n = 25

Groups	*Degree*	*Sum*	*Mean*	*Variance*
Group 1	BS	9	1.80	0.20
Group 2	MS	31	1.72	0.45
Group 3	Doctorate	5	2.50	0.50

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.09	2	0.54	1.34	0.281	3.443
Within Groups	8.91	22	0.41			

The null hypothesis is not rejected for this statement (F = 1.34; F-crit = 3.443)

RQ3: Does the grade level taught in education affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$3: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the grade level taught in education.

1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.

Table 34

Hypothesis 3, Statement 1; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	31	2.21	0.80
Group 2	Middle	8	2.00	0.67
Group 3	High	16	2.29	0.90

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.21	2	0.11	0.13	0.877	3.443
Within Groups	17.79	22	0.81			

The null hypothesis is not rejected for this statement (F = 0.13; F-crit = 3.443).

2. The role of the teacher is to tell students exactly what definitions, formulas and rules they should know and demonstrate how to use this information to solve mathematics problems.

Table 35

Hypothesis 3, Statement 2; n = 25

Groups	***Level***	***Sum***	***Mean***	***Variance***
Group 1	Elementary	33	2.36	1.32
Group 2	Middle	7	1.75	0.25
Group 3	High	16	2.29	0.24

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.17	2	0.58	0.66	0.526	3.443
Within Groups	19.39	22	0.88			

The null hypothesis is not rejected for this statement (F = 0.66; F-crit = 3.443).

3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

Table 36

Hypothesis 3, Statement 3; n = 25

Groups	***Level***	***Sum***	***Mean***	***Variance***
Group 1	Elementary	53	3.79	0.18
Group 2	Middle	16	4.00	0.00
Group 3	High	22	3.14	1.48

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	2.55	2	1.27	2.50	0.105	3.443
Within Groups	11.21	22	0.51			

The null hypothesis is not rejected for this statement (F = 2.50; F-crit = 3.443).

4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

Table 37

Hypothesis 3, Statement 4; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	54	3.86	0.13
Group 2	Middle	14	3.50	0.33
Group 3	High	26	3.71	0.24

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.42	2	0.21	1.11	0.348	3.443
Within Groups	4.14	22	0.19			

The null hypothesis is not rejected for this statement (F = 1.11; F-crit = 3.443).

5. Mathematics learning should focus on developing understanding of concepts and procedures through problems solving, reasoning, and discourse.

Table 38

Hypothesis 3, Statement 5; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	51	3.64	0.25
Group 2	Middle	13	3.25	0.25
Group 3	High	25	3.57	0.62

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.48	2	0.24	0.69	0.512	3.443
Within Groups	7.68	22	0.35			

The null hypothesis is not rejected for this statement (F = 0.69; F-crit = 3.443).

6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

Table 39

Hypothesis 3, Statement 6; n = 25

Groups	***Level***	***Sum***	***Mean***	***Variance***
Group 1	Elementary	30	2.14	1.05
Group 2	Middle	11	2.75	0.25
Group 3	High	17	2.43	0.29

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.26	2	0.63	0.86	0.438	3.443
Within Groups	16.18	22	0.74			

The null hypothesis is not rejected for this statement (F = 0.86; F-crit = 3.443).

7. Students can learn to apply mathematics only after they have mastered the basic skills.

Table 40

Hypothesis 3, Statement 7; n = 25

Groups	***Level***	***Sum***	***Mean***	***Variance***
Group 1	Elementary	32	2.29	0.68
Group 2	Middle	11	2.75	0.92
Group 3	High	16	2.29	0.57

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.72	2	0.36	0.53	0.596	3.443
Within Groups	15.04	22	0.68			

The null hypothesis is not rejected for this statement (F = 0.53; F-crit = 3.443).

8. Students can learn mathematics through exploring and solving contextual and mathematical problems.

Table 41

Hypothesis 3, Statement 8; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	51	3.64	0.40
Group 2	Middle	13	3.25	0.25
Group 3	High	25	3.57	0.29

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.48	2	0.24	0.69	0.512	3.443
Within Groups	7.68	22	0.35			

The null hypothesis is not rejected for this statement (F = 0.69; F-crit = 3.443).

9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Table 42

Hypothesis 3, Statement 9; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	54	3.86	0.13
Group 2	Middle	16	4.00	0.00
Group 3	High	25	3.57	0.62

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.57	2	0.29	1.16	0.333	3.443
Within Groups	5.43	22	0.25			

The null hypothesis is not rejected for this statement (F = 1.16; F-crit = 3.443).

10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.

Table 43

Hypothesis 3, Statement 10; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	27	1.93	1.15
Group 2	Middle	8	2.00	0.67
Group 3	High	13	1.86	0.14

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.05	2	0.03	0.03	0.967	3.443
Within Groups	17.79	22	0.81			

The null hypothesis is not rejected for this statement (F = 0.03; F-crit = 3.443).

11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

Table 44

Hypothesis 3, Statement 11; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	52	3.71	0.22
Group 2	Middle	16	4.00	0.00
Group 3	High	24	3.43	0.62

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.87	2	0.43	1.45	0.255	3.443
Within Groups	6.57	22	0.30			

The null hypothesis is not rejected for this statement (F = 1.45; F-crit = 3.443).

12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Table 45

Hypothesis 3, Statement 12; n = 25

Groups	*Level*	*Sum*	*Mean*	*Variance*
Group 1	Elementary	23	1.64	0.40
Group 2	Middle	7	1.75	0.25
Group 3	High	15	2.14	0.48

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.18	2	0.59	1.47	0.252	3.443
Within Groups	8.82	22	0.40			

The null hypothesis is not rejected for this statement (F = 1.47; F-crit = 3.443).

RQ4: Does the number of years in an educator's teaching position affect the results of the NCTM's Teaching and Learning Beliefs Survey?

$H_0$4: There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the number of years an educator's teaching position.

1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.

Table 46

Hypothesis 4, Statement 1; n = 25

Groups	*Years in Position*	*Sum*	*Mean*	*Variance*
Group 1	0-3 yrs.	19	2.38	0.55
Group 2	4-7 yrs.	16	2.67	1.47
Group 3	8-11 yrs.	12	2.00	0.40
Group 4	12+ yrs.	8	1.60	0.30

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	3.59	3	1.20	1.74	0.189	3.072
Within Groups	14.41	21	0.69			

The null hypothesis is not rejected for this statement (F = 1.74; F-crit = 3.072).

2. The role of the teacher is to tell students exactly what definitions, formulas and rules they should know and demonstrate how to use this information to solve mathematics problems.

Table 47

Hypothesis 4, Statement 2; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	23	2.88	1.55
Group 2	4-7 yrs.	11	1.83	0.17
Group 3	8-11 yrs.	11	1.83	0.57
Group 4	12+ yrs.	11	2.20	0.20

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	5.22	3	1.74	2.38	0.098	3.072
Within Groups	15.34	21	0.73			

The null hypothesis is not rejected for this statement (F = 2.38; F-crit = 3.072).

3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

Table 48

Hypothesis 4, Statement 3; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	30	3.75	0.21
Group 2	4-7 yrs.	21	3.50	1.50
Group 3	8-11 yrs.	23	3.83	0.17
Group 4	12+ yrs.	17	3.40	0.80

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	5.22	3	1.74	2.38	0.098	3.072
Within Groups	15.34	21	0.73			

The null hypothesis is not rejected for this statement (F = 2.38; F-crit = 3.072).

4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

Table 49

Hypothesis 4, Statement 4; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	31	3.88	0.13
Group 2	4-7 yrs.	21	3.50	0.30
Group 3	8-11 yrs.	24	4.00	0.00
Group 4	12+ yrs.	18	3.60	0.30

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	0.99	3	0.33	1.93	0.156	3.072
Within Groups	3.58	21	0.17			

The null hypothesis is not rejected for this statement (F = 1.93; F-crit = 3.072).

5. Mathematics learning should focus on developing understanding of concepts and procedures through problems solving, reasoning, and discourse.

Table 50

Hypothesis 4, Statement 5; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	28	3.50	0.29
Group 2	4-7 yrs.	21	3.50	0.30
Group 3	8-11 yrs.	24	4.00	0.00
Group 4	12+ yrs.	16	3.20	0.70

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.86	3	0.62	2.07	0.135	3.072
Within Groups	6.30	21	0.30			

The null hypothesis is not rejected for this statement (F = 2.07; F-crit = 3.072).

6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

Table 51

Hypothesis 4, Statement 6; n = 25

Groups	*Years in Position*	*Sum*	*Mean*	*Variance*
Group 1	0-3 yrs.	19	2.38	1.41
Group 2	4-7 yrs.	14	2.33	0.67
Group 3	8-11 yrs.	12	2.00	0.40
Group 4	12+ yrs.	13	2.60	0.30

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.03	3	0.34	0.44	0.727	3.072
Within Groups	16.41	21	0.78			

The null hypothesis is not rejected for this statement (F = 0.44; F-crit = 3.072).

7. Students can learn to apply mathematics only after they have mastered the basic skills.

Table 52

Hypothesis 4, Statement 7

Groups	*Years in Position*	*Sum*	*Mean*	*Variance*
Group 1	0-3 yrs.	21	2.63	0.55
Group 2	4-7 yrs.	13	2.17	1.37
Group 3	8-11 yrs.	12	2.00	0.40
Group 4	12+ yrs.	13	2.60	0.30

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1.85	3	0.62	0.93	0.443	3.072
Within Groups	13.91	21	0.66			

The null hypothesis is not rejected for this statement (F = 0.93; F-crit = 3.072).

8. Students can learn mathematics through exploring and solving contextual and mathematical problems.

Table 53

Hypothesis 4, Statement 8; n = 25

Groups	*Years in Position*	*Sum*	*Mean*	*Variance*
Group 1	0-3 yrs.	29	3.63	0.27
Group 2	4-7 yrs.	20	3.33	0.67
Group 3	8-11 yrs.	24	4.00	0.00
Group 4	12+ yrs.	16	3.20	0.20

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	2.15	3	0.72	2.51	0.087	3.072
Within Groups	6.01	21	0.29			

The null hypothesis is not rejected for this statement (F = 2.51; F-crit = 3.072).

9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Table 54

Hypothesis 4, Statement 9; n = 25

Groups	*Years in Position*	*Sum*	*Mean*	*Variance*
Group 1	0-3 yrs.	31	3.88	0.13
Group 2	4-7 yrs.	21	3.50	0.70
Group 3	8-11 yrs.	24	4.00	0.00
Group 4	12+ yrs.	19	3.80	0.20

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.83	3	0.28	1.12	0.365	3.072
Within Groups	5.18	21	0.25			

The null hypothesis is not rejected for this statement (F = 1.12; F-crit = 3.072).

10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.

Table 55

Hypothesis 4, Statement 10; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	19	2.38	1.13
Group 2	4-7 yrs.	10	1.67	0.67
Group 3	8-11 yrs.	9	1.50	0.30
Group 4	12+ yrs.	10	2.00	0.50

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	3.13	3	1.04	1.49	0.246	3.072
Within Groups	14.71	21	0.70			

The null hypothesis is not rejected for this statement (F = 1.49; F-crit = 3.072).

11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

Table 56

Hypothesis 4, Statement 11; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	30	3.75	0.21
Group 2	4-7 yrs.	21	3.50	0.70
Group 3	8-11 yrs.	24	4.00	0.00
Group 4	12+ yrs.	17	3.40	0.30

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	1.24	3	0.41	1.40	0.271	3.072
Within Groups	6.20	21	0.30			

The null hypothesis is not rejected for this statement (F = 1.40; F-crit = 3.072).

12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Table 57

Hypothesis 4, Statement 11; n = 25

Groups	***Years in Position***	***Sum***	***Mean***	***Variance***
Group 1	0-3 yrs.	15	1.88	0.41
Group 2	4-7 yrs.	11	1.83	0.57
Group 3	8-11 yrs.	8	1.33	0.27
Group 4	12+ yrs.	11	2.20	0.20

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F crit***
Between Groups	2.16	3	0.72	1.93	0.156	3.072
Within Groups	7.84	21	0.37			

The null hypothesis is not rejected for this statement (F = 1.93; F-crit = 3.072).

Summary

The NCTM's (2014) Teaching and Learning Beliefs Survey was analyzed to research the equity in mathematics through the instructional strategies that teachers tapped in to increase conceptual understanding of algebraic reasoning. Data sources included the teachers' number of years in education, the level of degree earned, the grade level taught and the number of years in the teaching position. Although most of the null hypotheses were not rejected, two of the hypotheses were rejected: Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse, and the role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

Chapter Five: Discussion and Reflection

The purpose of this study was to investigate equity in mathematics education through the conceptual understanding of algebraic reasoning in K-12 classrooms. Employers are increasingly asking for new hires to encompass critical skills, in which algebraic reasoning is the foundation. The United States math literacy score, according to PISA, has been lower than average when comparing with other countries. Males scored higher than females and whites scored higher overall. Algebraic reasoning can begin as early as elementary. Equity in education, especially mathematics, is needed in order for all to be successful. Most advanced math classes today include no students of color and are mostly males. Low algebraic reasoning skills can affect future mathematics classes, as well as damage critical thinking skills, which is needed in today's workforce. And due to COVID-19, education has influenced the way students learn, through virtual learning. Equity is more important now, more than ever (Talusan, 2022, preface).

Algebraic Reasoning is a mental process, a way of seeing and understanding relationships through general patterns. The Common Core Mathematical Standards was developed so that students and teachers can see a clearer and more coherent mathematical curriculum. The state of Missouri has adopted their own version of the Common Core Mathematical Standards to provide for its students. Algebraic reasoning was a particular concept that was not introduced to students until middle and high school. Trade schools are implementing algebraic reasoning into their classrooms, due to its deficit in their students. Colleges have also understood the significance and have created classes to teach critical thinking to prepare student for the workforce.

Employers are stating that employees do not possess the critical thinking, or soft skills, needed. They had the knowledge of the work required but do not know how to analyze, work with others or become self-starters. This places the economy in an unusual position.

So, what is educators doing? Educational systems are providing more ways for people to become teachers such a different certification programs. However, the professional development provided to many school districts are the same cookie cutter ideas. But nothing could have prepared educators for the COVID-19 pandemic. More classes were place online which did not help educators, especially math teachers, know if students were understanding the concepts taught. Students lost over a year in learning, African American and Hispanics lost more, due to schools being closed. Not only education was lost, but depressions, social anxieties increased during the pandemic.

Before the pandemic, best practices included technology on a surface level. However, after the pandemic, technology use in the math classes has risen significantly. Digital competence is almost the way of life. Students and adults must learn how to incorporate computer skills into their lives, i.e. for enjoyment and employment. In order to meet the demands of the new global economy, equity in education is the key. Understanding that every student must learn and every student must have the same opportunities will switch education and the world into a new way of living.

Participants were 25 teachers who teach elementary teachers, middle math teachers, and high school math teachers, from two urban school districts in the state of Missouri. The NCTM's (2014) Teaching and Learning Beliefs Survey was used in the investigation. The survey results were compared from the view of four data points: (1)

the number of years in education, (2) the highest degree earned, (3) the grade level taught in education, and (4) the number of years in the teaching position.

Research Questions

This study intended to answer four questions to investigate equity in the mathematics classrooms: (1) Does the number of years in education affect the results of the NCTM's Teaching and Learning Beliefs Survey? (2) Does the degree earned in education affect the results of the NCTM's Teaching and Learning Beliefs Survey? (3) Does the grade level taught in education affect the results of the NCTM's Teaching and Learning Beliefs Survey? (4) Does the number of years in an educator's teaching position affect the results of the NCTM's Teaching and Learning Beliefs Survey?

Importance of Study

The importance of this study is two-fold. The workforce is seeing a decline in qualified employees, due to the lack of critical thinking skills. Educators are not able to implement the deep contextual lessons, due to students' unavailability to reasoning and varied strengths of understanding mathematical content. Elementary students need teachers who have an understanding of mathematics and the ability to teach to reasoning.

The hypotheses and research questions supported the important reasons to conduct this study. Grade level serves as a way of examining whether teachers understand the importance of algebraic reasoning and critical thinking. The researcher found, when asking teachers to participate, that some responses from elementary teachers were that "they do not teach algebra," and therefore "could not provide any information to this study." Growth mindsets and perceptions of teachers shined a light on how students perceive algebraic reasoning. Degree level of a math teacher showed that when a teacher studied more about education, their mindset was changed about it. The number of years in a teacher's position and grade level taught can have an effect on the teacher's feeling about algebraic reasoning. While elementary teachers focused more on reading, mathematical ability in students got pushed onto the middle and high school teachers. Basic skills, such as operations on multi-digit numbers, were a hindrance to middle and high school teachers, which made it difficult to use DOK 2 and 3 level problems.

Conclusions

Data to support the four research questions were analyzed, and for the associated hypotheses it was concluded that: (1) There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the number of years in education, (2) There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the degree earned in education, (3) There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the degree earned in education, and (4) There is no significant difference in the results of the NCTM's Teaching and Learning Beliefs Survey when comparing the number of years an educator's teaching position.

There were two differences found in the study. In Hypothesis #1, according to the NCTM Teaching and Learning Beliefs Survey statement number five, Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning and discourse. Math teachers who have taught zero to five years and teachers who have taught 18 years or more agreed with this statement. Mathematics pre-service programs are now understanding and teaching this phenomenon to upcoming math educators. Math educators with 18+ years, either through on-the-job training, professional development, or degree earned, have accepted this and have incorporated into their instruction. In Hypothesis #2, according to the NCTM Teaching and Learning Beliefs Survey statement number four, the role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics. Math teachers with Doctoral degrees agree with this statement more that teachers with Bachelors' and Masters' degrees. Teachers with Doctoral degrees have committed themselves to researching articles, journals, and studies and have learned a great more about algebraic reasoning, critical thinking, and conceptual understanding.

Strengths and Weaknesses

One strength this study showed was the amount of teacher participation. Even with teacher burnout, the study showed two times more participants than expected. This led to more data being analyzed.

One weakness was the COVID-19 pandemic. Research was put to a halt; the study and hypotheses were adjusted, due to the pandemic. The researcher, being an educator, knew that teachers were mentally and physically drained. Another weakness was teacher burnout. The delivery method of the researcher's inventory had to be changed to meet the needs of the participants. Teachers did not want to submit student work and complete constructed response-like questions for the student. Most survey questions were changed to a Likert scale-type question in order to enhance the ability and interest of teachers to participate in the study.

Recommendations

One recommendation is to continue to educate present and future educators about growth mindsets and equity in education. Researchers and teachers should find and promote opportunities for discussion about increasing the understanding and use of algebraic reasoning among their students, as well as discussion of appropriate instructional strategies to support this. Additionally, utilizing this study, others can show that equity is an ongoing concept that has to be included in mathematics education in order for society to keep progressing.

The researcher developed an online journal for middle school students, based off of the book, *Culturally Responsive Teaching and the Brain: Promoting Authentic Engagement and Rigor among Culturally and Linguistically Diverse Students*, by Zaretta L. Hammond (2015), during COVID. It was designed to illustrate how the brain works when learning anything and that learning mathematics is no different from learning any other subject. It included an ELA, Science, and mathematical component. Students began their online journals with a math inventory, responding with true or false to the following statements:

(1) You must always know how to get the right answer.

(2) Boys are better at math than girls.

(3) Some people have a “math mind” and some people don’t.

(4) Math requires logic, not intuition.

(5) There is a best way to do math problems.

(6) Math is not creative.

(7) It is important to get the answer exactly right.

(8) It is bad to count on your fingers.

(9) Mathematicians do math quickly in their heads.

(10) Math requires a good memory. (The University of Alaska – Fairbanks, n.d.)

Students then read and discussed the three structures of effective learning: the thalamus, the hippocampus, and the amygdala (Hammond, 2015). The journal contained diagrams of the brain and a hyperlink to videos about the brain. They discovered that the recticular activating system scans for possible threats or rewards and sends it to the amygdala, the amygdala prepares the for fight, flight, or freeze when a threat is received and the hippocampus stores all of the knowledge (Hammond, 2015). They realized that neurons carried information back and forth across the brain and how dendrites are grown in response to new cognitive challenges, novel problem solving and increased physical activity (Hammond, 2015). Utilizing their ELA skills, students were placed in groups in breakout rooms and were asked to read and provide main ideas for the six core design principles used to interpret threats and opportunities:

- The brain seeks to minimize social threats and maximize opportunities to connect with others in the community.
- Positive relationships keep our safety-threat detection system in check.
- Culture guides how we process information.
- Attention drives learning.
- All new information must be coupled with existing funds of knowledge in order to be learned.
- The brain physically grows through challenge and stretch, expanding its ability to do more complex thinking and learning (Hammond, 2015).

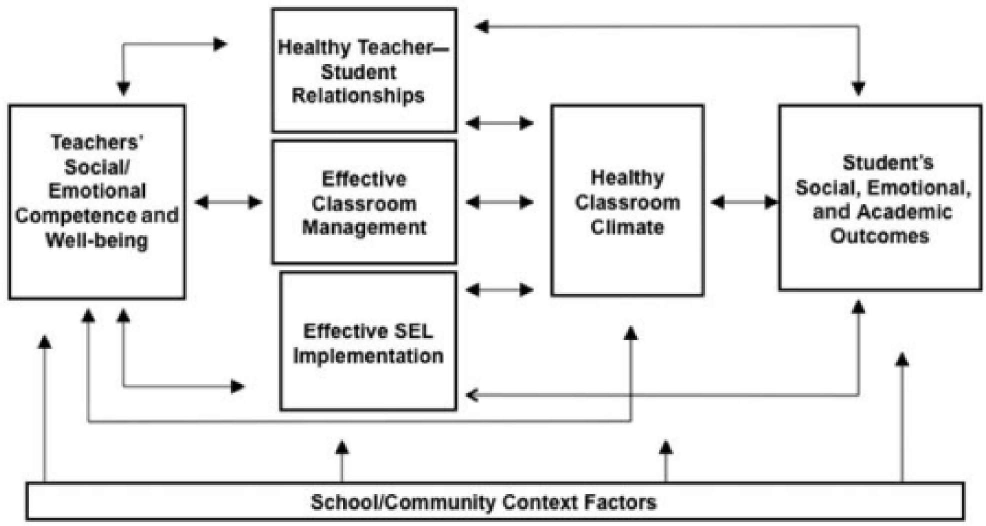
Based on the online blog, 10 Things Every Good Mathematician Should Do, students were to pick three attributes that they could work on during the school year. Then students created a mathematical goal for the quarter and three steps on how they were going to achieve the goal. Each section contained a reflection page to type any new learnings during this process. This was an attempt by the research to bring equity into the mathematics classroom.

While many districts incorporated equity into the learning space, one of the participating districts has created a plan on how to encompass equity reactively and proactively. It is described as SEEAL, Social Emotional Equity and Academic Learning. It was created in collaboration by the curriculum and instruction team, the student services team, and the well-being specialists in the district. Using the prosocial model from Jennings and Greenberg (2008), it was discovered how the model merges with the district's mission and vision.

Figure 1 indicates relationships between context factors found in the school and community.

Figure 1

School/Community Context Factors



Another recommendation to look at are the teacher educational programs being offered. These programs should include classes about equity in education and how to make sure every student receives an equitable education. Education was not offered to everyone at one point in the United States and certain groups had to create their own educational opportunities. In order for the United States to complete in the global workforce, every citizen must have an equal chance. Another type of class that should take place is a growth mindset. As stated previously, certain grade levels believe that teachers do not teach algebraic reasoning. This mindset has to change for students. Some participants in the study believed that algebraic reasoning is the stepping stone to critical thinking because it gives students the ability to observe the known facts and use them to figure out unknowns and the tenets of algebraic reasonings can be applied to other disciplines. Training future teacher how to create rich mathematical tasks is another was to improve the teacher educational programs. According to Principles to Actions: Ensuring Mathematical Success for All, one of the eight high leverage instructional practices is to implement tasks that promote reasoning and problem solving (p.17). Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and that allows for multiple entry points and varied solution strategies (NCTM, p. 17 2014). Table 58 shows the different level of demands in tasks at four levels of cognitive demand:

Table 58

Levels of Cognitive Demand

Levels of Demand	
Lower-level demands (memorization): • reproducing previously learned facts, rules, formulas, definitions or committing them to memory • Cannot be solved with a procedure • Have no connection to concepts or meaning that underlie the facts rules, formulas, or definitions	Lower-level demands (procedures without connections): • are algorithmic • require limited cognitive demand • have no connection to the concepts or meaning that underlie the procedure • focus on producing correct answers instead of understanding • require no explanations
Higher-level demands (procedures with connections): • use procedure for deeper understanding of concepts • broad procedures connected to ideas instead narrow algorithms • usually represented in different ways • require some degree of cognitive effort; procedures may be used but not mindlessly	Higher-level demands (doing mathematics): • require complex non-algorithmic thinking • require students to explore and understand the mathematics • demand self-monitoring of one's cognitive process • require considerable cognitive effort and may involve some level of anxiety b/c solution path isn't clear

Strategies for modifying tasks to increase the cognitive demand of the tasks can include but are not limited to:

- Ask students to create real-world stories for "naked number" problems. • Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context).

- Use a task "out of sequence" before students have memorized a rule or have practiced a procedure that can be routinely applied.
- Eliminate components of the task that confine student thinking or provide too much scaffolding.
- Create opportunities for repeated reasoning or pattern finding
- Create a prompt that asks students to write about the meaning of the mathematics concept.
- Add a prompt that asks students to make note of a pattern or to make a mathematical conjecture and to test their conjecture.
- Include a prompt that requires students to make a generalization.
- Include a prompt that requires students to compare solution paths or mathematical relationships and write about the relationship between strategies or concepts.
- Select numbers carefully so students are more inclined to note relationships between quantities (e.g., two tables can be used to think about the solutions to the four, six, or eight tables). (National Council of Teachers of Mathematics, n.d., p. 1)

References

25 In-demand jobs that require critical thinking and problem-solving skills. (2022, July). https://eggcellentwork.com/jobs-that-require-critical-thinking/

Abrami, P. C., Bernard, R. M., Borokhovski, E., Waddington, D. I., Wade, C. A., & Persson, T. (2014). Strategies for teaching students to think critically: A meta-analysis. *Review of Educational Research*, *85*, 275–314 https://doi.org/10.3102/0034654314551063

Alberta Government. (2002). Health and life skills for kindergarten to Grade 9: guide to implementation. https://open.alberta.ca>publication

Andrade, C. (2020, January 6). Sample size and its importance in research. Indian Journal of Psychological Medicine, 42(1), 103-104. https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6970301/pdf/UPsyM-42-102.pdf.

Apostolou, B., Dorminey, J., Hassell, J. & Rebele, J. (2014). A summary and analysis of education research in accounting information systems (AIS). *Journal of Accounting Education*, *32*(2): 99–112.

Aronson, B., & Laughter, J. (2016). The theory and practice of culturally relevant education: A synthesis of research across content areas. *Review of Educational Research*, *86*(1). P. 163-206. https://doi:10.3102/00346554315582066

Arranz, A. (November 2021). Technology in the math classrooms: Effective strategies for engaging your students. https://bolsteracademy/blog/technology-in-the-math-classroom/

Atweh, B., Forgasz, H., & Nebrez, B. (2001). Socio-cultural aspects of mathematics education: An international perspective. Mahwah, NJ: Lawrence Erlbaum.

Auxier, B., & Anderson, M. (March 2020). As schools close due to the coronavirus, some U.S. students face a digital 'homework gap'. Fact Tank. https://pewresearch.org/fact-tank/2020/03/16/as-schools-close-due-to-the-coronavirus-some-u-s-students-face-a-digital-homework-gap/

Ball, D. (2003). What mathematical knowledge is needed for teaching mathematics? University of Michigan. https://www.jwilson.coe.uga.edu/situatins/Framework%20Folder/FrameworkJan08/articles/Ball2003Math%20Summit.pdf

Bender, W. (2017). *20 strategies for increasing student engagement*. Learning Sciences International, West Palm Bach, FL.

Bethany. (March 2019). 10 things every good mathematician should do. https://mathgeekmama.com/10-things-every-good-mathematician-should-do/

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record*, *110*(3). 608-645.

Bolt-Lee, C. E. (2021). Developments in research-based instructional strategies: Learning-centered approaches for accounting education. *e-Journal of Business Education & Scholarship of Teaching 15*(2), 1-14. http://www.ejbest.org

Bonner, E. (2009). Achieving success with African American learners: A framework for culturally responsive mathematics teaching. *Childhood Education*, *86*(1). 2-6. Doi:10.1080/00094056.2009.10523110

Bowen, C. W. (2000). A quantitative literature review of cooperative learning effects on high school and college chemistry achievement. *Journal of Chemical Education*, 77(*1*), 116-119. http://dx.doi.org/10.1021/ed077p116

Bowman, M., Trowery, L., & Snyder, A. (n.d.). Reveal math k-5 research foundations: A research summary of program focused outcomes. https://www.mheducation.com/unitas/school/explore/sites/reveal-math/white-paper-program-focused-outcomes.pdf

Boyd D., Grossman, P., Lankford, H., Loeb, S., & Wyckoff, J. (2006). How changes in entry requirements alter the teacher workforce and affect student achievement. *Education Finance and Policy 1*(*2*): 176-216. https://doi.org/10.1162/edfp.2006.1.2.176

Boyd, D., Grossman, P., Hammerness, K., Lankford, H., Loeb, S., McDonald, M., Reininger, M, Ronfeldt, M., & Wyckoff, J. (2008). Surveying the Landscape of Teacher Education in New York City: Constrained Variation and the Challenge of Innovation. *Education Evaluation and Policy Analysis.* 30(*4*), 319-345. https://www.jstor.org/stable/25478675

Boyd, D., Grossman, P., Hammerness, K., Lankford, H., Loeb, S., Ronfeldt, M., & Wyckoff, J. (2010). Recruiting effective math teachers: How do math immersion teachers compare? Evidence from New York. *National Bureau of Economic Research*. Htttp://www.nber.org/papers/w16017

Brenner, M. (1998). Adding cognition to the formula for culturally relevant instruction in mathematics. *Anthropology and Education Quarterly*, *29*(2), 214-244. https://www.jstor.org/stable/3196183

Bümen, N. T. (2010). *Çoklu Zeka: Eğitimde Yeni Yönelimler*. Ankara: Pegem Akademi.

Carpenter, T., Fennema, E., Peterson, P., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26(*4*), 499-531. http://eric.ed.gov/?id=EJ406340

Carraher, D. W., & Schliemann, A. D. (2007). Early algebra and algebraic reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (2nd ed., pp. 669-705). National Council of Teachers of Mathematics.

Center for Digital Dannelse. [Website]. (2016). https://www.apollo.io/companies/Center-for-Digital

Chimoni, M., & Pitta-Pantazi, D. (2015). Connections between algebraic thinking and reasoning processes. Ninth Congress of the European Society for Research in Mathematics Education. 398-404. https://hal.archives-ouvertes.fr/hal-01286897/document

Common Core State Standards Initiative [website]. (n.d.). Home | Common Core State Standards Initiative https://corestandards.org

Curtain, M. (2018). 10 skills employers will want the most in 2020. *Insider*. https://www.businessinsider.com/10-skills-employers-will-want-the-most-in-2020-2018-1

Cvencek, D., O'Connor, K., Wischnia, S., Nasir, N. S., and Meltzoff, A. N. (2014). The development of math-race stereotypes: "They say Chinese people are the best at math." *Journal of Research on Adolescence*, *25*(4), 630- 637. https://doi.org/10.1111/jora.12151

Damarin, S. (1995). Gender and mathematics from a feminist standpoint. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 242–257). New York: Cambridge University Press.

Davidson, K. (August 2016). Employers find 'soft skills' like critical thinking in short supply. *The Wall Street Journal*. https://www.opportunityamericaonline.org/wp-content/uploads/2016/08/EMPLOYERS-FIND-'SOFT-SKILLS'-LIKE-CRITICAL-THINKING-IN-SHORT-SUPPLY.pdf

Davis, H., Summers, J., & Miller, L. (2012). What does it mean for students to be engaged? In H. A. Davis (Ed.), *An interpersonal approach to classroom management: Strategies for improving student engagement*. (pp. 21-33). Corwin.

Debter, L. (February 2014). From correspondence courses to MOOCS: the highlights of distance learning over the ages. Forbes. https://www.forbes.com/sites/laurengensler/2014/02/12/from-correspondence-classes-to-moocs-the-highlights-of-distance-learning-over-the-ages/?sh=7c6a38527690

De Bono, E. (1987). The direct teaching of thinking as a skill. M. Heiman & J. Slomianko (Eds.) Thinking skills instruction: Concepts and techniques (pp. 217-229). Washington DC: National Education Association.

Demirel, Ö. (2009). *Öğretme Sanatı: Öğretim İlke ve Yöntemleri*. Ankara: Pegem Akademi

Edelman, A. (2022, February 1). More than half of teachers looking to quit due to covid, burnout, poll suggest. NBC News. https://www.nbcnews.com/politics/politics-news/more-half-teachers-looking-quit-due-covid-burnout-poll-suggest-n1288365

Ellis, A. B. (2007). Connections between generalizing and justifying students' reasoning with linear relationships. *Journal for Research in Mathematics Education*, 38(*3*), 194–229. https://www.jstor.org/stable/30034866

Emprin, F. (2010). A didactic engineering for teacher's education courses in mathematics using ICT. In Durand-Guerrier, V., Soury-Lavergne, S. and Arzarello, F. (Eds.), Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education (CERME6). 1290-1299. Lyon, France: INRP. https://hal.archives-ouvertes.fr/hal-02182374

English, L. D., & Sharry, P. V. (1996). Analogical reasoning and the development of algebraic abstraction. *Educational Studies in Mathematics*, 30(*2*), 135–157. doi: 10.1007/BF0030262

Eragamreddy, N. (2013). Teaching creative thinking skills. International Journal of English Language & Translation Studies, *1*(2): 124-145.

Fennema, E. (2000, May). Gender and mathematics: What is known and what do I wish was known? Paper presented at the 5th annual forum of the National Institute for Science Education. Detroit, MI.

Ferguson, K., Frost, L., Hall, D. (2012). Predicting teacher anxiety, depression, and job satisfaction. *Journal of Teaching and Learning*, 8(*1*). 27 - 42. https://doi.org/10.22329/jtl.v8i1.2896

Fisher, D., Frey, N., & Hattie, J. (2021). Module 6: Engaging tasks. In *The distance learning playbook.* (pp. 101-121). Corwin.

Fitzpatrick, B., Berends, M. Ferrare, J., & Waddington, R. (2020). Virtual illusion: Comparing student achievement and teacher and classroom characteristics in online and brick-and-mortar charter schools. *Educational Researcher*. 49(*3*). 161-175. 10.3102/0013189X20909814

Florida National University. (n.d.). *The Evolution of Distance Learning*. Radio, television, and development. https://www.fnu.edu/evolution-distance-learning/

Freedberg, L. (2020, October). Sal Khan says distance learning can't fully replace in-person instruction. *EdSource*. https://edsource.org/2020/sal-khan-says-distance-learning-cant-fulky-replace-in-person-istrution/637541

Friel, S., Rachlin, S., Doyle, D., Nygard, C., Pugalee, D., & Ellis, M. (2001). *Navigating through algebra in grades 6–8. Principles and standards for school mathematics navigations series*. Reston, VA: National Council of Teachers of Mathematics.

Garet, M., Wayne, A., Stancavage, F., Taylor, J., Eaton, M., Walters, K., et al. (2011). Middle school mathematics professional development impact study: Findings after the second year of implementation (NCEE 2011–4024). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. http://eric.ed.gov/?id=ED519922

Garet, M., Wayne, A., Stancavage, F., Taylor, J., Walters, K., Song, M., et al. (2010). Middle school mathematics professional development impact study: Findings after the first year of implementation (NCEE 2010–4009). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. http://eric.ed.gov/?id=ED509306

Gautam, P. (2020, October 10). Advantages and Disadvantages of online learning. eLearning Industry. https://elearningindustry.com/advantages-and-disadvantages-online-learning

Gay, G. (1988). Designing relevant curricula for diverse learners. *Education and Urban Society*, *2*(4), 327-340. https://doi.org/10.1177/0013124588020004003

Gay, G. (2010). Acting on beliefs in teacher education for cultural diversity. *Journal of Teacher Education*, *61*(1-2). 143-152. https://doi.org/10.1177/0022487109347320

Gersten, R., Taylor, M. J., Keys, T. D., Rolfhus, E., & Newman-Gonchar, R. (2014). Summary of research on the effectiveness of math professional development approaches. (REL 2014–010). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southeast. http://ies.ed.gov/ncee/edlabs.

Gokhale, A. A. (1995). Collaborative learning enhances critical thinking. *Journal of Technology Education*, 7(*1*), 22-30. https://doi.org/10.21061/jte.v7i1.a.2

Grant, H. (July 2020). Keeping mathematics education real and fair; Connecting the disconnections. In C. Xenofontos [book review] Equity in mathematics education: Addressing a changing world. Springer. http://dx.doi.org/10.1007/s10649-020-09958-z

Greene, B., Miller, R., Crowson, H., Duke, B., & Akey, K. (2004, March 19). Predicting high school students' cognitive engagement and achievement: Contributions of classroom perceptions and motivation. *Contemporary Educational Psychology*. 29, pp. 462-468.

Gregory, E., Hardiman, M., Yarmolinskaya, J., Rinne, L., & Limb, C. (2013). Building creative thinking in the classroom: From research to practice. International Journal of Educational Research, *62*, 43–50.

Gust, H. Kühnberger, K., Rollinger, C., & Schmid, U. (2003, September 16). Algebraic models of reasoning. [workshop]. https://www.researchgate.net/publication/215992323_Algebraic_Models_of_Reasoning

Gutstein, E., Lipman, P., Hernandez, P., & De los Reyes, R. (1997). Culturally relevant mathematics teaching in a Mexican American context. *Journal for Research in Mathematics Education*, *28*(6), 709-737.

Hammond, Z. L. (2015). *Culturally Responsive Teaching and the Brain: Promoting Authentic Engagement and Rigor among Culturally and Linguistically Diverse Students*, Corwin.

Hanushek, E., Kain, J., & Rivkin, S. (2004). Why public schools lose teachers. *Journal of Human Resources*. 39(*2*) 326-254. https://www.nber.org/papers/w8599

Heijltjes, A., Van Gog, T., Leppink, J., & Paas, F. (2014a). Improving critical thinking: Effects of dispositions and instructions on economics students' reasoning skills. *Learning and Instruction*, *29*, 31–42. https://doi.org/10.1016/j.learninstruc.2013.07.003

Hertert, L. (1997). Investing in Teacher Professional Development: A Look at 16 Districts. Denver, Colorado: Education Commission of the States. https://files.eric.ed.goc/fulltext/ED416201.pdf

Hill, H. C. (2008). The Nature and Effects of Middle School Mathematics Teacher Learning Experiences. Manuscript. Harvard Graduate School of Education. https://eric.ed.gov/?id=EJ913421

Hill, H. (2009, March). Fixing teacher professional development. Phi Delta Kappan, 90(7). 470-476. https://doi.org/10.1177/003172170909000705

Horizon Research. (2002). The 2000 National Survey of Science and Mathematics Education: Compendium of Tables. Chapel Hill, N.C.

Howe, A., & Jones, L. (1993). *Engaging Children in Science*. New York: Macmillan.

Hu, W., Adey, P., Jia, X., Liu, J., Zhang, L., Li, J., & Dong, X. (2011). Effects of a "Learn to Think" intervention programme on primary school students. *British*

Journal of Educational Psychology, 81: 531–557. https://www.researchgate.net/publication/49722340

Illinois State Board of Education. (2020). Remote learning recommendations during COVID-19 emergency. https://www.isbe.net/Documents/RL-Reommendations-3-27-20.pdf

Inprasitha, M., Changsri, N., & Boonsena, N. (July 2021). Proceedings of the 44th conference of the international group for the psychology of mathematics education. The 44th Conference of the International Group for the Psychology of Mathematics Education. https://pme44.kku.ac.th/home/uploads/volumn/pme44_vol1.pdf#page=98

International Society for Technology in Education. (n.d.). https://www.iste.org/standards/iste-standards-for-teachers

Jackson, R. M. (1986). Thumbs up for the direct teaching of thinking skills. *Educational Leadership*, 43: 32-36. https://www.ascd.org/el/articles/thumbs-up-for-direct-teaching-of-thinking-skills

Jacobs, V., Franke, M., Carpenter, T., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education*, 38(*3*), 258-288. http://eric.ed.gov/?id=EJ765495

Jennings, P., & Greenberg, M. (2008). The prosocial classroom: Teacher social and emotional competence in relation to student and classroom outcomes. Review of Educational Research, 79:491-525. Doi:10.3102/0034654308325693

Johnson, M. A., & Lawson, A. E. (1998). What are the relative effects of reasoning ability and prior knowledge on biology achievement in expository and inquiry classes? *Journal of Research in Science Teaching*, 35 (*1*), 89-103. https://doi.org/10.1002/(SICI)1098-2739(199801)35:1<89::AID-TEA6>3.0.CO;2-J

Jones (2004). A review of the research literature on barriers to the uptake of ICT by teachers. *British Educational Communications and Technology Agency* (BECTA). http://dera.ioe.ac.uk/1603/1/becta_2004_barrierstouptake_litrev.pdf

Kaput, J. (1998). Transforming algebra from an engine of inequality to an engine of mathematical power by “algebrafying” the k-12 curriculum. In Fennell, S. (Ed.) The nature and role of algebra in the k-14 curriculum: Proceedings of a national symposium. Washington, DC. National Research Council, National Academy Press. https://files.eric.gov/fulltext/ED441664.pdf

Kaput, J. (1999). Teaching and learning a new algebra. In Fennema, E. & Romberg, T. (Eds) Mathematics classrooms that promote understanding. Mahwah, Nj: Erlbaum.

Kaput, J. (2000). Transforming algebra from an engine in Inequality to an engine of mathematical power by “algebrafying” the K-12 curriculum. [paper] National Center for Improving Student Learning and Achievement in Mathematics and Science. Darmouth, MA.

Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 5–17). New York: Routledge. https:/.doi.org/10.4324/9781315097435

Kaput, J., & Blanton, M. (1999). Algebraic reasoning in the context of elementary mathematics: Making it implementable on am massive scale. https://files.eric.ed.gov/fulltext/ED441663.pdf

Kieran, C. (1992). The learning and teaching of school algebra." In Grouws, D. (Ed.) *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan Publishing Co., and Reston, Va. National Council of Teachers of Mathematics.

Kieran, C. (1996). The changing face of school algebra. In C. Alsina, J. Alvarez, B. Hodgson, C. Laborde, & A. Pérez (Eds.), *8th International Congress on Mathematical Education*: *Selected lectures* (pp.271–290). Seville, Spain.

Killeen, K. M., Monk, D. H., & Plecki, M. L. (2002). School district spending on professional development: Insights available from national data (1992-98). *Journal of Education Finance* 28: 25-50. https://www.education.uw.edu/ctp/sites/default/files/ctpmail/PDFs/JEFArticle-KKDMMP.pdf

Koehler, M. (2012). TPACK explained. https://mattkoehler.com/tpack2/tpack-explained

Korkmaz, Ö. (2009). Öğretmenlerin Eleştirel Düşünme Eğilim ve Düzeyleri. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi* (KEFAD), 10 (*1*), 1-13. https://hdl.handle.net/20.500.12513/289

Kramarski, B. (2004). Making sense of graphs: Does metacognitive instruction make a difference on students' mathematical conceptions and alternative conceptions. *Learning and Instruction*, 14, 593–619.

Kuchemann, D. (1981). Algebra. *Children's Understanding of Mathematics*: *11*(16), 102–119.

Ladson-Billings, G. (2006). Yes, but how do we do it? N J. Landsman and C. Lewis (eds.), *White teacher/diverse classrooms: A guide to building inclusive schools. Promoting high expectations, and eliminating racism*. Sterling, VA: Stylus Publishing. P. 29-42

Lappan, G., & Briars, D. (1995). How should mathematics be taught? In I. M. Carl (Ed.), *Seventy-five years of progress: Prospects for school mathematics* (pp. 115-156). Reston, VA: National Council of Teachers of Mathematics.

Lawson, A. E., Banks, D. L. & Logvin, M. (2006). Self-efficacy, reasoning ability and achievement in college biology. *Journal of Research in Science Teaching*, https://doi.org/10.1002/tea.20172.

Leinwand, S., Brahier, D. and Huinker, D. (2014). Principles to Action. (pp. 18). Reston, VA: National Council of Teachers of Mathematics.

Leder, G. C. (1992). Mathematics and gender: Changing perspectives. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 597–622). New York: Macmillan.

Lempp, S. (2008). Lecture notes for math 135: Algebraic reasoning for teaching mathematics [Lecture Notes]. University of Wisconsin-Madison. https://people.math.wisc.edu/~lempp/teach/135.pdf

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (ed.), Multiple perspectives on mathematics teaching and learning. p. 19-44. Westport: Ables Publishing

Lerman, S. (2006). Cultural psychology, anthropology and sociology: The developing 'strong' social turn. In J. Maasz & W. Schloeglmann (eds.), New mathematics education research and practice. p. 171-188. Rotterdam: Sense

Lessard, L., & Schacter, H. (April 2020). Why the coronavirus crisis hits teenagers particularly hard: Developmental scientists explain. Education Week. https://www.edweek.org/leadership/opinion-why-the-coronavirus-hits-teenagers-particularly-hard/2020/04

Logical Thinking Helping Children to Become Smarter (2010). http://logizspark.com/general-logics/logical-thinking.

Macgregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11–15. *Educational Studies in Mathematics*, 33, 1 –19.

Mahalingam, M., Schaefer, F., & Morlino, E. (2008). Promoting student learning through group problem solving in general chemistry recitations. *Journal of Chemical Education*, 85(*11*), 1577-1581.http://dx.doi.org/10.1021/ed085p1577

Marr, B. (2019, October 28). The 10+ most important job skills every company will be looking for in 2020. Forbes. https://www.forbes.com/sites/bernardmarr/2019/10/28/the-10-most-important-job-skills-every-company-will-be-looking-for-in-2020/?sh=756310f167b6

Maryville University. (2022). Top 16 skills needed in 2022 and beyond. https://online.maryville.edu/blog/2020-skills/

Marzano, R. J. (1993). How classroom teachers approach the teaching of thinking. *Theory into Practice*, *32*(3): 154-160. https://www.jstor.org/stable/pdf/1476696

McMeeking, L. S., Orsi, R., & Cobb, R. B. (March 2012). Effects of a teacher professional development program on the mathematics achievement of middle school students. *Journal for Research in Mathematics Education*. *43*(*2*), 159-181. https://doi.10.5951/jersematheduc.43.2.0159

McCormick, N., Clark, L., & Raines, J. (2015). Engaging students in critical thinking and problem solving: A brief review of the literature. Journal of Studies in Education. 5(4), 100-113. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.960.810&rep=rep1&type=pdf

Merriam-Webster. (n.d.a). Conceptual. In Merriam-Webster.com dictionary. https://www.merriam-webster.com/dictionary/conceptual

Merriam-Webster. (n.d.b). Distance learning. In Merriam-Webster.com dictionary. https://www.merriam-webster.com/dictionary/distance%20learning

Merriam-Webster. (n.d.c). Inequalities. In Merriam-Webster.com dictionary. https://www.merriam-webster.com/dictionary/inequalities

Merriam-Webster. (n.d.d). Learning. In Merriam-Webster.com dictionary. https://www.merriam-webster.com/dictionary/learning

Merriam-Webster. (n.d.e). Understanding. In Merriam-Webster.com dictionary. https://www.merriam-webster.com/dictionary/understanding

Miles, K. H. (2003). Rethinking district professional development spending to support school improvement: Lessons from comparative spending analysis." In Margaret L. Plecki and David H. Monk (ed). *School Finance and Teacher Quality: Exploring the Connections*. Larchmont, N.Y.: Eye on Education.

https://gtlcenter.org/sites/default/files/docs/pa/7_PlenarySessions/PDSpending/RethinkingPDSpending.pdf

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A new framework for teacher knowledge. *Teachers College Record*, 108(*6*), 1017-1054. https://www.jwilson.coe.uga.edu/EMAT7050/areicles/KoehlerMishra.pdf

Morrison, K., Robbins, H., & Rose, D. (2008). Operationalizing culturally relevant pedagogy; A synthesis of classroom-based research. *Equity & Excellence in Education*, *41*(4), 433-452. https://doi.org/10.1080.10665680802400006

Missouri Department of Elementary and Secondary Education [MODESE]. (2022a). Missouri Learning Expectations. https://files.gabbart.com/1289/curr-mls-coding-sheet.pdf

Missouri Department of Elementary and Secondary Education [MODESE]. (2022b). Missouri Learning Standards. https://dese.mo.gov/college-career-readiness/curriculum/missouri-learning-standards

Missouri Department of Elementary and Secondary Education [MODESE]. (2022c). Missouri Show Me Standards.

https://dese.mo.gov/college-career-readiness/curriculum/show-me-standardsNational Assessment of Education Progress. (2019). *The nation's report card*. https://www.nationsreportcard.gov/mathematics/nation/scores

National Association of State Directors of Teacher Education and Certification [NASDTEC]. (2004). *Knowledgebase Table E1: Professional Development Description*. Whitinsville, Mass.: NASDTEC.

National Center for Education Statistics (NCES). (2001). *Teacher Preparation and Professional Development*: *2000.* NCES 2001-088. Washington D.C.: U.S. Department of Education. https://eric.ed.gov/?id=ED546035

National Council of Teachers of Mathematics. (2008, January). Equity in mathematics education. https://www.nctm.org/about/content.aspx?id=13490

National Council of Teachers of Mathematics. (2014a). *Teaching and Learning Beliefs Survey*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014b). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2020). What is procedural fluency, and how do we help students develop it? A position of the National Council of Teachers of Mathematics. http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/-Access and

equity in mathematics education. https://www.nctm.org/Sandards-and-Positions/Position-Statements/Access-and-Equity-in-Mathematics-Education

National Council of Teachers of Mathematics. (n.d.). Chart. https://www.nctm.org/uploadedFiles/Conferences_and_Professional_Development/Institutes/Supporting_Students_Productive_Struggle/Session3%20-%203-5.pdf

National School Board Association (December 2020). Equity. https://www.nsba.org/Advocacy/Equity

New York State Education Department [NYSED]. (2019). Culturally responsive-sustaining education. Retrieved from http://www.nysed.gov/crs

Odden, A., Archibald, S., Fermanich, M., & Gallagher, H. A. (2002). A cost framework for professional development." *Journal of Education Finance* 28: 51-74. http://www.jstor.org/stable/40704157

Oldridge, M. (September 2019). Why should teachers integrate technology in the math classroom? Learning.com. https://equio.learnig.com/integrate-technology-math-classroom

Organization for Economic Co-operation and Development. (2013). *First results from PISA 2003.* https://www.oecd.org/education/school/programmeforinternationalstudentassessmentpisa/34002454.pdf

Organization for Economic Co-operation and Development. (2013). *First results from PISA 2003.* https://nces.ed.gov/surveys/pisa/pisa2018/pdf/PISA2018_compiled.pdf

Osisioma, I. U., Kiluva-Ndunda, M. M., Van Sickle, M. L. (2008). Behind the masks: Identifying students' competencies for learning mathematics and science in urban settings. *School Science and Math, 108*(8), 389–400.

Paul, R. W. (1990). *Critical Thinking: How to prepare Students for a rapidly changing world*. 56-97. California: Dillon Beach.

Paul, R. W., & Elder, L. (2006). *Critical Thinking: Tools for Taking Charge of Your Learning and Your Life*. Upper Saddle River, NJ: Prentice Hall.

Perry, R. R., & Lewis, C. C. (2011). Improving the mathematical content base lesson study summary of results. https://www.lessonresearch.net/ISEAbstract10.pdf

Pena, C., & Almaguer, I. (2012). The use of online discussions to foster critical thinking in a teacher education program. *International Journal of Instructional Media*, 39 (1): 25-32.

Pressley, T. (2021, March 16). Factors contributing to teacher burnout during covid-19. *Educational Researcher.* 50(5), 325-327

Pintrich, P. R. (2003). Motivation and classroom learning. In W. M. Reynolds & G. E. Miller (Eds.), *Handbook of Psychology: Educational Psychology,* Vol. 7, pp. 103–122). John Wiley & Sons Inc.

Pykett, J. (2004). Using debate to promote critical thinking in citizenship education. A Report commissioned by the Training and Development Agency for Schools (TDA). https://www.citized.info/pdf/commarticles/Jessica_Pykett.pdf

Radford, L. (2008). Iconicity and contraction: a semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM*, 40(*1*), 83–96. doi: 10.1007/ s11858-007-0061-0

Rattan, A., Good, C., & Dweck, C. S. (2012). It's ok, not everyone can be good at math: Instructors with an entity theory comfort (and demotivate) students. *Journal of Experimental Social Psychology*, 48(*3*), 731-737. https://doi.org/10.1016/j.jesp.2011.12.012.

Rivera, F., & Becker, J. (2007). Abduction – Induction (generalization) processes of elementary majors on figural patterns of algebra. *Journal of Mathematical Behavior*, 26(*2*), 140–155. doi:10.1016/j.jmathb.2007.05.00

Romberg, T., & Kaput, J. (1999). Mathematics worth teaching, mathematics worth understanding. In E. Fennema & T. Romberg (Eds.). Mathematics classrooms that promote understanding. Mahwah, NJ: Erlbaum.

Saphier, J. (2017). The equitable classroom. *The Learning Professional, 38*(6), 28-3. https://eric.ed.gov/?id=EJ1166966

Schlechty, P.C. (2001). Shaking up the schoolhouse. San Francisco, CA: John Wiley and Sons.

Schlechty, P.C. (2011). Engaging students: The next level of working on the work. San Francisco, CA: John Wiley and Sons.

Sciarra, D., & Seirup, H. (2008). The multidimensionality of school engagement and math achievement among racial groups. *Professional School Counseling*, *11*(4), 218-228.

Scriven, M., & Paul, R. (2008). Defining critical thinking. https://www.criticalthinking.org/pages/defining-critical-thinking/766

Senemoğlu, N. (2004). *Gelişim Öğrenme ve Öğretim*, 46-56. Ankara: Gazi Kitabevi.

Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls reification—The case of algebra*.* *Educational Studies in Mathematics*, 26, 191–228.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(*2*), 4-14. https://www.jstor.org/stable/24636916

Simonson, M., & Schlosser, L. A. (2006). Distance Education: Definition and Glossary of Terms.

Singh, J., & Singh, J. (2020). COVID-19 and its impact on society. *Electronic Research Journal of Social Sciences and Humanities*. 2(*1*). www.eresearchjournal.com

Skinner, E. A., & Belmont, M. J. (1993). Motivation in the classroom: Reciprocal effects of teaching behavior and student engagement across the school year. *Journal of Educational Psychology, 85*(4), 571-581.

Skov, A. (2016). What is digital competence? https://digital-competence.eu/dc/front/what-is-digital-competence

Skovsmose, O., & Valero, P. (2001). Breaking political neutrality: The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, & B. Nebres (eds.). *Sociocultural research on mathematics education. An international perspective* (pp. 37-55). Mahwah: Erlbaum.

Skovsmose, O., & Valero, P. (2002). Democratic access to powerfull mathematical ideas. In L. D. English (ed.) Handbook of international research in mathematics education. Directions for the 21st century. p. 383-407. Mahwah: Erlbaum

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

Snyder, A., Trowery, L., & McGrath, K. (2019). Guiding principles for equity in education. mheonline.com/equity.

Spiteri, M., & Rundgren, S. N. C. (2020). Literature review on the factors affecting primary teachers' use of digital technology. Technology, Knowledge and Learning. 25, 115-128. https://doi.org/10.1007/s10758-018-9376-x

Stanford Encyclopedia of Philosophy. (n.d.). Critical thinking. https://plato.stanford.edu/entries/critical-thinking/#DefiCritThin

Stein, M. K., Grover, B. W., & Henningsen, M. A. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal* 33:455-488.

Stein, M., & Smith, M. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268-275.

Sternberg, R. (1985). Critical thinking, its nature, measurement and improvement. In Link, F. (ed.). Essays on the Intellect (pp. 46). Association for Supervision and Curriculum Development.

Sullivan, P., Clarke, D., & Clarke, B. (2013). *Teaching with tasks for effective mathematics learning*. New York: Springer.

Talusan, L. A. (2022). *The identity conscious educator,* Solution Tree Press.

Tobin, K. G., & Capie, W. (1981). The development and validation of a group test of logical thinking. *Educational and Psychological Measurement*, *41*, 413-423. https://doi.org/10.1177/001316448104100220

The University of Alaska – Fairbanks. Twelve math myths. https://uaf.edu/deved/math/help-for-math-anxiety/12-math-myths

U.S. Bureau of Labor Statistics. (2022). Occupational Outlook Handbook. https://www.bls.gov/ooh/

Utah Education Policy Center. (2012). Chronic absenteeism. [Research brief]. Uepc.utah.edu

Valero, P., & Zevenbergen, R. (2004). Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology. Boston: Kluwer Academic Publishers

Van Deursen, A. J. A. M. (2010). Internet skills. Vital assets in an information society. University of Twente. https://10.3990/1.9789036530866

Van de Walle, (2016). In Teaching mathematics for understanding in Kim Markworth, Jenni McCool, and Jennifer Kosiak problem solving in all seasons. Reston, VA. National Council of Teachers of Mathematics

Waddell, L. (2014). Using culturally ambitious teaching practices to support urban mathematics teaching and learning. *Journal of Praxis in Multicultural Education*, *8*(1), Article 2. https://digitalscholarship.unlv.edu/cgi/viewcontent.cgi?article=1069&context=jpme

Windschitl, M. (2019). Disciplinary literacy versus doing school. *Journal of Adolescent & Adult Literacy*, 63(*1*), 7-13.

Wood, E., Levinson, M., Postlethwaite, K., & Black, A. (2011). *Equity matters*. Brussels: EI. EN/ FR.

World Health Organization (2020). Coronavirus Disease 2019 (COVID-19) Situation Report-97. https://www.who.int/docs/default-sorce/coronaviruse/situation-reports/20200426-sitrep-97-covid-19.pdf

Wu, F., Zhao S., Yu B., Chen, Y. M., Wang, W., Song, Z. G., Hu, Y., Tao, Z. W., Tian, J. H., Pei, Y. Y., Yuan. M. L., Zhang, Y. L., Dai, F. H, Liu, Y., Wang, Q. M., Zheng, J. J., Xu, L., Holmes, E., & Zhang, Y. Z. (2020). A new coronavirus associated with human respiratory disease in China. Nature. 579(*7798*) p. 265–269. https://www.ncbi.nlm.nih.gov/pubmed/32015508

Zhou, P., Yang X. L., Wang, X. G., Hu, B., Zhang, L. Zhang, W., Si, H. R., Zhu, Y., Li, B., Huang, C. L., Chen, H. D., Chen, J., Lou, Y., Guo, H., Jiang, R. D., Liu, M. Q., Chen. Y., Shen, X. R., Wang, X., Zheng, X. S., Zhao, K., Chen, Q. J., Deng, F., Liu, L. L., Yan, B., Zhan, F. X., Wang, Y. Y., Xiao, G. F., & Shi, Z. L. (2020). A pneumonia outbreak associated with a new coronavirus of probable bat origin. *Nature*. 579(*7798*) p. 270–273. https://www.ncbi.nlm.nih.gov/pubmed/32015507

Zilvinskis, J., Masseria, A. A., & Pike, G. R. (2017). Student engagement and student learning: Examining the convergent and dicriminant validity of the revised National Survey of Student Engagement. *Research in Higher Education 58*, 880–903. https://doi.org/10.1007/s11162-017-9450-6

Appendix A

Beliefs About Teaching and Learning Mathematics

Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Source: NCTM (2014b)

Appendix B

Teacher Survey Prompts

1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.

2. The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.

3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.

5. Mathematics learning should focus on developing understanding of concepts and procedures through problems solving, reasoning, and discourse.

6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

7. Students can learn to apply mathematics only after they have mastered the basic skills.

8. Students can learn mathematics through exploring and solving contextual and mathematical problems.

9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.

11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Source: Beliefs about Teaching and Learning Mathematics. (NCTM, 2014a)

Appendix C

Participant Characteristics

Table 59-A

Participant Characteristics

Participant	# Years Taught	Highest Degree Earned	Grade Level Taught	# Years in Position
1	12 – 17	Bachelor’s	Elementary	8 – 11
2	0 – 5	Bachelor’s	Elementary	0 - 3
3	0 – 5	Bachelor’s	Elementary	0 - 3
4	6 – 11	Bachelor’s	Elementary	0 – 3
5	6 – 11	Bachelor’s	Middle	4 – 7
6	12 – 17	Doctorate	High	4 - 7
7	18+	Doctorate	Middle	12+
8	12 – 17	Master’s	Elementary	4 - 7
9	6 – 11	Master’s	Elementary	4 - 7
10	18+	Master’s	Elementary	0 - 3
11	6 – 11	Master’s	Elementary	8 - 11
12	6 – 11	Master’s	Elementary	0 - 3
13	12 – 17	Master’s	Elementary	8 - 11
14	6 – 11	Master’s	Elementary	0 – 3
15	12 – 17	Master’s	Elementary	0 - 3
16	0 – 5	Master’s	Elementary	0 - 3
17	12 = 17	Master’s	Elementary	12+
18	12 – 17	Master’s	High	8 - 11
19	12 – 17	Master’s	High	12+
20	6 – 11	Master’s	High	8 - 11
21	18+	Master’s	High	12+
22	6 – 11	Master’s	High	8 - 11
23	6 – 11	Master’s	High	4 - 7
24	6 – 11	Master’s	Middle	4 - 7
25	18+	Master’s	Middle	12+