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An Investigation of Success Factors in a High School Algebra I Program

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Declaration of Originality

I do hereby declare and attest to the fact that this is an original study based solely upon my own scholarly work here at Lindenwood University and that I have not submitted it for any other college or university course or degree here or elsewhere.

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Abstract

The purpose of this study was to investigate a high school Algebra I program through an examination of potential relationships among teachers' beliefs about teaching and learning mathematics, teachers' instructional styles, students' academic self-concept in mathematics, and students' mathematics achievement in an Algebra I course. While a significant amount of research has focused on individual components of the study (e.g., instructional practices, academic self-concept), little research has been done to identify potential relationships between factors. Additionally, much of the available research focused on the elementary level leaving a gap in understanding how various factors were related to the success of high school students. Results of this study will provide teachers and administrators at the research site with information regarding the relationship between several factors shown to impact student achievement in mathematics in order to evaluate the current Algebra I program. Findings will also enhance the understanding of the relationships among teacher and student components, specifically, at the secondary level.

For this quantitative study, both survey and observational tools were used to collect information from teachers and students in the 2016-2017 school year. Teachers were surveyed using the National Council of Teachers of Mathematics Teaching and Learning Beliefs Questionnaire to identify their beliefs about teaching and learning mathematics, and the researcher utilized the Reformed Teaching Observation Protocol (RTOP) to determine how closely instruction in Algebra I classrooms aligned with constructivist practices. Students were given the Academic Self-Description Questionnaire II (ASDQII) as both a pre-and post-measure of their academic self-concept in relation to mathematics, and student scores on common semester final exams provided additional data for study.

Using a Pearson Product Moment Correlation Coefficient (PPMCC) to determine if a relationship existed between various factors studied revealed few statistically significant relationships between student achievement and factors, such as teacher beliefs about teaching mathematics and the teaching style used in the classroom. Despite a large amount of research in education showing the importance of the teacher in relation to achievement, this study did not support a similar conclusion. Instead, the only statistically significant relationship shown was between students' academic self-concept in mathematics and their overall mastery of Algebra content, as measured by scores on semester finals. These results encourage educators and administrators to consider the impact of student grouping for instruction and how to best support students for success in mathematics.

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Chapter One: Introduction

Purpose of the Dissertation

The purpose of this quantitative study was to investigate the implementation of an Algebra I program at a suburban high school by examining various factors including; teacher beliefs about teaching math; students' academic self-perception related to math; instructional practices in algebra classrooms; and the amount of time a student receives intervention support in addition to his/her regularly scheduled Algebra I course to determine which factors, if any, have an impact on student achievement in Algebra I. Several sources of data including scores on teacher-created common assessments, results of a student academic self-perception survey, a teacher beliefs survey, the number of times a student works with the math interventionist, and analysis of instructional practices via classroom observations were used to determine possible relationships between the various factors.

This study was the result of the researcher's experience with the supervision of the Algebra I program at the high school level and administration's frustration with lower-than-anticipated scores on Algebra I End of Course Exams (EOCs) despite the school being considered one of the top in the state and efforts to provide support for students in Algebra I through a multi-tiered system of support (MTSS).

Statement of the Problem

Although research generally supported Algebra for 8th graders, some students found early Algebra challenging (Clotfelter et al., 2015; National Council of Teachers of Mathematics [NCTM], 2014; Stein et al., 2011). On a national level, despite legislation meant to ensure high school graduates in the United States were prepared for post-secondary endeavors including two- and four-year colleges, military service, and workforce employment, only a slight improvement in mathematics scores was evident (Mullis et al., 2020; U.S. Department of Education, 2016). National Assessment of Educational Progress (NAEP) reported a drop in mathematics scores in 2015 with high school seniors scoring (Mullis et al., 2020) lower than seniors who took the test in 2013 (The Nations Report Card, 2016). Additionally, professional organizations, such as the National Council for Teachers of Mathematics (NCTM), in an effort to increase achievement, revised standards to outlining not only content, but process standards including, "problem solving, reasoning and proof, communication, connections, and representation" (National Council of Teachers of Mathematics, n.d., para. 5).

Despite these efforts, gains in mathematics scores on international and national assessments failed to rise significantly (OECD, 2018; Mullis, et.al., 2020). Likewise, despite being recognized as a Top Ten school in the state in which the research site was located, scores on Algebra I EOCs were not improving at the expected rate despite efforts at student support (Missouri Department of Elementary and Secondary Education, n.d.). It was important to the researcher to identify which factors included in the study, if any, were related to student achievement in order to provide information for future program revisions, if needed.

Rationale

The research site, out of a desire to improve achievement for students struggling to meet grade level expectations in Algebra I, responded by restructuring the way in which Algebra I students could access instructional supports. After receiving an additional full-time-employee (FTE), the high school reviewed both reading and math data, and determined the greatest need to be in supporting mathematics achievement though a focus on what was often a gate-keeper class, Algebra I (Burdman, 2018; Cortez et al., 2013; Fisher & Frey, 2013; Loveless, 2013). The site added an Algebra I support teacher who had a background in special education and structured this teacher's day in a way that provided time for analysis of achievement data for all students in Algebra I and time for this additional teacher to meet with students who showed signs of struggle, as identified by classroom data, Algebra I teachers, or the students themselves. Changes to the Algebra I curriculum and instructional methods of the teachers were not addressed.

At the time of the study, the support role had been in place for two years. Standardized test data showed little change in the overall achievement of students on the Algebra I state assessment, and the researcher wanted to more-closely examine the program by studying the relationships among student self-concept, teacher beliefs about teaching math, instructional styles, and overall achievement in Algebra I. Identification of relationships, or lack thereof, could then be used to inform potential next steps regarding the structure of the Algebra I program.

Hypotheses

Because the goal of the research was to identify which elements, if any, among student, teacher, and instructional factors were related, a quantitative approach was selected for the research design (Fraenkel et al., 2012; University of Southern California Libraries Research Guide, 2019).

Hypothesis 1: There is a relationship between students' academic self-concept, as measured by the ASDQII, and teachers' beliefs about the teaching of algebra as measured by the Teacher Belief Survey.

Hypothesis 2: There is a relationship between students' academic self-concept in math, as measured by the ASDQII and the instructional practices in the classroom as measured by the Reformed Teaching Observation Protocol.

Hypothesis 3: There is a relationship between Algebra I mastery, as measured by student results on Algebra I semester comprehensive finals and teachers' beliefs about the teaching of algebra, as measured by the Teacher Belief Survey.

Hypothesis 4: There is a relationship between Algebra I mastery, as measured by student results on Algebra I comprehensive semester finals and the instruction used in the Algebra I classrooms, as measured by the Reformed Teaching Observation Protocol.

Hypothesis 5: There is a relationship between teacher beliefs, as measured by the Teacher Belief Survey and instructional practices in the Algebra I classroom, as measured by the RTOP.

Hypothesis 6: There is a relationship between student academic self-concepts, as measured by the ASCDII and Algebra I mastery, as measured by scores on Algebra I comprehensive semester finals.

Research Participants and Research Site

The research population included four teachers and 203 students. The four teachers were selected because they were the only teachers assigned to teach Algebra I at the research site. The students were selected using a random sample described below.

The math department consisted of 15 teachers, with four teachers responsible for Algebra I instruction taking part in the study. Of the four teachers, one was in the second year of teaching; one had nine years of experience; the third had been teaching for ten years, and the fourth brought 23 years of experience to the classroom. None of the four teachers had taught other content areas outside of math, nor had any taught at a level different than high school or grades 9 to12. Two of the four teachers had experience in another district and two of the four teachers achieved National Board Certification in Adolescence and Young Adulthood in Mathematics. Of the two teachers with National Board Certification, one was recognized as a Teacher of the Year at a high school in a neighboring district.

Teachers at the research site had the opportunity to regularly participate in professional development at the district level, department level, and independently. The district provided professional learning during two full-day sessions for all teachers in the district on the topic of equity, which was a district-wide focus at the time of the study. The district was also leading a mathematics curriculum cycle review and one teacher in the study served as a high school representative on the district committee. In addition, teachers in their first and second years of teaching were provided a mentor and regularly attended district-level new teacher meetings with their mentors. Topics for new teacher support followed the state requirements and included classroom management, student engagement, professional communication, and school law. One teacher in the study was in the second year of the district mentoring program.

The research site was in the beginning stages of learning to create common assessments and utilize student responses to inform instruction and respond to students' needs. Teachers participated in this professional development during after school faculty meetings and monthly early-release days. Due to the nature of the data team work, teachers typically worked either as a department or according to the courses they were teaching, which meant that the Algebra I teachers met multiple times throughout the year to develop common assessments and to discuss data generated by the assessments. In addition, teachers were able to request funds for external workshops and conferences, and many of the math teachers in the department attended the National Council of Teachers of Mathematics (NCTM) conference, along with other independently selected single day events.

While participation at district and building-level professional development was expected by administrators, follow up differed depending on the type of professional development provided. District-level learning did not typically include follow up to ensure application of the new learning leading to little or no change in teacher practice. At the building level, the administrators of the research site frequented department meetings to support application of new learning around common assessments and data teams, and there was an expectation for each course to develop four common assessments during the time span of the study, which the Algebra I teachers accomplished. Teachers were not required to provide evidence of application of new learning from the independently selected learning opportunities.

The student research population consisted of 203 high school students enrolled in an Algebra I course at the research site. Students' enrollment in Algebra I was based on several factors, including recommendation by an eighth-grade teacher or counselor, parent request, lack of Algebra I credit due to having transferred from a private school or other institution that did not offer Algebra I to eighth-grade students, and repeating the course due to earning a failing grade.

A random sample of students enrolled in Algebra I at the high school was selected by sharing an informational video with students and asking students' teachers to hand out assent and consent forms. In addition, the researcher attended parent open house in the fall of 2016 in order to answer any questions and encourage participation in the study. All students enrolled in Algebra I at the high school were eligible to participate, and, based on having returned both student assent forms and parental consent forms, the resulting sample included 71students.

As shown in Table 1, students ranged in age from 14 to 17 with the majority of participants reported as Caucasian. In addition, most scored proficient or advanced on the state standardized assessment taken when students were in eighth grade. Due to transfers from private or parochial schools, some students did not have an eighth-grade standardized test score, as they were not required to take the assessment.

Table 1

Study Sample Demographic Characteristics

Characteristic	Number	Percentage of Sample
Age		
14	49	72
15	14	21
16	3	4
17	2	3
Race		
African American	7	10
Asian	4	6
Caucasian	51	75
Hispanic	3	4
Multi-racial	3	4
Gender		
Female	37	54
Male	31	46
8th Grade Missouri Assessment in Mathematics Proficiency Score		
Below Basic	6	10
Basic	7	10
Proficient	38	56
Advanced	5	7
No Score Available	12	17
Transfer		
From Public School	4	6
From Private School	8	12

Limitations

Because there were only four teachers assigned to teach Algebra I during the study, one limitation was the small population size accessible to the researcher. While the results of the study may not be easily generalized to all Algebra I teachers in all settings, according to Fraenkel et al. (2012), “a population can be any size” (p. 92) and the population of four teachers can be appropriately applied to all Algebra I teachers at the study site during the 2016-2017 school year. Additionally, the population of teachers in the study was largely, if not solely, responsible for the mathematics instruction of the student sample throughout the 2016-2017 school year, meeting with the students every day for a minimum class period of 55 minutes to a maximum period of 90 minutes, thereby strengthening the results of the relationship between the various factors studied.

Another limitation to the study was mortality, which occurred in the student sample over the course of the school year. Initially, the sample included any student for whom both the student assent and parental consent form were completed and returned to the researcher, which resulted in 71 out of 203 students in the initial sample. The final phase of the study asked students to complete the ASCDII a second time as a post-test, and 23 of the 71 students participated. Completion of the ASCDII took place during students’ academic homeroom periods. The pre-test required that students convene in the cafeteria, where they were given the details of the survey and took the assessment in a large-group setting.

Because the second ASCDII was given during the final months of school, when standardized assessments and finals were a focus for both teachers and students, the researcher was asked to make the assessment available electronically, so teachers could provide access during class when time allowed instead of pulling students out of academic support during homeroom. This change in the way the survey was provided to students may have impacted the number of students taking the assessment. Because the delivery of the survey changed, location threat may also be a reason for the decrease in the number of students taking the post survey. The novelty of the first data collection occurring in a large group setting could result in more positive scores on the first assessment as compared to the second assessment, as well as account for the fewer number of students taking the post-survey. It is important to note that one teacher admitted to forgetting to share the post-survey with her students, indicating the lower participation may not be due to either mortality nor location threat.

Although the Reformed Teaching Observation Protocol (RTOP) has been shown to be reliable (Sawada et al., 2002) and the researcher completed the suggested online calibration activities, using only one person to document classroom observations may pose some threat to validity (Piburn & Sawada, n.d.).

Student-related limitations included the process for identifying subjects for the study and the design of the student survey. Because both parent consent and student assent forms were required for each subject, due to age, teachers were asked to explain the study and hand out consent forms to parents who attended open house. All students were provided an assent form while in class. Due to the large number of consent forms signed by parents at open house, subjects were selected who initially had both the parent and student permission forms signed. As a result, all student subjects in the study had parents who attended open house, which could be an influential factor in student achievement, but was not part of this study.

Because data from the ASDQII assessment was self-reported, accuracy may be compromised. In addition, accuracy of information from the self-perception survey may be impacted as a result of giving the same survey both as a pre-assessment and a post-assessment. Students, by nature of the pre/post design may identify what is being studied and/or may score differently on the post assessment, due to having seen the questions before instead of the change indicating an actual change in self-perception.

The Hawthorn Effect could impact the responses of the teachers on the Teacher Perception Survey, as well as their behavior in the classroom while being videotaped and/or observed, simply because they knew they are part of a study and the survey tool provided some information regarding what was being studied (American Psychology Association, 2020).

Despite the limitations, it is assumed that the participants' responses were offered honestly and willingly. It is also assumed that the semester final scores provided to the research were accurate.

Definition of Terms

ASDQII: A student survey used to measure students' academic self-concept in five academic areas. For the purpose of this study, only the questions related to mathematics were used (Marsh, 1990). Permission for use was granted by Australian Catholic University.

At-Risk Math Student: For the purpose of this study, the researcher defined this term as students who were identified as needing extended time to learn Algebra and who are enrolled in a two-year Algebra class, and/or students who are enrolled in an Algebra I class in high school and who scored Basic or Below Basic on the 8th grade Missouri Assessment Program for math.

Intensity of Intervention: For the purpose of this study, intensity of intervention was measured by the number of times and the total amount of time a student worked with the math interventionist in addition to his or her regularly scheduled algebra class.

Intervention: For the purpose of this study, students enrolled in Algebra I as 9th graders were considered to be receiving a math intervention.

Mathematics Self-Concept: “A person’s self-related perceptions in the area of mathematics that are formed through experience with others and one’s own interpretation of their environment” (Lazarides & Ittel, 2012, p. 1).

Reformed Teaching Observation Protocol (RTOP): A classroom observation tool designed by researchers at Arizona State University, which has been shown to be valid and reliable in measuring the impact of mathematics teaching practices on student achievement (Sawada et al., 2000). Permission for use was granted by the Arizona Collaborative for Excellence in Preparation of Teachers.

Response to Intervention (RTI):

> RTI is a structure to enhance instructional effectiveness through the use of evidence-based practice, systematic data collection and data-based decision making. The framework is a tiered model of providing intervention service to students that is systematic and data-driven. The level or intensity of the intervention is based upon the specific academic or behavioral needs of the student. Student progress is monitored during all points in the system in order to

> provide information on the response of the student to the intervention implemented. (Department of Elementary and Secondary Education, 2014, para.1)

Teaching and Learning Beliefs Survey: A survey designed to measure teachers' level of agreement or disagreement with statements based on the National Council of Teachers of Mathematics principles of learning (Leinwand et al., 2014). Permission for use was granted by National Council of Teachers of Mathematics.

Summary

This study was designed to examine the relationship of various teacher and student factors, including teachers' beliefs about teaching and learning mathematics, teachers' instructional styles, students' academic self-concept in mathematics, and students' mathematics achievement in an Algebra I course. The relationships between the student and teacher factors were analyzed using a Pearson Product Moment Correlation (PPMC).

A sample of 71 students and four teachers was selected from a random group who returned all required consent and assent forms. Data from the student sample included a pre- and post-ASDQII to identify student self-concept in mathematics, and scores on semester Algebra I finals. Teacher data were gathered from teacher surveys and classroom observations.

Data collection took place over the course of one school year. Several changes to the data collection procedures were necessary, however, due to changes in assessment practices at the research site, and the poor quality of data from formative assessments in the Algebra I classrooms.

Chapter Two: The Literature Review

This literature review begins with a broad discussion of related concepts, including academic self-concept, instructional styles, teacher professional development and measurement tools, including those used in this study and national and international standardized testing. Because of the many variables at play in a broad concept, such as Algebra I, this literature review then divided the multiple factors related to Algebra I into four categories: systemic factors, student factors, instructional factors, and teacher factors. Systemic factors included how and when students are placed in Algebra I courses, along with subsequent mathematics achievement. An examination of the literature related to student factors focused on academic self-concept in mathematics, including understanding the construct of academic self-concept, forces that impact academic self-concept, and how academic self-concept relates to achievement in Algebra I. Instructional factors were grounded in a constructivist approach to teaching in alignment with National Council of Teaching Mathematics (2017) recommendations. Finally, individual teacher beliefs about best practices for teaching and how students best learn mathematics served as the focus for teacher factors. Both seminal and contemporary research was included to provide a broad scope of reference.

Systemic Factors Impacting Achievement in Algebra: Equity

While mathematics has historically been part of a standard curriculum in most schools and districts in the United States, the focus on mathematics curricula and instructional design took on new importance in conjunction with an increased focus on science, technology, engineering, and mathematics (STEM) programs meant to prepare students for jobs, which required the ability to apply reasoning and understand complex mathematical applications (Common Core State Standards, 2018; National Council of Teachers of Mathematics [NCTM], 2008). Additionally, changes to the Elementary and Secondary Education Act (ESEA) in 2015 brought the inception of the Every Child Succeeds Act (ESSA), which included a focus on college and career readiness and equitable access to high quality curriculum (Kostyo et al., 2018; U.S. Department of Education, n.d.). Historically, ESSA's roots were based in what was essentially a civil rights issue, which mandated that all students would be provided with high quality educational opportunities (U.S. Department of Education, 2016). The current iteration of the law, at the time of this writing, includes a focus on high-quality assessment along with a call to action that schools focus on ensuring all students are prepared to take high level math courses, and that students who are enrolled in Algebra in 8th grade are assessed using state tests given to high school students in the same course (U.S. Department of Education, 2016).

With the push toward higher-level courses in mathematics came a philosophical shift from a plug and chug understanding toward a conceptual understanding of math processes (Common Core State Standards, 2018). Stoelinga and Lynn (2013) described the cognitive shifts necessary as, "an important transition point in the learning of mathematics, requiring the use of generalized models, mathematical abstractions, and understandings of variables and symbols, all of which are particularly challenging for many students" (p. 3).

Because those who take Algebra in 8th grade are able to take more mathematics courses in high school, the focus on college and career readiness shone a spotlight on algebra achievement while at the same time drawing attention to a series of inequities related to students' Algebra achievement. Those who struggled in math did not always have access to high quality math courses nor was high quality instruction consistently provided for students who were behind their peers in mathematics and had not completed Algebra by the time they entered high school (Clotfelter et al., 2015; Knuth et al., 2016). Widely known as a gatekeeper course (Desilver, 2017; Kieran et al., 2016) because of the potential to determine the long-term success of an individual based on his or her achievement in what is typically an early course in the mathematics sequence offered in high school, Algebra has become an issue of equity as much as it is a core subject As such, Algebra has been viewed as a civil rights issue, in part because minorities and low-income students are under-represented in early Algebra courses in general, and because poor performance in algebra keeps students from completing higher level mathematics courses in high school, effectively limiting opportunities in higher education mathematics courses (Stein et al., 2011; Venenciano, Heck, 2016).

Systemic Factors Impacting Achievement in Algebra: Early Algebra

Algebra is considered a gatekeeper course, because access to the course and the level of achievement by students while in the course has the potential to determine the success of an individual in upper mathematics classes, such as geometry and calculus, as well as post-high school mathematics subjects (Burdman, 2018; Clotfelter et al., 2015; Hanover Research, 2016; Kostyo et al., 2018; The Nations Report Card, 2016). According to Loveless (2013), "Researchers have documented that readiness for both college-level mathematics and technically-oriented employment hinges on students gaining, at least by the end of high school, a basic knowledge of algebra" (p. 1). Additionally, algebra has been recognized as an important indicator of subsequent success in mathematics courses, because it is often the first mathematics course that requires students to think abstractly and to understand multiple representations of an idea (Star et al., 2015). On the Trends in International Mathematics and Science Study (TIMSS), only 7% of twelfth graders taking the TIMSS assessment scored in the highest achievement level, and those students reported taking advanced courses, such as Advanced Placement Calculus, which supported the importance of access to advanced coursework (Provasnik et al., 2019).

In response to a growing understanding of the importance of Algebra, many systems moved toward enrolling more students in Algebra in middle school than was previously the norm (Dougherty et al., 2017). Additionally, the move toward enrolling students in Algebra prior to high school may have been the result of ESSA, which required schools to ensure all students were ready for advanced math classes (U.S. Department of Education, 2016). Instead of improving the outlook for students, researchers reported that students who were enrolled in Algebra I courses who were also considered at risk due to previously documented poor math performance did not benefit from early Algebra enrollment (Clotfelter et al., 2015; Domina et al., 2015). It appeared that more was needed to increase students' content mastery than simply enrolling students in a high-level math course. This finding was also supported by Stein et. al. (2011), who determined that requiring students to take Algebra in 8th grade only resulted in success when strong supports were in place for struggling students. However, there was some conflicting evidence that providing students with an additional year of general math before enrolling them in Algebra I did not improve outcomes; and, in fact, widened some gaps in students' numeracy skills (Cirino, Tolar, & Fuchs, 2019). These findings were in alignment Venenciano and Heck’s (2016) research which found that rather than students lacking skills, the written and taught curriculum was at fault for students’ struggle with Algebra which resulted in the recommendation to embed Algebra skills throughout elementary mathematics courses.

In partnership with NCTM, The National Association for the Education of Young Children (NAEYC) released a position paper advocating for a student-centered approach to teaching mathematics, along with the inclusion of a focus on problem solving and algebra-based concepts, such as recognizing patterns for three to six-year-old children (National Association for the Education of Young Children, 2010).

To address the achievement issue from both an equity and an academic standpoint, algebra shifted from a course typically taken in middle school by only the highest achieving students, to a standard middle school course for the majority of students, with many systems requiring students to take algebra in 8th grade (Loveless, 2013, Stein et al., 2011). The move toward early algebra was a somewhat recent shift since, according to Gojak (2013), “As recently as twenty years ago, most students took algebra in 9th grade” (para. 4). According to Loveless (2013), “Researchers have documented that readiness for both college-level mathematics and technically-oriented employment hinges on students gaining, at least by the end of high school, a basic knowledge of algebra” (p. 1). The president of the National Council of Teachers of Mathematics, in 2013, agreed with the idea of Algebra as a potential gatekeeper, but disagreed with requiring all students to enroll in Algebra in eighth grade, citing the need for clear enrollment requirements, including the ability for students to think abstractly (Gojak, 2013). NCTM’s standards advocated for embedding algebraic thinking throughout kindergarten to twelfth grade curriculum (National Council of Teachers of Mathematics, n.d.).

Results of the push for Algebra before high school were contradictory. Overall, Gojak (2013) reported low-performing students saw no real benefit from Algebra in eighth grade. Interestingly, positive outcomes occurred when low achieving students waited to take Algebra until high school. Stein et al. (2011) revealed that the unintended consequence of early Algebra was a widening of the learning gap with high performing students benefitting the most from early Algebra and low performing students falling further behind (2011). As a result of the widening gap, standardized assessment scores were not positively impacted by more students taking Algebra prior to high school, as was shown in a state-level analysis by the Brookings Institute (2013), which revealed no relationship between the number of students who participated in higher math courses before ninth grade and improved NAEP scores. However, Sorenson et al. (2018) found when students who had low readiness for Algebra according to seventh grade math scores were placed in eighth-grade Algebra, their achievement initially had a negative impact on their grade point average, but achievement rebounded by the end of ninth grade with little to no negative consequences. Positive results were also reported when students who were initially placed in an early Algebra class were allowed to retake the course while in high school, if they had not achieved mastery of course objectives (Clotfelter et al., 2014; Gamoran & Hannigan, 2000). Likewise, Stein et. al. (2011) reported positive results, such as a more positive feeling toward math for students taking Algebra in middle school as opposed to waiting until high school. A few bright spots existed in the research, which revealed that when strong instructional supports were in place for struggling students, or

Algebra skills were embedded throughout early mathematics, students were more likely to be successful (Stein et al., 2011; Venenciano & Heck, 2016). For high-performing students, the research pointed to a distinct advantage as those who took early Algebra were more likely to take high-level math courses such as calculus in high school (Clotfelter et al., 2014). The negative impact on mathematics instruction did not just impact students, but had negatives effects on teacher quality as well.

Another systemic approach to addressing the Algebra achievement gap was to enroll students in a double dose of Algebra, which increased the amount of Algebra instruction students received, as compared to students who were viewed as being on-target for mathematics achievement. Overall, results of double dose Algebra were mixed. A study of Chicago Public Schools' use of double dosing showed some success for students in schools that did not track students according to achievement level for the double-dose class; and, either no impact or a negative impact for students identified as in need of remediation, who were in double dose classrooms with similarly identified peers (Nomi & Raudenbush, 2016). Because the students placed in a double-dose Algebra class were typically those identified as needed more support, a high school in Florida focused its double-dose structure on the curriculum and delivery of instruction and reported improved achievement for students in classrooms with high-fidelity implementation of grade-level content along with a focus on mind set and perseverance (Tidd, Stoelinga, Bush-Richards, De Sena, & Dwyer, 2018).

The need for more Algebra teachers, as opposed to those who taught a more general mathematics course in eighth grade, may have resulted in a decline in teacher quality (Clotfelter et al., 2014). Historically, the Coleman Report, the first study of teacher and instructional factors that influenced academic achievement, reported that while teacher quality was important, quality was not related to years of experience, but to verbal abilities and level of education (Coleman et al., 1966). While the increase in teachers new to the subject of Algebra was not shown to have a negative impact on overall algebra end of course scores, the U.S. Department of Education recommended teachers have, “domain-specific knowledge (e.g., deep knowledge of the subject being taught, and facility with a range of instructional strategies” (2009, p. 17). The shift to early Algebra may have made it more difficult to find teachers considered highly quality under these definitions, which also highlighted an additional systemic need according to research; time for teacher collaboration.

Students in China consistently outperformed students in the United States on international assessment (OECD, 2020). An examination of instruction revealed that Chinese instructors who were most successful with students cited collaboration with others as valuable for improving student outcomes (Pepin, Xu, Trouche, & Wang, 2017).

The debate on early Algebra resulted in two schools of thought regarding the approach to increasing success in Algebra, as well as different outcomes related to student achievement. For some systems, early Algebra meant moving Algebra curriculum typically reserved for high school students to the middle grades, along with a push for the majority of students to enroll in the course. While research showed that both high and low performing students benefitted by taking more advanced math courses in high school, high performing students showed improved performance in math while lower performing students did not show mastery of course material and may have been harmed by the accelerated approach (Cirino et al., 2018; Clotfelter et al., 2015; McEachin et al., 2017). More recently, a distinction was made between accelerated Algebra, or moving content to lower grades, and early Algebra, supported by NCTM, which embedded algebraic concepts throughout the kindergarten to twelfth grade curriculum (Knuth et al., 2016; National Council of Teachers of Mathematics, n.d.). Studies of programs with algebraic concepts embedded throughout elementary and secondary math at all levels showed students improved their abilities to utilize algebraic thinking, such as reasoning, generalizing, and justifying (Blanton et al., 2018; Kieran et al., 2016).

It is difficult to determine if the necessary shifts in thinking are taking hold among U.S. students, as scores on national and international mathematics assessments are somewhat contradictory according to the National Center for Educational Statistics (National Center for Educational Statistics, n.d.) The most recent National Assessment of Educational Progress (NAEP) assessment for 8th grade students showed no significant change in mathematics scores from the previous testing in 2015 (National Center for Educational Progress, 2019; National Center for Educational Statistics, n.d.), and a widening gap between low and high performers (2017 NAEP Mathematics and Reading Assessment, n.d.; Tucker, 2018). When compared on an international scale, students in the United States ranked below the Office for Economic Cooperation and Development's (OECD) average on the Program for International Student Assessment (PISA), and scores dropped significantly from the 2012 PISA mathematics assessment (Office of Economic and Cooperative Development [OECD], 2016; Compare Your Country, n.d.). Boaler in her interview with Mosley (2017), described the PISA as having a focus on problem-solving over rote memorization. Interestingly, when students were asked about their method of learning math, responses on the PISA student survey indicated U.S. students often selected memorization over other choices, despite options, such as making connection and self-monitoring, which were more closely aligned with problem solving (Boaler & Zoido, 2016). In contradiction to the PISA results, scores on the Trends in International Mathematics and Science Study (TIMSS) provided an alternate perspective, as average scores for eighth grade students in the United States increased (Mullis et al., 2016). Prior to these assessments, the National Governors Association issued a wake-up call, which stated,

> This nation must understand that a post-secondary degree or relevant workforce certificate is the "new minimum" and without it, our young people will not be able to access the middle class, the American dream and the opportunities and responsibilities of citizenship and a fuller life. (National Governors Association. 2013, para. 4)

Even with a slight rise in scores on the PISA, the overall achievement of U.S. students in mathematics in eighth grade provided a reminder that the majority of students are not proficient in mathematics (Carr, 2016; NAEP Mathematics and Reading Assessment, n.d.; National Center for Educational Statistics, n.d.).

Response to Intervention

As researchers and educators sought ways to better support students and increase achievement in literacy and mathematics, a move toward Multi-Tiered Systems of Support (MTSS), also known as Response to Intervention (RTI) took hold. The goal of Response to Intervention (RTI) was to improve student achievement through a series of data-based decisions (Jimerson et al., 2016). This multi-tiered approach was designed to deliver research-based instruction informed by data, including individualized instruction with remedial opportunities made available in the general education setting (Fuchs & Fuchs, 2017).

RTI at the secondary level, however, faced unique challenges. According to the National Comprehensive Center for Teacher Quality, “RTI applications in elementary schools are generally well-known and widely endorsed, but RTI applications in the middle and high school classrooms are less well-developed and implemented” (U.S. Department of Education, 2009, p. 1). A key difference between RTI at the elementary and secondary levels was a lack of vetted assessment tools. As a result, teachers were often responsible for creating assessments and monitoring achievement within a self-developed system (Fuchs & Fuchs, 2017; U.S. Department of Education, 2009, p. 5).

The lack of specific programs and assessments for secondary students meant high school teachers were on their own to develop programs and interventions to meet the needs of students who were considered to be in danger of falling behind. Kin, Lemins, and Hill (2012) provided some guidance by giving secondary schools permission to develop new models that included scheduling changes and student-centered problem-solving. A review of RTI practices at the secondary level identified teacher-created assessment probes as the main source of progress monitoring, as opposed to pre-made or purchased assessments (Johnson et al., 2012). A challenge to secondary RTI development and implementation was that in schools where teachers were generally satisfied with their assessment results, it may have been more difficult to achieve buy-in for RTI practices and realize systemic change (U.S. Department of Education, 2009, p. 1). Fuchs and Fuchs (2017), in response to the challenges faced by secondary schools, recommended high schools should not worry about conforming to the same practices as elementary schools, and should configure their RTI systems in a way that fit within the current schedule and staffing capabilities; citing evidence that some implementation of RTI is better than no implementation.

In addition to system-wide goals of identification and student support, RTI has also been described as having different goals in terms of desired learner outcomes from elementary to the high school level (Bouck & Cosby, 2019). In “How RTI Works in Secondary Schools,” the goal of RTI in elementary schools was described as being focused on preparing students for secondary school success (Johnson et al., 2009). Secondary schools, according to Johnson et.al. (2009), used RTI to help all students graduate and to ensure all students were provided with effective instruction. NCTM issued a position paper supporting interventions and high-quality teaching, saying,

> we endorse the use of increasingly intensive and effective instructional interventions for students who struggle in mathematics. Teachers must use a variety of formative assessments to target strategic instructional techniques that are tailored to meet individual students' needs. When implementing appropriate interventions for all mathematics learners, teachers must possess strong backgrounds in mathematical content knowledge for teaching, pedagogical content knowledge, and a wide range of instructional strategies. (National Council of Teachers of Mathematics [NCTM], 2011, para. 1)

Regardless of the purpose of the interventions, some proved more effective than others. Double dosing, or increasing the amount of time a student was assigned to a math class for instruction proved successful when combined with strong pedagogy (Nomi & Allensworth, 2013); but, Cortez et al. (2013) revealed that only a subset of students who struggled were actually a good fit for the double dosing strategy, as only students in the upper end of the intervention achievement range benefitted from the additional time. This research points to the importance of looking beyond systemic factors and more closely examining student factors related to success.

Student Factors

Students, regardless of their levels of achievement or need for intervention support, have developed an academic self-concept that may provide insight into aspects of mathematics achievement. The validity of academic self-concept has long been the topic of research and has resulted in multiple definitions, which reveal the evolving nature of understanding around how academic self-concept functions and its relationship to various factors in education. Ali-Hilal and Nasser (2013) broadly defined self-concept as, "General feelings of doing well or poorly in mathematics" (p. 577) and "students' self-perception of ability" (p. 574). Prior research resulted in a refined model of academic self-concept as a construct that was both "descriptive/affective (I like; I enjoy) and evaluative/comparative/cognitive (good at; do well at) beliefs that people have about specific characteristics," (Burnett et al., 1999, p. 2). Moller and Pohlman (2010) described self-concept as, "an important mediating construct that facilitates the attainment of desirable outcomes" (p. 436). Bong et al. (2012) attempted to describe the differences among self-concept, and self-efficacy and determined self-concept to be related to "how individuals evaluate themselves in comparison with others . . . [and] self-efficacy [as] one's belief in his/her ability to accomplish a goal" (p. 337). Unlike self-efficacy, which is a general belief in one's ability to do well, and self-esteem, which is one's overall feeling of self-worth, academic self-concept is related to concrete factors, such as academic content and how a student performs in relation to his or her peers (Marsh et al., 2008; McInerey et al., 2012; OECD, 2018). Marsh and Shavelson developed a framework for examining self-concept that included both academic and non-academic constructs, and showed the two constructs were independent of one another (Marsh, 1990). Where previous studies on self-concept tended to include a variety of components in one holistic rating or score making it difficult to determine specific areas of impact, Shavelson et al. (1976) separated self-concept into two domains, academic and non-academic, which helped researchers identify specific factors that influenced academic self-concept separate from a subject's overall self-concept. Additionally, earlier research suggested a flexible notion of academic self-concept related to specific courses of study, which led Marsh to recommend researchers utilize subject-specific self-concept scales (Marsh, 1990; Marsh & Martin, 2011). Subsequent research confirmed the domain-specific nature of academic self-concept (Niepel et al., 2014).

While important as a stand-alone factor, academic self-concept was found to be directly related to the courses students selected, and students' achievement in academic courses; particularly English and mathematics (Lazarides & Ittel, 2012; Morony et al., 2013). Although an important mediating factor within the discreet subjects of math and English, there was no relationship between academic self-concept and achievement across disciplines (Niepel et.al., 2014). Liem et al. (2015) studied self-concept in tracked levels of students, including above-average, average, and below-average tracks and reported the strongest correlation between academic self-concept and achievement in math to be within the lower achieving track. This phenomenon was earlier identified by Coleman et al. who found that students in less diverse classrooms had lower achievement scores and that the impact was more profound on minority students than on white students (Coleman et al., 1966).

In addition to the impact of students' assigned track or level in school, students' peers have been shown to impact self-concept in a unique fashion, known as the Big Fish Little Pond Effect (BFLPE), which was described by Marsh et al. (2008) as the result of students comparing themselves to their peers. The result of the BFLPE, found to exist regardless of gender or culture, was that students assigned to more selective schools or higher-level math classes with high achieving peers reported a lower academic self-concept (Niepel et al., 2014; Loyalka et al., 2018). In relation to mathematics classes, this might mean that students enrolled in an Algebra I class in eighth grade may have a lower mathematics self-concept, due to a larger number of peers who were high achieving. The reverse may also hold true with students who were enrolled in Algebra in ninth grade having higher mathematics self-concepts, due to comparisons with peers who were not as high achieving. As Coleman et al. (1966) put it, "Attributes of other students account for far more variation in the achievement of minority group children than do any attributes of facilities and slightly more than do attributes of staff" (p. 302). This finding aligned with the seminal research of Marsh and Shavelson (1990) which indicated self-concept was strongly connected to students' peer groups in both positive and negative ways.

Student responses on the 2018 PISA reflected this phenomenon as more than 75% of U.S. 15-year-olds indicated they planned to complete education beyond high school compared to only 48% of 15-year-olds from other countries (OECD, 2020). Considering U.S. students scored below the OECD average in mathematics (PISA 2018, n.d.), U.S.

students revealed much more confidence in their ability than peers in higher performing countries; a global magnification of the BFLP effect.

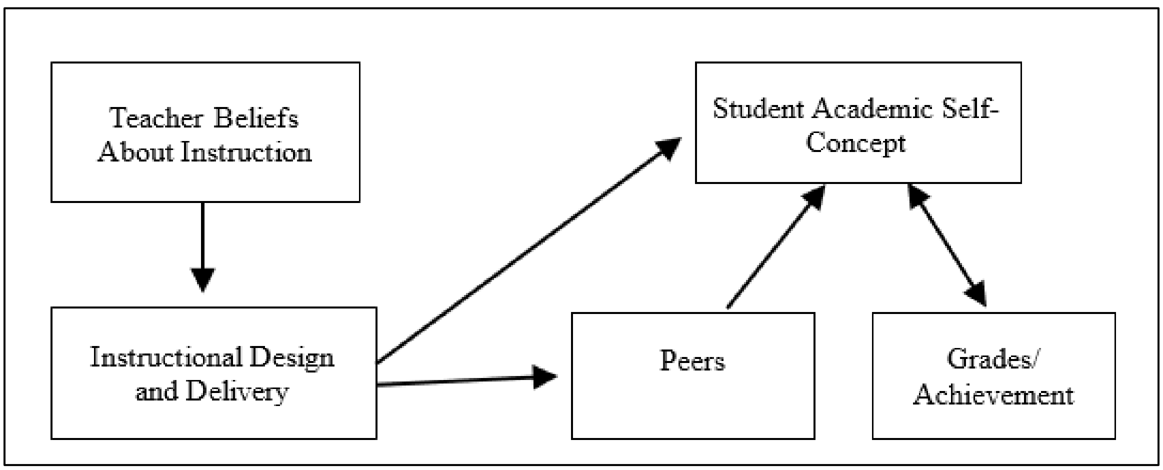
Also of importance was research by Liem et al. (2015), which revealed students assigned to lower academic tracks in mathematics showed the strongest relationship between academic self-concept and achievement. However, Gray and Mannahan (2017) showed, more often than not, students over-estimated their abilities to do well in a class which, when considered along with Liem et al.'s research, suggested the potential for a positive reciprocal correlation when a student received a lower grade than anticipated on course work, thus negatively impacting his academic self-concept.

The reciprocal relationship between academic achievement and self-concept was recognized in studies showing when students' academic achievement improved, their self-concept also increased (Niepel et al., 2014; Moller & Pohlman, 2010). Studies identified a positive relationship with a third component, adding in the amount of effort exerted by students. In short, when students put in more effort, the better they felt about themselves as math students, and their achievement positively increased (Abu-Hilal & Nasser, 2012; McInerey et al., 2012). Furthermore, self-concept was not only found to be related to achievement, but predictive of both academic and non-academic success (Ben-Eliyahu et al., 2017).

Also, of importance to an understanding of academic self-concept is identifying practices that enhance students' perceptions of themselves as students who do well in academic subject areas. Praise and feedback were two noteworthy teacher/classroom practices that positively impacted academic self-concept (Burnett et al., 1999). Additional factors related to academic self-concept included the types of strategies taught by teachers and employed by students to master mathematics content. Surface strategies, such as rote memorization had less of a positive effect on achievement and therefore academic self-concept as opposed to more in-depth strategies in which students had to organize and make sense of information and connect new learning to previously learned material (McInerney et al., 2012). As shown in Figure 1, several external factors have the potential to influence students' academic self-concept.

Figure 1

Influential Factors on Academic Self-Concept



Several tools, including the ASDQII, a secondary version of the original ASDQ, have been developed and validated to measure both self-concept in general and academic self-concept as it relates to a specific content area such as mathematics (Marsh, 1990; Pena et al., 2015; Sawada et al., 2002). In alignment with the descriptive/affective and evaluative/comparative nature of academic self-concept (Burnett et al., 1999), the ASDQII asks questions regarding perceived ease of the subject and how well students feel they do in comparison to peers.

Having a high academic self-concept may be perceived as a desirable outcome, but research revealed a downside to a high academic self-concept. Henderson et.al., (2017) painted a cautionary picture regarding a high academic self-concept as they suggested that students with a strong positive academic self-concept may give less effort in school due to a false sense of confidence.

Additionally, Veiga et al. (2015) found that academic self-concept declined as students went from early adolescence to middle/late adolescence; likely due to peer influence.

Instructional Factors

In order to excel in mathematics courses, experts agreed that moving students beyond rote memorization of facts toward conceptual understanding and the ability to utilize problem-solving skills was essential for mastery and success in subsequent courses (Boaler, 2016; NCTM, 2017; Star et al., 2019). The National Council of Teachers of Mathematics (NCTM) advocates for teaching algebraic concepts such as "recognizing patterns. . . representing and analyzing mathematical structures. . . and analyzing change" throughout students' school experience beginning with kindergarten (National Council of Teachers of Mathematics, n.d., para. 1). Stephens et al. provided guidance in this area advocating for promoting student questions such as, "Why did this work?" and, "Do you think this will always work?" (2015, p. 98). This inquiry-based approach to Algebra emphasized making connections, using process-oriented thinking, and generating multiple strategies to solve a problem, in short, moving away from one correct answer and one correct way to get there (Boaler, 2013; Gonser, 2021; Star, et.al., 2019). Research also supported instruction that helped students build understanding through interactions with the content, which, according to Kieran et.al, required teachers go beyond simply asking for correct answers and instead, “Analyze their students’ ideas in the moment…[and] learn the types of questions and responses that will draw students’ attention to the content to be explored and help them make new connections” (2016, p. 22).

In a 2017 policy brief, the International Association for the Evaluation of Educational Achievement (IEA) identified a world-wide shift, or mega-trend, toward a constructivist approach to teaching based on four cycles of TIMMS results which included surveys of instructional practices (Rozman & Klieme, 2017). Similarly, the OECD, advocated for instruction focused on cognitive engagement and that:

> Students should be intellectually enticed and challenged by the content in the classroom if they are to move forward with their learning and prepared for a world of problem-solving and application. Engaging students in work that demands analysis, creation, evaluation, thoughtfulness, whilst promoting multiple ways of reasoning and emphasizing the rationale behind processes and methods, can be powerful ways to stretch and challenge students and build deep conceptual understanding. (Organisation for Economic Co-operation and Development, para. 1)

In a study of Chinese instructors Pepin, Xu, Trouche, and Wang (2017) noted that the most successful students came from a classroom in which the instructor focused more on students’ independent thining ability than on rote memory of processes and mathematical facts, further supporting the effectiveness of a constructivist approach. Constructivist teaching has its roots in the teachings of both Piaget and Vygotsky. Piaget’s theory of learning puts the child at the center as a key agent in constructing knowledge (as cited in Cohen & Waite-Supiansky, 2017) which has implications for instructional design. Likewise, the social nature of learning posed by Vygotsky, is also child-centric, but with the addition of a social and cultural aspect. Constructivism, in Vygotsky's view, requires social interaction with both peers and teachers in order for students to be within the zone of proximal development (as cited in Berkeley University of California, 2021). Both Piaget and Vygotsky's frameworks for learning align with the NCTM in that they advocate for interaction that engages students in inquiry and problem solving (NCTM 2017, NCTM 2014).

In an attempt to more closely align mathematics outcomes with college and career readiness, the National Council for Teachers of Mathematics (2008) revised its math standards to include more problem solving, critical thinking, and problems connected to the real world. Additionally, many state standards revised their mathematics outcomes resulting in adoption of Common Core Standards which stress critical thinking and problem solving (Common Core State Standards, 2018). The National Council of Teachers of Mathematics' standards included "problem solving, reasoning and proof, communication, connections, [and] representations" (National Council of Teachers of Mathematics, n.d., para. 5), which aligned with the fundamental practices outlined in the constructivist approach to mathematics instruction measured by the Reformed Teaching ~~and~~ Observation~~al~~ Protocol (Piburn & Sawada, n.d.). Reformed teaching, or teaching that is constructivist in nature, is identified as instruction that calls upon students' prior knowledge, allows for multiple pathways toward solutions, and helps students make connections both between mathematical concepts and with other content areas (Boston et al., 2015).

Instruction, according to McInerney et al. (2012), can be described as both deep and surface learning, "with deep strategies addressing key concepts as well as connections for thorough mastery, and surface strategies as rote and addressing only bare essentials" (p. 252). Kim (2005) described this type of teaching as a method that encourages internalization of concepts and knowledge, which move far beyond rote memorization. McInerney (2014) further defined constructivist teaching as requiring, "active involvement of the learner and a shift in focus from what the teacher may do to influence learning to what the learner does" (p. 7). Active involvement played out in constructivist classrooms as students taking ownership of their learning. One way this might have been observed in a classroom is that students would have time to solve problems on their own using teacher feedback as opposed to the teacher providing the answer and/or providing little to no feedback (Young, 2014). Eighth grade students taking the National Assessment of Educational Progress Mathematics Assessment must be able to apply their learning to new concepts in order to be considered proficient while the advanced level requires students to, reach beyond the recognition, identification, and application of mathematical rules in order to generalize and synthesize concepts and principles (National Assessment of Educational Progress, 2017).

Not all research pointed to the success of constructivist approaches to teaching mathematics, however. A study by Anderson and Anderson (2017) identified constructivism as having, "a negative impact on academic achievement in general, and for students with low parental education in particular (para. 1). Hattie's (2021) research, while supportive of a constructivist approach when considering instructional strategies such as problem-solving, cooperative learning and questioning, also showed the seemingly contradictory approach of direct teaching to have a high effect size.

Despite the mixed research results on the effectiveness of a constructivist approach to mathematics instruction, The National Council of Teachers of Mathematics (NCTM) supported this student-centered approach and issued the following statement regarding constructivism:

> Constructivism focuses our attention on how people learn. It suggests that math knowledge results from people forming models in response to the questions and challenges that come from actively engaging math problems and environments - not from simply taking in information, nor as merely the blossoming of an innate gift. The challenge in teaching is to create experiences that engage the student and support his or her own explanation, evaluation, communication, and application of the mathematical models needed to make sense of these experiences. (National Council of Teachers of Mathematics [NCTM], 2017, para. 4)

NCTM's statement regarding constructivism aligned with Piaget's Constructivist Theory, in that both argued for learning that builds on prior knowledge and experience and is best done in social contexts in which learners take an active role in shaping their learning experiences (Alanazi, 2016).

Nomi, et. Al. (2016) pointed to the importance of both cognitive and social emotional supports as integral components of instruction supporting student success in Algebra. For example, in the double-dose Algebra program studied, students who were in classrooms with high-fidelity implementation of the Intensified Algebra program which included a focus on mind set, self-reflection and goal setting, and persistence were more successful than those who were in classrooms which followed a more traditional approach to instruction focused solely on the academic content. Non-cognitive skills such as a belief in one's ability to succeed, ability to persist when facing challenge and an overall enjoyment of mathematics have been increasingly identified in the research as important to mathematics success (Boaler, 2013; Gonser 2021).

Teacher Factors

As shown by Coleman, teacher quality played a significant role in student success; more important than school resources such as innovative materials, and the teacher's educational background (1966). As such, professional development became an important tool for school systems. In order to support teachers' professional development in regards to their newly revised standards, NCTM identified both productive and unproductive thinking patterns. Unproductive thinking, as outlined by NCTM (2014), is grounded in a linear approach to content that requires memorization and reproduction of skills. Productive thinking patterns, on the other hand, put the student at the center of the learning and require students to utilize inquiry skills to solve relevant problems reflected in the Common Core State Standards and the Missouri State Standards. In accordance with the NCTM recommendations, McInerey et al. (2012) identified a relationship between academic self-concept and student learning strategies, noting that students who were able to use deep learning strategies had higher academic self-concepts.

Whether teaching Algebra in middle or high school, the NCTM supported a constructivist approach as reflected in their Teacher Beliefs Survey (National Council of Teachers of Mathematics [NCTM], 2014). Expertise in content and in pedagogy was important and required teachers to "understand that [Algebra] is not a body of content "to master," but that they and their students will continue to discover new connections through the act of teaching" (Kieran et al., 2016, p. 22). As Vintere (2018) stated, teaching algebra using a constructivist approach, "changes the role of a teacher in mathematics education from an instructor to a leader or facilitator" (p. 6). In addition, data from teacher surveys on the TIMSS indicated a shift in teacher focus from simple procedural skills to reasoning and problem-solving (Rozman & Klieme, 2017).

There is a need for teacher professional development that supports the recommended approaches to teaching mathematics. Recent teacher survey results from the TIMSS assessment given to students in 4th and 8th grades indicated 69% of 8th grade teachers globally indicated a need for professional development in improving critical thinking and problem-solving skills and 60% needed support for mathematics pedagogy and instruction. Results for the United States were 71% and 49% respectively (IEA TIMSS & PIRLS International Study Center, 2019).

According to Burnett, et. al (1999), teachers' interactions with students also had the potential to positively or negatively impact student academic self-concept with actionable feedback and praise showing the highest potential for a positive influence. Feedback was also recognized by Hattie as a high impact instructional strategy with an effect size of 0.73 (2021).

Math Performance Over Time

Despite increased focus on mathematics, student achievement in mathematics, and algebra in particular, has not shown much improvement over time. International, national, and state assessments in mathematics revealed the existence of gaps in mathematics learning for students in the United States and Missouri, and a lack of significant progress in addressing those gaps (Missouri Department of Elementary and Secondary Education, 2020; National Assessment of Educational Progress [NAEP], 2019; OECD, 2018). Internationally, two assessments, the Trends in International Math and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) provided information on how students in the United States performed compared to international students. The National Assessment of Educational Progress (NAEP) results revealed a national perspective on students' mathematics achievement, and locally, the state-required standardized assessments provided a measure of mathematics understanding and growth over time.

The TIMSS study, an international assessment covering math and science, is given to students in 4th and 8th grade every four years (National Center for Educational Statistics, 2020). Four levels of achievement are identified; Low, Intermediate, High and Advanced (IEA TIMSS & PIRLS International Study Center, 2019).

Statistically significant growth has occurred since the first test in 1995, and eighth grade students in the United States showed statistically significant growth since the most recent administration of the TIMSS. Despite this growth, students in the United States scored significantly lower than students in eight countries, statistically similar to students from nine countries and significantly higher than students in 14 countries (Mullis, et. al., 2020).

A deeper dive into the TIMSS data revealed U.S. students' relative deficit in the ability to reason as opposed to a basic knowledge of mathematics as shown in Table 2.

Table 2

A Multi-Year Comparison of U.S. Students' TIMSS Average Scale Score by Cognitive Domain

Year	Overall Average Scale Score	Knowing	Applying	Reasoning
2019	515	522	515	507
2015	518	528	515	514

Note: Information from: IEA TIMSS & PIRLS International Study Center, 2019.

Although average scale scores remained relatively the same from 2015-2019, two statistically significant changes were of note for 2019. First, the difference between the average scale score and the score in the cognitive domain of knowing represented a statistically significant increase. However, in the domain of reasoning, the difference between the 2019 score of 507 and the overall average scale score of 515 was a statistically significant negative difference in scores indicating that U.S. students' ability to reason was lower than their knowledge and the ability to apply skills (IEA TIMSS & PIRLS International Study Center, 2019).

Twelfth grade students in the United States also participated in the international TIMSS Advanced assessment, which is given every four years to a select group of students, who are typically enrolled in advanced mathematics courses and focuses on Algebra, calculus, and geometry content. Results of the TIMSS Advanced showed students in the United States struggled when asked to apply their learning to real-life contexts (Provasnik et al., 2019).

Also, an international assessment, the PISA was given to 15-year-olds in participating countries every three years in the areas of reading, science and mathematics

(OECD, 2020). The assessment was designed to measure more than simple knowledge of mathematics. According to the National Center for Educational Statistics (2020),

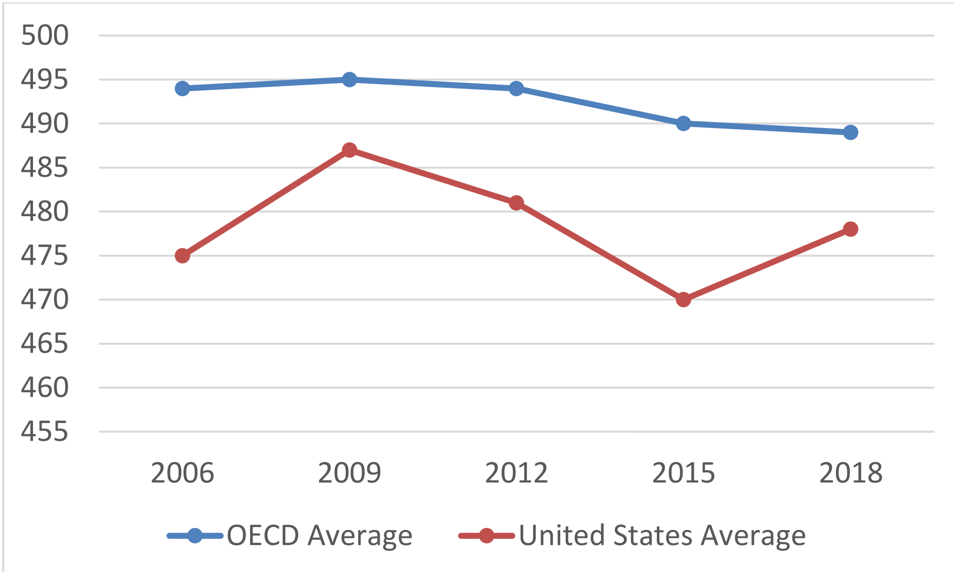
> In PISA, the assessment of mathematics literacy focuses on students' capacity to formulate, use, and interpret mathematics in a variety of contexts. Proficiency in mathematics is more than the ability to reproduce the knowledge of mathematical concepts and procedures; it is conceptualized as students' ability to extrapolate from what they know and apply their knowledge in both familiar and unfamiliar situations. (p. 1)

PISA described the test as assessing students' "ability to use their reading, mathematics and science knowledge and skills to meet real-life challenges" (OECD, 2018, para. 1).

Historically, U.S. students performed below average on the PISA, compared to other countries as shown in Figure 2.

Figure 2

Comparison of Average PISA Scores Over Time



Note: OECD, 2020; IEA TIMSS & PIRLS International Study Center, 2019.

While the gap between students in the United States and OECD countries has recently narrowed, a comparison of scores over time revealed the consistently low scores of U.S. students on the PISA over time. According to the OECD (2020), students in the United States showed no statistically significant decrease or increase on overall mathematics scores from the 2015 to 2018 assessment cycle.

While results of international assessments such as TIMMSS and PISA provided insight into the performance of U.S. students, they must be carefully considered. Because the results, which are often reported as a ranking of countries' results, appear easy to understand, Serino (2017) advised careful analysis of results to avoid false conclusions. Likewise, in the report *International Education Assessments: Cautions, Conundrums, and Common Sense*, Singer et. al (2018) outlined multiple ways interpretation of international assessments could go wrong, including a lack of understanding regarding the context of the assesments in relation to a county's culture, and missing information regarding the education infrastructure in assessed countries.

Nationally, students also showed gaps in mathematics learning on the National Assessment of Educational Progress (NAEP). The NAEP assessment for math and reading was given to students in grades four and eight every other year. An assessment was also given to twelfth graders in reading and math every four years (NAEP, 2019). No significant change was reported on the number of students scoring proficient on eighth grade mathematics assessment between the 2017 and 2019 scores with proficient defined as students who can, "conjecture, defend their ideas, and give supporting examples. They should understand the connections between fractions, percents, decimals, and other mathematical topics such as algebra and functions" (National Center for Educational Statistics, 2019, p. 1).

International assessments also revealed information related to teachers' professional development needs and the overall perception of teachers as professionals. According to the Trends in International Mathematics And Science Study (OECD, 2020), there was variation across countries in both the perceived professional development needs reported by teachers and the professional development teachers participated in according to responses on the Classroom Contexts Questionaire as evidenced in Table 3.

Table 3

U.S. Teachers' Perceived Need for Professional Development

	Percent of U.S. Teachers Who Reported a Need for Professional Development in This Area	Average Percent of International Teachers Who Reported a Need for Professional Development in This Area
Improving Students' Critical Thinking and Problem-Solving Skills	71	69
Mathematics Pedagogy/ Instruction	49	60

Teachers in the United States reported a similar need as their international peers for professional development in improving students' critical thinking abilities, but a much lower percent of teachers in the United States indicated a desire to learn more about pedagogy or how to teach mathematics (see Table 4).

Table 4

U.S. Teachers Self-Reported Professional Development Activity

	Percent of U.S. Teachers Who Reported Participating in Professional Development in This Area	Average Percent of International Teachers Who Reported Participating in Professional Development in This Area
Improving Students' Critical Thinking and Problem-Solving Skills	60	46
Mathematics Pedagogy/ Instruction	73	60

A larger percentage of teachers in the United States participated in professional learning in improving students' critical thinking and problem-solving skills than did their international peers even though teachers in the United States did not indicate they desired professional development in this area. Based on teachers' responses to the survey, teachers in the United States also spent more time in professional learning focused on mathematics pedagogy than their international peers. The Teaching and Learning International Survey (TALIS), in the report Teachers and School Leaders as Valued Professionals (OECD, 2020), cited professional learning as one of five aspects related to teachers' job satisfaction which indicated that teachers valued the opportunity to learn further supporting the need for ongoing professional development.

Locally, students in the state of the research site, showed little improvement on the state-required standardized assessments given for eighth grade mathematics and for

Algebra I. It is important to note that in the state where the research site resides, many students take Algebra in 8th grade, therefore, the scores for eighth grade Algebra reflect achievement levels for students who did not take Algebra I. Students taking eighth grade math were either in schools that did not offer Algebra I to eighth graders, or they were identified by school staff as not ready for Algebra I.

Tools for Measurement of Student Progress

TIMMS

The Trends in International Mathematics and Science Study (TIMSS), launched in 1995, is an assessment of mathematics and science knowledge for grades four and eight. It is administered every four years. Data are gathered from students, teachers, and principals concerning learning activities in mathematics and science. The United States has participated in each administration of the assessment since its launch. The National Center for Education Statistics (NCES) administered the assessment, which is sponsored by the International Association for the Evaluation of Educational Achievement. TIMSS allows academic comparisons across international education systems, as well as longevity comparison within its seven cycles of administration (National Center for Education Statistics [NCES], 2021a).

PISA

The Program for International Student Assessment (PISA) provides an assessment of reading, mathematics, and science literacy for 15-year-old students. The assessment was established in 2000 and is conducted every three years. During each administration of the assessment, the major domain concentrated upon rotates between reading, mathematics, and science. The NCES administered the assessment, which is sponsored by the Organization for Cooperation and Economic Development. Approximately 80 countries participated in the 2018 administration of the PISA, which featured reading as the major domain (NCES, 2021b).

NAEP

The National Assessment of Educational Progress (NAEP) measures the academic knowledge of United States students across the nation, states, and some urban areas. Established in 1969, the assessment is also known as the Nation's Report Card. The NAEP is sponsored by the U.S. Department of Education and the Institute of Education Sciences and is administered by NCES. National aggregate results are available in all tested subject areas, and results are available on the state and urban level for reading and mathematics, and in some cycles, science and writing. NAEP is administered to representative samples of students grouped by characteristics, such as gender, race, ethnicity, and school location (NCES, 2021c).

RTOP

The Reformed Teaching Observation Protocol (RTOP) was developed by the Evaluation Group of the Arizona Collaborative for Excellence in the Preparation of Teachers (ACEPT), with the intent to measure "reformed" teaching. This assessment was developed in 1998 and 1999, with its first administration in the Fall of 1999. Reformed teaching was related to the reformation of mathematics and science teaching taking place at the time of development. Piburn and Sawada (2000) describe the basis of this observational tool as constructivism. A constructivist classroom could be described as "one in which people are working together to learn. This has been called a knowledge-building community" (Piburn & Sawada, 2000, p. 4). The classroom would be one in which inquiry and diversity would be honored.

ASDQII

The Academic Self-Description Questionnaire II (ASDQII), developed by Marsh, was one of three sets of inventories for measuring self-concept. The intended population was junior and senior high school students aged 13 through 17 years (Boyle, 1994, p. 632).

The tool measured both individual and group differences in self-concept. The ASCDII measures the original seven categories established in ASCDI, with an additional four new categories. The eleven scales covered concept of Reading, Mathematics, General School, Physical Abilities, Physical Appearance, Peer Relations (Split into Same and Opposite Sex), Parent Relations, General Self, Emotional Stability, and Honesty-Trustworthiness (Boyle, 1994, p. 633).

Administration of the ASDQII to adolescents takes between 15 and 25 minutes. "It is a useful tool for the educator who wishes to investigate why a particular student is not achieving academically" (Boyle, 1994, p. 634). The instrument is helpful in measuring self-esteem, self-worth, and psychological adjustment.

National Council of Teachers of Mathematics Teaching and Learning Beliefs Questionnaire

The Teaching Learning Beliefs Questionnaire divided beliefs into the categories of unproductive and productive. The productive beliefs, centered on the learning of mathematics, included a focus on developing understanding through problem solving, reading, and discourse. When problem solving, students need a wide range of strategies at their disposal. Learning mathematics can involve exploring and solving both contextual and mathematical problems. The role of the teacher is to engage students in a way to promote reasoning and discourse among the students. Students should actively make connections to prior knowledge, contexts, and experiences and also should consider the reasoning offered by others. The teacher should challenge, encourage, and support the mathematical student (“Beliefs About Teaching”, n.d.).

Chapter Three: Methodology

Purpose

This study was designed to investigate multiple aspects of an Algebra I intervention program in a suburban high school as a way to determine which factors, such as mathematics self-concept, teacher instructional style, scores on semester finals, teacher beliefs about teaching mathematics, and the intensity of intervention a student received, if any, were related. Because the factors studied were observed and not manipulated in any way, a correlational research design was used (Price et al., 2017) to determine relationships among the variables. Additionally, the use of standardized observation and survey tools suggested a quantitative research design would be optimal (Fraenkel et al., 2018).

The research site was the only high school in the district of 5000 students and served approximately 1800 students in grades nine through twelve. Recognized as an exemplary high school by several entities, including the state in which the school existed, the school was generally thought to provide a rigorous education. Despite the school's overall success, teachers and administrators identified a group of students who were behind their peers in either the level of mathematics courses taken or in relation to grades in previous math classes and scores on standardized tests. The intervention program studied was put in place as a way to support students who did not take Algebra I prior to entering high school, which could put them at risk for completing fewer math courses than their peers (Cirino et al., 2018; McEachin et al., 2017). The majority of students who attended the research school took Algebra I in 8th grade. Thus, students who were not enrolled in Algebra I prior to entering high school were behind their peers in coursework, and were deemed ‘at risk’ in terms of math achievement. This must be kept in mind when examining the results of this study, as it was designed to identify potential relationships among student and teacher factors in an Algebra I intervention class, in order to better understand the variables at work in student success.

Research Design

Null Hypotheses

Null Hypothesis 1: There is no relationship between students’ academic self-concept, as measured by the ASDQII, and teachers’ beliefs about the teaching of algebra as measured by the Teacher Belief Survey.

Null Hypothesis 2: There is no relationship between students’ academic self-concept in math, as measured by the ASDQII and the instructional practices in the classroom as measured by the Reformed Teaching Observation Protocol.

Null Hypothesis 3: There is no relationship between Algebra I mastery, as measured by student results on Algebra I semester comprehensive finals and teachers’ beliefs about the teaching of algebra, as measured by the Teacher Belief Survey.

Null Hypothesis 4: There is no relationship between Algebra I mastery, as measured by student results on Algebra I comprehensive semester finals and the instruction used in the Algebra I classrooms, as measured by the Reformed Teaching Observation Protocol.

Null Hypothesis 5: There is no relationship between teacher beliefs, as measured by the Teacher Belief Survey and instructional practices in the Algebra I classroom, as measured by the RTOP.

Null Hypothesis 6: There is no relationship between student academic self-concepts, as measured by the ASCDII and Algebra I mastery, as measured by scores on Algebra I comprehensive semester finals.

Population and Sample

Two hundred and three high school students were enrolled in Algebra I at the research site. Enrollment was based on several factors, including recommendation by an eighth-grade teacher or counselor, parent request, and lack of Algebra credit due to having transferred from a private school or other institution that did not offer Algebra to eighth grade students. Some students were repeating the course, due to earning a failing grade.

The researcher met with teachers to explain the purpose of the study and secure all Algebra I teachers as voluntary participants in the study. To recruit student participants, the researcher created an informational video, which was shared during Algebra I class time, and teachers handed out assent and consent forms to students. In addition, the researcher attended a parent open house in the fall of 2016 in order to answer any questions and encourage participation in the study. All students enrolled in Algebra I at the high school were eligible to participate, and, based on returning both student assent forms and parental consent forms, the resulting sample included 71 students.

Instrumentation and Data Collection

Student Factors

The Academic Self-Description Questionnaire II, designed to measure academic self-concept of students in seventh through twelfth grades, was used as both a pre- and post-test. According to researchers, the ASDQII was a valid and reliable method for understanding students' academic self-concept (Marsh, 1990; Pena et al., 2015). A quantitative survey, the ASDQII utilized an eight-point Likert scale, which asked students to reflect on how true or false each statement was in relation to students' feelings about mathematics. Statements connected to each point on the scale included: Definitely False, Mostly False, False, More False Than True, More True Than False, Mostly True, True, and Definitely True. Because research confirmed that academic self-concept was domain specific (Liem et al., 2015; Marsh, 1990; Niepel et al., 2014), the researcher modified the ASDQII survey to include only questions related to mathematics.

Seventy-one students completed the ASDQII as a pre-test during a single homeroom period at the beginning of the study. When administered as a post-test in the spring of the same school year, only 23 students participated, due in part to one teacher who failed to provide the assessment to students. Because all but one statement on the survey was written in the affirmative, the negative statement was reverse scored for purposes of analysis. Additionally, student scores on common semester finals were collected for the population sample of 71 students. Created collaboratively by the Algebra I teachers, the semester finals were cumulative, covering one year of Algebra I content.

Teacher Factors

Teachers' beliefs about teaching and learning were collected using the NCTM's Teaching and Learning Beliefs Survey, which consisted of 12 questions regarding aspects of both teaching and learning and aligned with what NCTM called productive and unproductive thinking (2014). Respondents selected from a four-point Likert scale to indicate their level of disagreement or agreement with each statement. The mean score for both productive and unproductive beliefs was computed, with high scores indicating strong alignment to productive or unproductive beliefs and low scores indicating little or no alignment.

In order to gather data on teaching styles implemented by the teachers in the study, the researcher observed each of the four teachers three times during the course of the study, as outlined in Table 5. Using the Reformed Teaching Observation Protocol (RTOP), the researcher rated three broad categories: lesson planning and implementation, the content of the lesson, and the classroom climate and culture on a five-point scale to indicate how accurately each statement reflected the observed instruction.

Table 5

Teacher Observation Dates

Teacher	Observation 1	Observation2	Observation3
A	11.8.16	11.15.16	11.29.16
B	1.31.17	2.10.17	2.27.17
C	1.31.17	2.10.17	2.27.17
D	2.3.17	2.10.17	2.27.17

The National Council of Teachers of Mathematics' standards included "problem solving, reasoning and proof, communication, connections, [and] representations" (National Council of Teachers of Mathematics, n.d., para. 5) which aligned with the fundamental practices outlined in the constructivist approach to mathematics instruction measured by the Reformed Teaching ~~and~~ Observation~~al~~ Protocol (Piburn & Sawada, n.d.). Reformed teaching, or teaching that is constructivist in nature, was identified as instruction that called upon students' prior knowledge, allowed for multiple pathways toward solutions, and helped students make connections both between mathematical concepts and with other content areas (Boston et al., 2015).

Designed to determine how well instruction aligned with reformed teaching practice (defined as constructivism for the purposes of this study) in mathematics and science, the RTOP provided a quantitative description for three broad categories, including lesson design and implementation, content, and classroom culture (Classroom Observation Project, 2018; Piburn & Sawada, n.d.; Sawada et al., 2002). Sawada et al. (2002) confirmed the reliability of the RTOP by computing Chronbach's alpha for both subsets of the observation tool and the assessment as a whole, with the resulting alpha of 0.97, indicating strong internal reliability.

Twenty-five items were scored using a rubric scale of zero to four, with zero indicating the item was not observed and a score of four indicating the item is "very descriptive of the lesson" (Boston et al., 2015, p. 156), and total scores were used to indicate the degree of constructivist approaches used by the instructor as outlined in Table 6 (Classroom Observation Project, 2018; Science Education Resource Center at Carleton College, 2014). Teachers were observed three times, and a mean score for each teacher was calculated.

Table 6

Interpreting Reformed Teaching Observation Protocol Scores

Score	Type of Instruction
0-29	Traditional Lecture
30-49	Active Lecture
50+	Constructivist

Student Assessment Scores and Intensity of Intervention

Initially, the researcher planned to utilize standardized End of Course exam scores from the Algebra End of Course exam thought to be required by the state. However, the teachers and administrators at the research site learned mid-year that the requirement did not specify Algebra I as the required assessment. Instead, schools were only required to assess high school students in math once during students' high school enrollment. As a result of this understanding, a decision was made to forgo the Algebra End of Course Exam for these students.

The researcher determined that data from teacher-created common formative assessments would be used in place of the end of course assessment data. Teachers at the research site had started collaborating to create common formative assessments around agreed-upon essential elements of an Algebra I course, and the expectation by administration in the building was that teachers would begin collecting data and using it to inform instruction. A database was created for collection of data. Teachers administered and scored assessments, and students were responsible for entering their scores using a Google form. Although the collection and use of data was an expectation for all teachers in the study, the use of the database itself was voluntary. After receiving data from the formative common assessments, the researcher determined that the data were not useful for this study due to inconsistencies in format and frequency of reporting, and instead used scores from common finals given both first and second semester in the Algebra I course. It should be noted that in addition to the change in the type of assessment scores used, the use of data related to intensity of intervention was not included in the final data analysis, as none of the students in the sample were identified as having spent additional time with the interventionist.

Ethical Considerations

In order to ensure confidentiality, all consent and assent forms were secured in a locked cabinet in the researcher's home office. Electronic files were housed in a password-protected folder and on a thumb-drive secured in a locked cabinet in the researcher's home office. Upon completion of the study, records will be kept for three years and will then be destroyed to protect confidentiality.

The researcher protected participants' anonymity through the random assignment of a numerical code in place of first and last names. For the student sample, the sample size of 71 made it unlikely any individual response would be identifiable. However, the small teacher sample size required the researcher to inform participants that identifiable information was possible, due to the limited number of teacher participants.

All participants received Informed Consent forms describing the purpose and duration of the study, along with a description of the type of data that would be collected. Students who had not yet reached the age of consent were also given Assent forms to be signed by a parent or guardian in compliance with IRB standards. Although the researcher was, at one time, an administrator in the building, she was no longer employed at the research site during the study, thus no conflict of interest existed.

Delimitations, Limitations, and Assumptions

The time frame of the study was designed to take place starting in the Fall of the 2015 school year and ending in September of 2016, which would allow for permissions to be collected during the start of school open house, observations to be completed throughout the school year, and standardized assessment scores from the Spring of 2016 to be used as one variable. Due to a change in assessment practices by the school, the data collection process was adjusted to include results of teacher-created formative assessment data collected electronically via a data dashboard that all teachers were required to use. This change did not initially impact the timeframe for data collection. Upon receipt of the data, it became evident that there were problems with the data including large sets of missing data from multiple teachers, and data that were deemed unlikely due to patterns of results. The data were supposed to be entered by students themselves after each formative assessment and used for reflection and goal setting purposes. Due to the missing data, and potentially inaccurate data, the researcher determined it best to again change the data source to teacher-created summative semester finals. These data were based on common summative assessments given to all students in Algebra I, and the results were entered into the school's official grade book by teachers. This change did impact the timeframe of the study as it took several months to secure the data from the research site.

The research site was located in a suburban area outside of a major metropolitan city and was the sole high school in the district of approximately 5000 students. The researcher met with the Algebra I teachers to explain the study, and to provide both consent and assent forms. These were given to students by their teachers, and students were asked to return both the assent and consent forms. As a result, a sample of 71 returned all necessary forms and made up the initial student sample. The majority of student participants were Caucasian followed by African American students, Asian, Hispanic and multi-racial students respectively. Fifty-four males and 46 females were included with the majority of students scoring proficient or advanced on the state standardized math assessment taken when they were in eighth grade. During the study, some students were not given the opportunity to participate in the ASDQII post-test. As a result, 23 students returned both required permission forms and participated in all aspects of the study. Of the 23 students, 78% were 14-years old and 22% were 15-years old. Sixteen percent of the study participants were white, 4% were black, 1% were Hispanic, 6% were Asian, and 1% identified as multi-racial. Fifty-seven percent were male and 43% were female. Two students transferred from private schools and two transferred from other public schools in the region. Within this smaller sample, scores from the eighth-grade Missouri Assessment Program (MAP) mathematics assessment showed that 78% of the sample group scored in the Basic range with 5% at Below Basic, 11% scoring Proficient, and 5% in the Advanced range.

Teacher participants were selected based on their teaching assignment; all teachers who were assigned at least one Algebra I section were invited to participate, and all teachers signed consent forms. Of the four teachers assigned a section of Algebra I, three were female and one was male. Two teachers each had more than 15-years of experience, one teacher had more than 10-years of experience and one teacher was in the third year of teaching.

Summary

Using a correlational design to study the relationship between multiple student and teacher-related factors in an Algebra I intervention program allowed the researcher to determine if a relationship existed and the strength of any relationships. Variables studied included student self-concept in mathematics, student scores on Algebra I semester finals, teacher beliefs about teaching and learning mathematics, and the instructional style utilized by teachers in the Algebra I classrooms. The research design included a student survey given at two points in the year and a teacher survey. In addition, multiple classroom observations were conducted for each teacher participant.

Tools used for data collection included the ASDQII to measure academic self-concept in subjects thirteen years of age or older. A questionnaire from the NCTM was given to teachers, and classroom observations were conducted using the RTOP. An initial sample size of 71 student participants was randomly selected from a population of 203 students, and four teachers voluntarily took part in the study. Standard steps were taken to protect both the confidentiality and anonymity of participants by utilizing randomly assigned numerical identifiers, keeping forms in locked cabinets, and digital files in password-protected folders. The researcher was not connected with the research site throughout the study limiting the possibility of a conflict of interest.

Chapter Three outlined the tools and processes used for data collection. Chapter Four includes the results of the data collection and an analysis of the data in relation to the hypotheses which framed the study.

Chapter Four: Results

This study took place at a suburban high school in the Midwest and was designed to study various student, teacher, and instructional factors of an Algebra I intervention program in order to determine which factors were related. 221 students were part of the initial sample, and 71 students participated in all aspects of the study. In addition, four Algebra I teachers took part in the study.

Null Hypotheses

The Null Hypotheses addressed in analysis were:

Null Hypothesis 1: There is no relationship between students' academic self-concept, as measured by the ASDQII, and teachers' beliefs about the teaching of algebra as measured by the Teacher Belief Survey.

Null Hypothesis 2: There is no relationship between students' academic self-concept in math, as measured by the ASDQII and the instructional practices in the classroom as measured by the Reformed Teaching Observation Protocol.

Null Hypothesis 3: There is no relationship between Algebra I mastery, as measured by student results on Algebra I semester comprehensive finals and teachers' beliefs about the teaching of algebra, as measured by the Teacher Belief Survey.

Null Hypothesis 4: There is no relationship between Algebra I mastery, as measured by student results on Algebra I comprehensive semester finals and the instruction used in the Algebra I classrooms, as measured by the Reformed Teaching Observation Protocol.

Null Hypothesis 5: There is no relationship between teacher beliefs, as measured by the Teacher Belief Survey and instructional practices in the Algebra I classroom, as measured by the RTOP.

Null Hypothesis 6: There is no relationship between student academic self-concepts, as measured by the ASCDII and Algebra I mastery, as measured by scores on Algebra I comprehensive semester finals.

A correlational research design was used to identify which elements, if any, among student and teacher factors in a mathematics intervention program were related. In order to compare the variables and generalize the results to a population, a quantitative approach was selected for the research design (Fraenkel et al., 2018; University of Southern California Libraries Research Guide, 2019). Degree of correlation between the variables was determined using a Pearson Product Moment Correlation Coefficient (PPMCC), which identified the strength of the relationship between two variables using a range from negative one to positive one. Results of zero or close to zero indicated a weak or non-existent relationship (Bluman, 2013).

Multiple variables were studied, including student self-concept in mathematics both at the beginning of the school year and at the end of the school year, student scores on semester finals, teacher beliefs about teaching mathematics, and teacher instructional style. Data on student factors were collected via the ASDQII survey and from scores on Algebra I first and second semester finals.

In addition to data related to student factors, data on teacher-related factors were collected via NCTM's Teaching and Learning Beliefs survey and classroom observations conducted by the researcher. All data collected were numerical as part of the quantitative study design. When interpreting the PPMCC, the researcher used the boundaries shown in Table 7 as suggested by Laerd Statistics (2018, para. 4)

Table 7

Interpretation Guidelines for PPMC Coefficient, r

Strength of Association	Positive	Negative
Small	0.1 to 0.3	-0.1 to -0.3
Medium	0.3 to 0.5	-0.3 to -0.5
Large	0.5 to 1.0	-0.5 to -1.0

Data Analysis

Null Hypothesis 1

There is no relationship between students' academic self-concept as measured by the ASDQII, and teachers' beliefs about the teaching of Algebra I as measured by the Teacher Belief Survey.

Pearson Product Correlation Coefficients are displayed in Table 8.

Table 8

Correlation Between Students' Self Perception in Mathematics and Teachers' Beliefs About the Mathematics Instruction

		ASDQII Pre-Scale Sum	ASDQII Post-Scale Sum	Teacher Belief Unproductive Sum	Teacher Belief Productive Sum
ASDQ II Pre-Scale Sum	Pearson Correlation	1.0	0.739**	0.110	0.191
	n	71	23	71	71
ASDQII Post-Scale Sum	Pearson Correlation	0.739**	1.0	-0.197	0.064
	n	23	23	23	23
Teacher Belief Unproductive Sum	Pearson Correlation	0.110	-0.197	1.0	.558
	n	71	23	71	71
Teacher Belief Productive Sum	Pearson Correlation	0.191	0.064	.558**	1.0
	n	71	23	71	71

** Correlation is significant at the 0.01 level (2-tailed).

Results of the PPMCC revealed no significant relationship between students' pre or post scores on the survey of academic self-concept in mathematics and teachers' beliefs regarding the best way to teach math, both productive or unproductive, as classified by the NCTM Teacher Belief Survey. Statistical results for the ASDQII Pre-Score to Teacher Belief Unproductive were n = 71, α = .01, r-critical = .303, and r = .110, and n = 71, α = .05, r-critical = .232, and r = .110. The results for the ASDQII Post-Score to Teacher Belief Productive were n = 23, α = .01, r-critical = .505, and r = .064 and n =

23, $\alpha = .05$, r-critical = .396, and $r = .064$. Therefore, the researcher failed to reject the null hypothesis.

Within this data set, statistically significant relationships between pre- and post-scores on the ASDQII were evident. With a correlation of 0.739 at a significance level of 0.01 (two-tailed), student survey results showed a positive relationship between the pre- and post-surveys; that is, high scores on the ASDQII given as a pre-survey correlated with high scores on the same survey given as a post-survey. Statistical results for the ASDQII pre-survey to the ASDQII post-survey were $n = 23$, $\alpha = .01$, r-critical = .505, and $r = .739$ and $n = 23$, $\alpha = .05$, r-critical = .396, and $r = .739$. Therefore, the null hypothesis was not rejected for this comparison.

Similarly, a correlation of 0.558 at a significance level of 0.01 (two-tailed) between productive and unproductive beliefs about teaching mathematics indicated a statistically significant positive relationship. In other words, when scores on unproductive beliefs about teaching math increase, there was also an increase in scores on productive beliefs about teaching math. Likewise, if one variable decreased the other followed suit. Statistical results for the Productive Belief Survey to the Unproductive Belief Survey were $n = 71$, $\alpha = .01$, r-critical = .303, and $r = .558$ and $n = 71$, $\alpha = .05$, r-critical = .303, and $r = .558$. Therefore, the null hypothesis was not rejected for this comparison.

A small inverse relationship was revealed between unproductive teacher beliefs and students' self-concept in mathematics. The results for the ASDQII Post-Score to Teacher Belief Unproductive were $n = 23$, $\alpha = .01$, r-critical = .505, and $r = -.197$ and $n = 23$, $\alpha = .05$, r-critical = .396, and $r = -.197$. While the r-value was not high enough to cause the researcher to reject the null hypothesis, descriptively the relationship suggests that as teachers' levels of agreement with unproductive math statements on the NCTM survey increased, students became less confident in their ability to do well in mathematics.

Null Hypothesis 2

There is no relationship between students' academic self-concept in mathematics as measured by the ASDQII and the instructional practices in the classroom as measured by the Reformed Teaching Observation Protocol.

Pearson Product Correlation Coefficients are shown in Table 9.

Table 9

Students' Self Perception in Mathematics and Instructor's Teaching Style

	ASDQII Pre Scale Average	ASDQII Post Scale Summary	Teaching Style (RTOP)
ASDQII Pre Scale Average	1.0	1.000**	0.269
ASDQII Post Scale Summary	1.000**	1.0	0.269
Teaching Style (RTOP)	0.269	0.269	1.0

** Correlation is significant at the 0.01 level (2-tailed). n=71, all correlations.

Using boundaries suggested by Laerd (2018), the correlation between students' self-perception about mathematics and the style of teaching used by their instructors was small at 0.269 when looking at both the ASDQII pre survey and ASDQII post survey, which indicated no relationship between these factors. Statistical results for the ASDQII Pre-Score to Teaching Style were $n = 71$, $\alpha = .01$, r-critical = .303, and $r = .269$. The results for the ASDQII Post-Score to Teaching Style were $n = 23$, $\alpha = .01$, r-critical = .505, and $r = .269$ and $n = 23$, $\alpha = .05$, r-critical = .396, and $r = .269$. As a result, the researcher failed to reject the null. The researcher looked at the relationship between specific questions within the ASDQII and teachers' teaching styles, which revealed some statistically significant results as shown in Table 10, but the results did not impact the overall null hypotheses.

Table 10

Students' Self Perception in Mathematics and Specific RTOP Scores

	RTOP Score
RTOP Score	1
ASDQII Pre Q1: I am hopeless when it comes to mathematics N=71	-0.340**
ASDQII Pre Q5: Work in mathematics is easy for me N=71	0.257*
ASDQII Pre Q6: I get good marks in mathematics classes N=71	0.278*

* Correlation is significant at the 0.05 level (2-tailed);
** Correlation is significant at the 0.01 level (2-tailed). n=23, all correlations.

The correlation between students' responses on specific questions within the ASDQII and their teachers' scores on the RTOP revealed three statistically significant results from the pre-survey and no statistically significant results on the post survey. Statistical results for the ASDQII Pre-ScoreQ1 to RTOP $n = 23$, $\alpha = .01$, r-critical = .505, and $r = -.340$, and $n = 23$, $\alpha = .05$, r-critical = .396, and $r = -.340$. Results for the ASDQII Pre-ScoreQ5 to RTOP $n = 23$, $\alpha = .01$, r-critical = .505, and $r = .257$, and $n = 23$, $\alpha = .05$, r-critical = .396, and $r = .257$. Results for the ASDQII Pre-ScoreQ6 to RTOP $n = 23$, $\alpha = .01$, r-critical = .505, and $r = .278$, and $n = 23$, $\alpha = .05$, r-critical = .396, and $r = .278$.

Question one revealed an inverse correlation, -0.340, which indicated as an individual teacher's level of constructivism increased, students in that class had decreasing levels of hopelessness related to their ability to do well in math. The inverse was also true; as a teacher's level of constructivism decreased, students in that classroom had increasing feelings of hopelessness related to doing well in math. Questions five (0.257) and six (0.278) on the ASDQII pre-survey were positively correlated to their teachers' level of constructivism as assessed by the RTOP indicating that as a teacher's RTOP score increased, students' perceptions of the ease of work in math class and their perceived ability to achieve high grades in math also increased.

No statistically significant results existed between student academic self-perception scores on the post survey and their teachers' score on the RTOP, which is likely due to the change in sample size from 71 students to 23. As discussed in chapter three, the mortality rate in the study was the result of one teacher failing to provide the post survey to students.

Null Hypothesis 3

There is no relationship between Algebra I mastery as measured by student results on Algebra I semester comprehensive finals and teachers' beliefs about the teaching of algebra as measured by the Teacher Belief Survey. These results can be seen in Table 11.

Using a PPMC, the researcher determined that there was no statistically significant relationship between student scores on semester finals and productive teacher beliefs as defined by the NCTM Teacher Belief survey. The researcher analyzed each question on the NCTM survey in relation to students' scores on semester finals, and found that while none of the teacher perceptions were significantly related to student assessment scores, three questions did have a small association when using Laerd's boundaries for analysis (2018).

For both first and second semester course finals, questions five, nine, and eleven showed a small positive association with student scores (Table 8). These questions specifically called out the importance of students as active participants in learning mathematics. As teachers' scores on these questions more strongly indicated agreement with the statements, students' scores on semester finals would also increase. Likewise, as teachers' scores on questions five, nine, and eleven decrease indicating less agreement with the statements, students' scores on semester finals would likely decrease. A similar approach was applied to unproductive beliefs, as shown in Table 9.

Table 11

Relationship Between Semester Finals and Productive Teacher Beliefs

		1	2	3	4	5	6	7	8	9
1. Semester 1 Final Percentage	Pearson Correlation	1.0	0.884**	0.954**	0.022	*b*	0.129	0.053	0.134	0.134
	n	69	69	69	71	71	71	71	71	71
2. Semester 2 Final Percentage	Pearson Correlation	0.884**	1.0	0.951**	-0.025	*b*	0.127	0.071	0.191	0.191
	n	69	69	69	71	71	71	71	71	71
3. Semester 1 & 2 Final Average	Pearson Correlation	0.954**	0.951**	1.0	0.018	*b*	0.16	0.050	0.177	0.177
	n	69	69	69	71	71	71	71	71	71
4. Productive Tchr Beliefs Q3	Pearson Correlation	0.022	-0.025	0.018	1.0	*b*	0.690**	.466**	-0.393**	-0.393**
	n	69	69	69	71	71	71	71	71	71
5. Productive Tchr Beliefs Q4	Pearson Correlation	*b*	*b*	*b*	*b*	1.0	*b*	*b*	*b*	*b*
	n	69	69	69	71	71	71	71	71	71
6. Productive Tchr Beliefs Q5	Pearson Correlation	0.129	0.127	0.160	0.690**	*b*	1.0	0.675**	0.393**	0.393**
	n	69	69	69	71	71	71	71	71	71
7. Productive Tchr Beliefs Q8	Pearson Correlation	0.053	0.071	0.050	0.466**	*b*	0.675**	1.0	0.266*	0.266*
	n	69	69	69	71	71	71	71	71	71
8. Productive Tchr Beliefs Q9	Pearson Correlation	0.134	0.191	0.177	-0.393**	*b*	0.393**	0.266*	1.0	1.0**
	n	69	69	69	71	71	71	71	71	71
9. Productive Tchr Beliefs Q11	Pearson Correlation	0.134	0.191	0.177	-0.393**	*b*	0.393**	0.266*	1.0**	1.0
	n	69	69	69	71	71	71	71	71	71

**. Correlation is significant at the 0.01 level (2-tailed); *correlation is significant at the 0.05 level (2-tailed); *b*=cannot be computed because at least one variable is constant.

Table 12

Relationship Between Semester Finals and Unproductive Teacher Beliefs

		1	2	3	4	5	6	7	8	9
1.Semester 1 Final Percentage	Pearson Correlation	1	0.884**	0.954**	*b*	-0.061	-0.061	0.022	0.129	*b*
	n	69	69	69	71	71	71	71	71	71
2.Semester 2 Final Percentage	Pearson Correlation	0.884**	1	0.951**	*b*	-0.090	-0.090	-0.025	0.127	*b*
	N	69	69	69	71	71	71	71	71	71
3. Semester 1 & 2 Final Average	Pearson Correlation	0.954**	0.951**	1	*b*	-0.097	-0.097	0.018	0.160	*b*
	n	69	69	69	71	71	71	71	71	71
4. Productive Tchr Beliefs Q1	Pearson Correlation	*b*	*b*	*b*	1	*b*	*b*	*b*	*b*	*b*
	n	69	69	69	71	71	71	71	71	71
5. Productive Tchr Beliefs Q2	Pearson Correlation	-0.061	-0.090	-0.097	*b*	1	1.0**	0.711**	0.275*	*b*
	n	69	69	69	71	71	71	71	71	71
6. Productive Tchr Beliefs Q6	Pearson Correlation	-0.061	-0.090	-0.090	*b*	1.0**	1	0.711**	0.275*	*b*
	n	69	69	69	71	71	71	71	71	71
7. Productive Tchr Beliefs Q7	Pearson Correlation	0.022	-0.025	0.018	*b*	0.711**	0.711**	1	0.690**	*b*
	n	69	69	69	71	71	71	71	71	71
8. Productive Tchr Beliefs Q10	Pearson Correlation	0.129	0.127	0.160	*b*	0.275*	0.275*	0.690**	1.0	*b*
	n	69	69	69	71	71	71	71	71	71
9. Productive Tchr Beliefs Q12	Pearson Correlation	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	1.0
	n	69	69	69	71	71	71	71	71	71

**. Correlation is significant at the 0.01 level (2-tailed); *correlation is significant at the 0.05 level (2-tailed); *b*=cannot be computed because at least one variable is constant.

As was the case with semester finals and productive beliefs, using a PPMC revealed no statistically significant relationship between students' scores on semester finals and unproductive teaching beliefs as defined by the NCTM Teacher Belief Survey (Table 12). As a result, the researcher failed to reject the null hypothesis.

One question, categorized as unproductive, revealed a small, nonsignificant, relationship. Question number ten on the Teacher Belief Survey states, "The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes and tests" (NCTM, 2014, p. 11). The relationship between first and second semester finals and question ten was 0.129 and 0.127, respectively. Using Laerd's boundaries (2018), the results indicated a positive relationship. In other words, as teachers' levels of agreement with the statement increased, students' scores on semester finals also increased and as teachers' levels of agreement with question ten decreased, students' scores on semester finals would also decrease. This interested the researcher as the statement is defined as unproductive, yet data implied that higher levels of agreement with the statement yielded improved scores on teacher-created assessments.

Because no statistically significant relationships were shown between scores on semester finals and teacher productive and unproductive beliefs about teaching, the research failed to reject the null.

Null Hypothesis 4

There is no relationship between Algebra I mastery as measured by student results on Algebra I comprehensive semester finals and the instruction used in the Algebra I classrooms as measured by the Reformed Teaching Observation Protocol.

Pearson Product Moment Correlation Coefficients are displayed in Table 13.

Table 13

Relationship Between Semester Final Scores and RTOP Scores

		1st Semester Final Percentage	2nd Semester Final Percentage	Average of Semester Final Scores	RTOP Observation Score
1st Semester Final Percentage	Pearson Correlation	1.0	0.884**	0.954**	0.135
	n	69	69	69	69
2nd Semester Final Percentage	Pearson Correlation	0.884**	1.0	0.951**	0.211
	n	69	69	69	69
Average of Semester Final Scores	Pearson Correlation	0.954**	0.951**	1.0	0.169
	n	69	69	69	69
RTOP Observation Score	Pearson Correlation	0.135	0.211	0.169	1.0
	n	69	69	69	71

**Correlation is significant at the 0.01 level (2-tailed).

To assess the relationship between student scores on semester finals and the teacher observation score as measured by the RTOP, the researcher used a PPMCC. First and second semester final scores were each examined individually along with an average of the semester scores. All student scores were presented as a percentage. The p value was significant at the 0.01 level (2 tailed) with a critical value (r) of 0.354. As shown in Table 10, there is no significant correlation between students' scores on semester finals and the degree of constructivist teaching employed by teachers in the Algebra I classroom. This was true for both first and second semester finals individually and the average of the two scores, therefore, the researcher failed to reject the null.

Using an alpha 0.01 and a critical value of 0.354, the r scores indicated only a small, observable, yet nonsignificant, direct relationship between scores on semester finals. As a result, the researcher failed to reject the null hypothesis. Additionally, data from second semester finals showed a higher correlation to the instructional style of the teacher than did scores on first semester finals, but the correlation was not statistically significant.

Null Hypothesis 5

There is no relationship between teacher beliefs as measured by the Teacher Belief Survey and degree of constructivism evident in instructional practices in the Algebra I classroom as measured by the RTOP.

The researcher used scores from the NCTM's Teaching Beliefs Survey and the RTOP score gleaned from classroom observations to examine the relationship between teacher beliefs about teaching mathematics and the style of instruction used in the classroom. Data from the Teaching Beliefs Survey were separated into productive and unproductive beliefs for analysis. Because none of the correlations fell into the critical area, the researcher failed to reject the null. See Table 14 for results for unproductive beliefs and styles. See Table 15 for results for productive beliefs and styles.

Table 14

Unproductive Teacher Beliefs and Teaching Style

	1	2	3	4	5	6	7	8
1	1.0	-0.182	*a*	-0.190	-0.190	-0.172	-0.058	*a*
2	-0.182	1.0	*a*	0.888**	0 .888**	0.924**	0.665**	*a*
3	*a*	*a*	1.0	*a*	*a*	*a*	*a*	*a*
4	-0.190	0.888**	*a*	1.0	1.000**	0.711**	0.275*	*a*
5	-0.190	0.888**	*a*	1.000**	1.0	0.711	0.275*	*a*
6	-0.172	0.924**	*a*	0.711**	0.711	1.0	0.690**	*a*
7	-0.058	0.665**	*a*	0.275*	0.275*	0.690**	1.0	*a*
8	*a*	*a*	*a*	*a*	*a*	*a*	*a*	1.0

Note. **Correlation is significant at the 0.01 level (2-tailed); *correlation is significant at the 0.05 level (2-tailed); *a*=cannot be computed because at least one variable is constant; n=71.

Table Legend: 1. RTOP Score; 2. Teacher Belief Scale Summary; 3. Teacher Belief Survey Q1: Mathematics learning should focus on practicing procedures and memorizing basic number combinations.; 4. Teacher Belief Survey Q2: The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.; 5. Teacher Belief Survey Q6: An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.; 6. Teacher Belief Survey Q7: Students can learn to apply mathematics only after they have mastered the basic skills; 7. Teacher Belief Survey Q10: The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes and tests.; 8. Teacher Belief Survey Q12: Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Table 15

Productive Teaching Beliefs and Teaching Style

	1	2	3	4	5	6	7	8
1	1.0	-0.026	-0.172	*a*	-0.058	-0.089	0.145	0.145
2	-0.026	1.0	.484**	*a*	0.949**	0.795**	0.591**	0.591**
3	-0.172	0.484**	1.0	*a*	0.690**	0.466**	-.393**	-0.393**
4	*a*	*a*	*a*	1.0	*a*	*a*	*a*	*a*
5	-0.058	0.949**	.690**	*a*	1.0	0.675**	0.393**	0.393**
6	-0.089	0.795**	.466**	*a*	0.675**	1.0	0.266*	0.266*
7	0.145	0.591**	-.393**	*a*	0.393**	0.266*	1.0	1.000**
8	0.145	0.591**	-.393**	*a*	0.393**	0.266*	1.000**	1.0

**Correlation significant at the 0.01 level (2-tailed); *Correlation significant at the 0.05 level (2-tailed); *a*=cannot be computed because at least one of the variables is constant; n=71.

Table Legend. 1. RTOP Score; 2. Teacher Belief Scale Summary; 3. Teacher Belief Survey Q3: All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures; 4. Teacher Belief Survey Q4: The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics; 5. Teacher Belief Survey Q5: Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning and discourse.; 6. Teacher Belief Survey Q8: Students can learn mathematics through exploring and solving contextual and mathematical problems.; 7. Teacher Belief Survey Q9: An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.; 8. Teacher Belief Survey Q11: The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.

Null Hypothesis 6

There is no relationship between student academic self-concept as measured by the ASDQII and Algebra I mastery as measured by scores on Algebra I comprehensive semester finals.

Pearson Product Correlation Coefficients are displayed in Table 16.

Table 16

Student Self-Concept and Scores on Semester Finals

	1st Semester Final %	2nd Semester Final %	Avg. of Semester Finals
ASDQII Pre-Survey n=71	0.447**	0.449**	0.445**
ASDQII Post-Survey n=23	0.428*	0.461*	0.452*
Difference Between ASDQII Pre and Post	-0.033	-0.054	-0.044

**Correlation significant at the 0.01 level (2-tailed); *Correlation significant at the 0.05 level (2-tailed).

Student self-concept in mathematics was measured using a modified version of the ASDQII, which included only questions related to math. The survey was given both at the beginning and the end of the study, although one teacher in the study did not give the survey to students, which resulted in a smaller sample size for the post-survey results. Results from both the pre and post survey were examined for a possible relationship with student scores on both first and second semester finals using a PPMCC.

Direct correlations between student self-concept in mathematics pre-survey and scores on both first and second semester finals were significant at the .01 level which indicated as students' self-concept in math increased, scores on Algebra finals for both semesters also increased (Table 11). Statistics results for ASDQII Pre to Scores on Semester Finals were n = 71, α = .01, r-critical = .303, and r = .445, and n = 71, α = .05, r-critical = .232, and r = .445. Results for ASDQII Post to Scores on Semester Finals were n = 23, α = .01, r-critical = .505, and r = .445, and n = 23, α = .05, r-critical = .396, and r = .452. The inverse was also true meaning that as students' academic self-concept decreased, scores on Algebra I semester finals also decreased.

Post-survey results were significant at the 0.05 level and again showed a direct positive relationship between student self-concept in math and scores on Algebra I finals, meaning that as students' academic self-concept in math increased, scores on semester finals also increased.

Laerd's boundaries (2018) indicated a medium positive relationship between students' academic self-concept in mathematics and their grades on semester finals for both the pre- and post-survey. As shown by the data, when students' self-perception ratings regarding their ability to do well in math class increased, their scores on Algebra I finals also increased. The more time students spent in Algebra I classrooms, the strength of the relationship increased, as shown by increased correlation strength of the post survey and second semester finals. However, the difference between the pre and post ASDQII surveys was not statistically significant.

As a result of the positive medium relationship between the pre- and post-survey results and scores on semester finals, the researcher rejected the null hypothesis. There is a relationship between students' self-perception in mathematics and achievement as measured by Algebra I semester finals.

Summary

Quantitative data sources were used to examine the potential relationship between several variables in the Algebra I intervention classroom, including student academic self-perception in math and teachers' beliefs about teaching, students' academic self-concept in math and the instructional style of teachers, students scores on Algebra I semester finals and teachers' beliefs about teaching, students' scores on Algebra I semester finals and the instructional style of teachers, teacher beliefs about teaching math and instructional style, and students' academic self-concept in math and scores on semester finals. Few statistically significant relationships, as stated in the null hypotheses, were found with the exception of the direct relationship between student self-concept in mathematics and grades on semester finals.

Although not statistically significant in relation to the null hypotheses, small positive associations existed between students' scores on final assessments and teacher beliefs about teaching mathematics related to specific questions for both productive and unproductive beliefs on the NCTM's Teacher Belief Survey.

Chapter Five: Discussion and Reflection

Introduction

The purpose of this study was to investigate which factors in a high school Algebra I class, if any, were related. The factors were separated into student and teacher categories and included student academic self-concept in mathematics; student scores on teacher-created, comprehensive semester finals; teacher beliefs about teaching mathematics; and teacher observations designed to identify where individual teachers fell on a scale from a traditional approach to teaching to a constructivist approach to instructional design and delivery. The initial study also included the number of times a student worked with the intervention teacher. However, none of the students in the study saw the intervention teacher, so this factor was not included in the final analysis.

Quantitative measures were utilized to collect data from both students and teachers. The ASDC II, a survey for students aged 15 and older designed to measure self-concept in mathematics, was given to students both at the beginning and at the conclusion of the study. Student scores on semester finals were collected via access to teachers' grade books. Collection of teacher-related data included administration of a teacher belief survey grounded in the NCTM's standards (2014). Additionally, the researcher observed each teacher three times throughout the study and used the RTOP observation format to record and score the type of instruction observed on a scale which indicated the level of constructivism evident in the instructional design and delivery. To determine if a relationship existed between various factors and the strength of the relationship, the researcher used a PPMMC.

With mathematics courses undergoing a renaissance of sorts due to factors such as a focus on STEM skills in schools, and the realization that advanced math courses in high school set the stage for success in high-demand college courses and applied technical fields (US Department of Education, 2018), understanding the relationships among student and teacher factors was important in determining effectiveness of the current intervention program and planning for future changes. Although long recognized as a gate-keeper course (Burdman, 2018; Loveless, 2013; U.S. Department of Education Office for Civil Rights, 2018), Algebra I continues to be the source of much debate (Clotfelter et al., 2014; McEachin et al., 2017). Should all students take Algebra I prior to high school? If not, what is the impact on students' self-concept and skill development? What type of instruction impacts student achievement? And, among teacher and student factors, which are most impactful in a high school Algebra course? This study attempted to examine the relationship between these elements, and determine which, if any, were related in order to inform instructional practices for students taking Algebra I at the high school level.

Hypotheses

Hypothesis 1: There is a relationship between students' academic self-concept as measured by the ASDQII, and teachers' beliefs about the teaching of algebra as measured by the Teacher Belief Survey.

Hypothesis 2: There is a relationship between students' academic self-concept in math as measured by the ASDQII and the instructional practices in the classroom as measured by the Reformed Teaching Observation Protocol.

Hypothesis 3: There is a relationship between Algebra I mastery as measured by student results on Algebra I semester comprehensive finals and teachers' beliefs about the teaching of algebra as measured by the Teacher Belief Survey.

Hypothesis 4: There is a relationship between Algebra I mastery as measured by student results on Algebra I comprehensive semester finals and the instruction used in the Algebra I classrooms as measured by the Reformed Teaching Observation Protocol.

Hypothesis 5: There is a relationship between teacher beliefs as measured by the Teacher Belief Survey and instructional practices in the Algebra I classroom as measured by the RTOP.

Hypothesis 6: There is relationship between student academic self-concepts as measured by the ASCDII and Algebra I mastery as measured by scores on Algebra I comprehensive semester finals.

Student Academic Self Concept

Academic self-concept has been shown to be an important factor that can positively or negatively impact student achievement (Liem, G. A., McInerney, D. M., & Yeung, A. S., 2015; Marsh, H.,1990). This study examined students' academic self-concept in mathematics in relation to several factors including: teachers' beliefs about the teaching of mathematics, the instructional practices used by teachers in the math classroom, and student achievement on teacher-created semester finals.

The domain-specific nature of academic self-concept revealed by Shavelson et al. (1976) and Marsh (1990) confirmed academic self-concept differed depending upon the content being studied. Later study also showed that academic self-concept was not a fixed construct, but could be influenced by experiences (Majeed et al., 2013). Therefore, it was feasible that students' academic self-concept in mathematics could be related to factors such as teacher beliefs about teaching mathematics, a teacher's instructional practices, and overall achievement in mathematics. Of the three relationships examined in this study, only academic self-concept and scores on semester finals were correlated. The study design did not allow for identification of interdependence of variables which may reveal more in-depth understanding of the relationships studied. Recommendations for study design are included later in this chapter.

Academic Self-Concept and Teacher Beliefs

Although academic self-concept was shown to be malleable, little change was evident in the results of this study. When considering that teachers' beliefs have the potential to impact the kind of instruction provided, it seemed logical that these two factors would be related. However, data did not support the existence of a relationship. Instead, data indicated that students who ranked themselves high on the ASDQII pre-survey also had high scores on the post-survey, and students who ranked themselves low on the ASDQII pre-survey were likely to rank themselves low on the post-survey. In other words, teacher beliefs were not related to students' academic self-concept when examined in isolation. As noted later in this discussion, the research also indicated that teachers' beliefs about teaching mathematics were not aligned with their instructional practice which could have been a tangential factor in the relationship between academic self-concept in mathematics and teacher beliefs.

Academic Self-Concept and Instructional Practices

Although classroom observations during the study indicated teachers' instructional practices were not aligned with the NCTM's recommendations for best practice in a mathematics classroom (2014), the misalignment did not impact students' academic self-concept in mathematics either positively or negatively as there was no significant relationship between the two factors.

Research on effective instructional practices in mathematics indicated an overall preference for a constructivist approach which focused on conceptual understanding over rote memorization of mathematics concepts and processes (Kiernan et.al, 2016; Kim, 2005; NCTM, 2017; Rozman and Klieme, 2017). However, RTOP results indicated the majority of Algebra I teachers in the study scored very low on the scale of reformed teaching or constructivist practices. The researcher noted that much of the talking in a classroom was done by the teachers who were often asking questions which consisted of a right or wrong answer. When students responded with an incorrect answer, the teachers' next steps were typically to show the students the correct way to solve the problem instead of asking additional questions as to how the student arrived at his or her answer or to allow time for students to further explore their own thinking and develop a revised response. Additionally, very little peer collaboration occurred in the observed classrooms. Most interaction was from teacher to student and student to teacher.

The results of this study indicated that despite a more traditional approach to instruction, students' academic self-concept in mathematics was neither positively nor negatively impacted contradicting previous research on the relationship between the two variables (Venenciano & Heck, Proposing and testing a model to explian traits of algebra preparedness, 2016).

Academic Self-Concept and Achievement

Research has indicated a solid relationship between academic self-concept and achievement (McInerney et al., 2012), and this relationship was supported by the results of this study which indicated students' academic self-concept was related to their achievement in Algebra I as measured by teacher-created, comprehensive semester finals. The relationship was positive in that as student scores on semester finals increased, so, too, did their academic self-concept. However, due to the change in available data as a result of standardized assessment scores not being available as the researcher originally planned, the results may be misleading. As noted, the measure of achievement for the purpose of this study was based on how well students performed on teacher-created comprehensive semester finals. Teachers were not provided with any guidelines regarding the content of the assessments, nor was their knowledge of assessment design evaluated. For the purpose of this study the researcher did not measure the quality of the semester assessments so the validity and reliability of the assessments cannot be determined.

Additionally, students' academic self-concept in mathematics and its relationship to academic achievement is likely highly nuanced as suggested by Gray and Mannahan (2017) who identified a need for studies to focus on the student phenomenological experience as opposed to outside factors. Also of importance when considering academic self-concept, is the direction in which the relationship flows. For example, Niepel (2014) suggested that effort, leading to mastery, is what caused self-concept to rise; self-concept alone did not positively impact achievement. These findings suggested a focus on teacher instructional style as it relates to structures for student mastery may be beneficial in more fully understanding the interplay between academic self-confidence and achievement.

Algebra I Mastery and Teacher Beliefs About Teaching

Algebra I Mastery and Instructional Practices

The instructional style of the teacher, specifically, how much the instruction aligned with a constructivist approach to teaching, was not correlated with an increase or decrease on students' self-perception in math. Results contradicted research which showed the importance of the teacher on student achievement (Nilsen & Gustafsson, 2016; Opper, 2019), and suggest that the relationship between mastery of content and the approach taken by a teacher is multi-faceted.

One explanation for the contradiction could be that studies focused on teacher quality in a broad sense, collecting data on elements, such as teachers' level of education and years of experience (Coleman et al., 1966; Nilsen & Gustafsson, 2016) instead of narrowing in on specific instructional moves made with students. Results of this study suggested a necessary defining feature of a teacher quality included a teacher's ability to positively impact student academic self-concept in mathematics, which could be explored in future research.

Teacher Beliefs and Teacher Instructional Practices

There was no relationship between teacher beliefs and the style of instruction used by the teachers, according to hypothesis five. Results suggested that teachers' instructional practices are out of alignment with their beliefs. Results showed that teachers valued student-centered practices, such as problem-solving, collaboration, reasoning, and discourse; practices at the center of a constructivist classroom. Classroom observations, however, revealed teacher-centered practices being used in the majority of classrooms. Students were given little time to problem-solve. Emphasis was placed on the correct answer instead of valuing the process a student used to determine the answer. Collaboration among students was minimal, and the teachers generally did the majority of the talking during observed sessions.

Interestingly, for the teachers in the study, a positive relationship between productive and unproductive beliefs existed. Teachers indicated agreement with statements about constructivist teaching practices and they agreed that more traditional practices, such as memorization were important in the teaching of mathematics. Although NCTM identified some statements as productive and others as unproductive in relation to effective mathematics instruction, the teachers in the study did not view the statements as mutually exclusive. As agreement with unproductive statements increased, agreement with productive statements followed suit. The inverse was also true. This may be related to a teacher's level of understanding a constructivist approach to teaching. Teachers may aspire to be more student centered, but lack deep knowledge regarding the 'how' of designing instruction that reflects constructivist beliefs. In addition, teachers may not have realized that their practice was out of alignment with their beliefs. For these reasons, a study focusing on initial alignment of teachers' beliefs and practices and the impact of professional development to enhance alignment might prove beneficial to teachers in regards to satisfaction with their work, and for students in regards to a positive impact on achievement.

Based on research by Gray and Mannahan (2017), which suggested that after spending time in a course, students attributed success in the course to factors outside of their control, research comparing student-centered classrooms designed with opportunities for reflection and goal setting, to more traditional teacher-centered classrooms may also inform researchers on how to more positively impact student achievement by putting students at the center of the teaching and learning cycle.

Hypothesis three showed no relationship between teacher beliefs about teaching mathematics and student achievement in Algebra I, as measured by teacher-created finals. When considering hypothesis five, the connection may be that the teachers' approaches to instructional design were also reflected in their approaches to assessment development. Because teacher practice appeared to value rote memorization of facts and processes in Algebra, assessments were likely to ask students to utilize the same low-level approaches to thinking by way of the types of questions that were included on the test and what students were asked to do to show mastery of the material.

This misalignment between belief and practice suggests the need for teacher professional development that includes exposure to effective constructivist practices, opportunities to record themselves teaching and reflect on current practice with the additional steps of goal setting and self-monitoring advancement toward those goals.

Recommendations

Systemic Factors

Decisions regarding when students take Algebra, how students are grouped for Algebra, and the type of curriculum provided to Algebra teachers are systemic factors that should be addressed to improve achievement for all.

When deciding between either eighth grade or ninth grade enrollment in Algebra, the research does not provide clear guidance. However, because Algebra is a gate-keeper course, and because research has shown that students who are not placed in Algebra in

8th grade are less likely to take higher level math courses, placement in Algebra in eighth grade is recommended along with curriculum and instructional supports included in this section. Additionally, schools should avoid tracking students by perceived readiness and instead create classrooms inclusive of all readiness levels.

While the historical debate on when students should take Algebra I has been a dichotomous argument focusing on either eighth or ninth grade, in light of analysis of international assessment results and an examination of successful programs, withholding algebraic concepts and focusing on rote memorization of facts and processes in the early grades is an antiquated approach to preparing students for mathematics success. When students should take Algebra may not be the right question, and should instead shift to when do we introduce various algebraic concepts to students starting with the early years of learning math?

Delivering a guaranteed and viable Algebra curriculum isn't enough to ensure all students learn Algebra at high levels and are ready for advanced mathematics courses in high school and beyond. School systems must consider both academic content social emotional content when designing curriculum. For example, Intensified Algebra, which showed success in Florida, included mathematics and literacy content on equal footing with learning about the "malleability of intelligence, metacognition and goal setting, self-efficacy, and productive persistence—through explicit exercises embedded within content lessons as well as through the routines woven throughout the course" (Tidd, Stoelinga, Bush-Richards, De Sena, & Dwyer, 2018, p. 99). Mathematics curriculum should be reviewed through both a lens of rigor and social emotional elements. When students are provided with challenging curriculum built around prioritized standards addressing the big ideas of mathematical thinking the opportunity exists to also build in more time to allow for inquiry, high-level discourse, and problem-solving. These components cannot be left to chance; they must be explicitly stated in the required curriculum. Perhaps most importantly, students should not have to wait until eighth or ninth grade to be exposed to algebraic concepts. In alignment with NCTM recommendations (2014), abstract and algebraic concepts should be embedded throughout mathematics courses at the elementary and early middle school level in order to build problem-solving skills. At the secondary level eliminating what many call the "geometry sandwich" and providing students with integrated math courses which include concepts from typically discreet courses of algebra, geometry, statistics and data science is more in line with curriculum from the nations who do well on international assessments such as PISA (Gonser, 2021; PISA 2018).

Student Factors

The results of this study indicated a significant correlation between students' academic self-concept and grades which is also supported by previous research (Burnett et al., 1999; Liem, G. A., McInerney, D. M., & Yeung, A. S., 2015; Marsh, H., & Martin, A. J. 2011). In light of these findings, it is imperative for teachers to understand the construct of academic self-concept and how it differs from self-esteem. Simply praising students for work and effort is not enough to positively impact academic self-concept leading to higher achievement and mastery of course content. Instead, school administrators can support student achievement by ensuring that teachers understand and apply actionable feedback, use positive responses to students, and design instruction that allows for student questioning, inquiry, problem-solving and relevant connections. Teacher observations should focus on the effective use of these strategies.

To help students build non-cognitive skills such as problem-solving and perseverance, students should be given relevant problems to solve and taught that struggle is part of the learning process. Helping students to develop an understanding of how their brains learn has the potential to mitigate effects of the Big Fish Little Pond Theory as students may look less to their peers to determine their own level of competence, and rely more on their own understanding of how they learn.

Although data in this study indicated no relationship between students' academic self-concept in mathematics and neither teachers' beliefs about teaching math, nor teachers' instructional style, these results may have been the result of a misalignment between teachers' beliefs and their instructional styles, or the minimal data related to the students who received the instruction most aligned with constructivist teaching. Based international testing data and the push for more interactive, student-centered approaches to education, the researcher recommends schools move to a more constructivist approach to instruction as a means for strengthening students' academic self-concept.

Instructional Factors

The teachers in this study indicated they valued the use of diverse strategies for problem-solving and the opportunity for students to investigate mathematical concepts, however, the instruction observed by the researcher included little, if any, opportunities for students to apply reasoning and problem-solving skills. Instead, the instruction was teacher-centered, and followed what can be considered a more traditional approach with the teacher showing students how to solve a problem, giving students time to practice the technique, and then asking questions of students to check for understanding. In China, which consistently ranks high in mathematics (OECD, 2020), successful teachers pointed to the value of time to collaborate and reflect on their instructional practices (Pepin, Xu, Trouche, & Wang, 2017). In light of this, the researcher recommends the study site consider professional development for teachers that emphasizes constructivist approaches to teaching math, and provides teachers with time and practice around how to effectively collaborate.

To improve the academic achievement of students in Algebra, focusing on how teachers teach is imperative.

Teacher Factors

Assuring the quality of teacher-made assessments by helping teachers understand and apply concepts of high-quality assessment design and validity and reliability measures would increase the likelihood of assessments that are aligned with grade level standards which would require assessments to focus on conceptual understanding over rote application of algebraic rules. Additionally, such assessments would allow for teachers to better understand the curriculum and design instruction that supports the kind of thinking required for students to demonstrate mastery.

Just as teachers need to be knowledgeable about assessment design principles, results of this study indicate a need for educators to understand the construct of academic self-concept.

Recommendation for Future Research

In order to make the results of future studies applicable across a wide range of classrooms, and to dig more deeply into the relationships and causes of success, the researcher suggests future studies consider more in-depth research design, as well as additional studies focusing on singular factors to better understand the individual variables.

First, the researcher suggests going beyond a correlational design to using a statistical model that allows for testing of the interdependence of the variables. In addition to the statistical model used, securing a larger sample by adding multiple study sites would providing a more robust statistical foundation on which to base conclusions.

Because the student data were looked at as a collective whole, it is not clear if the results would have been different by looking at student data according to subgroups based on results of the ASDQII. The study was limited by the mortality rate of students, which resulted in a significant drop in the number of students completing the ASDQII post-test.

Further exploration of teacher beliefs might explore how often and in which ways a teacher's belief system is communicated to students both overtly and covertly. In this study, teacher beliefs were self-reported and utilized as a static measure for correlational study. Studying how mathematics teachers use language to convey their beliefs about teaching and learning mathematics may lead to a deeper understanding of how beliefs influence instructional design. Because the lack of a statistically significant relationship between student self-concept in math and teacher beliefs and instructional styles may have been the result of a misalignment between teacher beliefs and practice, a study focused on the alignment of beliefs and practice in mathematics instruction and factors that support alignment may also yield interesting results. It would also serve students well for future studies to explore what influences teachers' beliefs about teaching mathematics and how to change beliefs from unproductive to productive.

Summary

In summary, understanding how to improve student achievement in Algebra will help set the course for students to take more advanced mathematics courses, as well as be prepared to meet the demands required for success in STEM careers. Supporting student success will require eliminating tracking of low-performing students, ensuring that teachers have time for collaboration and reflection on their instruction, along with professional development grounded in constructivist practices. It will also require teachers to have a deep understanding of mathematical content in order to provide supports for those students who have not mastered grade-level content. As shown in this study, it's imperative that teachers also have an understanding of the importance of academic self-concept as it relates to achievement, and that teachers have concrete strategies for building academic self-concept, while at the same time requiring high levels of thinking and perseverance. In this way we can also support alignment between teachers' beliefs and their instructional practices. This study, while initially focused on student success factors, revealed the need for a shift to teacher learning and teacher support; a shift that ultimately serves students by creating an environment where success is more likely to occur.

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