1990

A Workshop for Teachers Using Manipulative Math Materials

Connie Rauch Burbes

A WORKSHOP FOR TEACHERS USING MANIPULATIVE MATH MATERIALS

By

CONNIE RAUCH BURBES

Submitted in partial fulfillment of the requirements

for the Master of Arts in Education degree

Lindenwood College

1990

Accepted by the faculty of the Department of Education, Lindenwood College, in partial fulfillment of the requirements for the Master of Arts in Education degree.

Jane Henderson
Advisor

Virginia Boyle
Reader

DEDICATION

Marianne and Bob Rauch, my parents

Jerry, my husband

Brian, Craig, Carrie and Matt, my children

Cathy Healy, my sister and camerawoman

Peter Rauch, my brother and video editor

Nancy Long, my typist

Dr. Gene Henderson, my advisor

ABSTRACT

The purpose of this paper was to justify the need for teachers to realize the value and accessibility of manipulatives in their classrooms. Principals in the Francis Howell School District felt the need from the comments made from their respective teaching staffs. A flyer sent to all K-2 district teachers describing the purpose of the workshop, substantiated the principals' feelings. A workshop with funding for each participant was set up for July 18. Research of literature and attendance at manipulative workshops by the presenter helped to build the knowledge and techniques that would be used in the presentation. An outline of the math areas to be covered was devised and then many hours were spent in making and preparing the manipulatives thought to be most useful for each workshop concept. Careful thought was given to the arrangement of the room and the ease and accessibility of the materials so that there would be optimum learning time. Surveys completed before the presentation and after, along with an evaluation, proved helpful in assessing the six hour workshop. The presentation netted a 100% response from the participants that using manipulatives is valuable to students and they could stimulate thinking.

TABLE OF CONTENTS

CHAPTER ONE

INTRODUCTION

The purpose of this workshop was to show teachers the importance and value of using manipulatives with their students in math, to whet teachers' instincts, to build their confidence in using manipulatives, and to demonstrate the ease with which everyday items can be gathered and used as manipulatives. It was intended that they carry these manipulative concepts back to their everyday teaching in the classroom. Participants were shown the importance of a hands on approach to developing mathematical concepts and how to build on familiar experiences to extend children's understanding and appreciation of mathematics. Recent evaluations of children's ability in math concepts and their application and research in the way teachers are presenting math lessons have been the topic of numerous articles and the cause of much concern, nationally.

Evidence Of Teacher Need For A Workshop

Carpenter (1988) cited results of the fourth mathematics assessment of the National Assessment of Educational Progress indicating that a greater emphasis needed to be placed on helping students become mathematical problem solvers able to reason and communicate mathematically, as these were "most critically in need of reform" (p. 41). The National Advisory Committee on Mathematics Education indicated in 1975 that extensive and detailed information about classroom practice was an urgent need. Suydam and Higgins (1977) highlighted the fact that "through most of this century, and in particular since the mid-1930's, the importance of meaningful instruction in the learning of mathematics has been espoused and accepted" (p. 61). Weaver and Suydam (1972) suggested that using concrete manipulative teaching materials is an integral part of developing such meaningful mathematics instruction.

Scott (1983) reports a survey taken in a large urban school district to gain information on the current use of concrete, manipulative, teaching materials and to get some feel for teacher in-service desires in the use of mathematics materials. Seventy-five elementary schools in the district received the survey and there were responses from 88% of the schools and over 60% of the teachers.

Over twenty-five materials were listed on the survey. Rulers and flash cards were two of the items included on the list of manipulatives. These two items were used by over 50% of the teachers. While flash cards may not be considered as a "concrete manipulative" the developers and administrators of the survey felt that it was necessary to include a range of materials that teachers could respond to positively on the survey. The steady increase in the percent of teachers who indicated that they used none of the selected materials as the grade level increased, was accompanied by a decrease in total use. The average total use by first grade teachers was significantly higher than each of the grades two through five (at the 0.001 level). The fact that most teachers apparently used few materials might lead one to assume the heavy reliance on textbooks. In fact, 86% reported using textbooks and those teachers who did not use texts did use significantly more materials (at the 0.0001 level). Another statistical procedure used was to look at the correlation between years of teaching experience and total material use. The Pearson correlation was -0.13 (significant at the 0.0009 level) which does seem to indicate that perhaps more recently trained teachers are using more manipulative materials. The percentage of teachers indicating that they would like to have in-service on the use of math materials was 52.5%, with only the fifth grade level teachers being below 50%. It should also be noted that 17.7% of the teachers did not

respond to this item about inservice. It was also found that teachers requesting inservice and more materials did tend to use more materials. Although teachers requesting more materials and in-service had less experience on the average, the differences were not statistically significant.

The Assistant Superintendent in charge of Elementary Education and Special Services in the Francis Howell School District was asked about offering a workshop on manipulative math. There was a unanimous agreement from the principals that there was a need, and plans were set into motion for setting a date. The Assistant Superintendent asked if the workshop could be set in July because funds would be available then to provide substitutes for those teachers who wanted to attend. Initially, the workshop was set up to handle ten participants, but requests continued to come in and nine other spots were added and funded. Finally, plans were made for two additional workshops during the school year, enabling forty more participants to attend, and it seemed likely that more workshops would be organized for the future.

Justification of Workshop Content

Anxieties in other subject contents such as history, social studies and science aren't as commonplace as in math. There is a need to help teachers to stay knowledgeable of math concepts and

skills students need. The widespread concern about achievement in mathematics should lead us to consider the evidence from the Suydam and Higgins (1977) research which stated that:

1. Lessons using manipulative materials have a greater probability of increasing mathematics achievement; and

2. achievement is enhanced across a variety of topics at every grade level (K-8) with the use of manipulatives. (p. 26)

Sirotnik; Goodlad and Klein; and Medley assert that most teachers believe the research about manipulatives, but this belief is not always translated into action. In fact, the research shows that most teachers use a narrow range of practices and that they expand their repertoire only when they are given substantial, carefully designed training.

Math Their Way is a manipulative math program originally developed for pre-school to third graders by Mary Baratta-Lorton. It is based on the idea that through an active involvement with materials, the child will draw out the generalizations within the material. Lorton's first book *Workjobs* was published in 1972 and her ideas continued to grow and in 1976 her book *Math Their Way* was published by the Addison-Wesley book company.

Laura Choate, a second grade teacher in Fallbrook, California, and Lynda Holman, a kindergarten teacher in Westminister, California, travel presenting workshops to continue Mary Baratta-

Lorton's *Math Their Way* manipulative-based math program. At the Learning Center Workshop they advocated that the *Math Their Way* program is consistent with any math curriculum since all math textbooks cover the following content areas. These areas were listed on a chart at their workshop:

1. Numbers
2. Measurement
3. Geometry
4. Patterns and Functions
5. Statistics and Probability
6. Logic
7. Algebra

All of the above categories except Algebra were covered in the workshop. Numerous activities in each were explained, manipulated and discussed so teachers could have a chance to see what a child experiences manipulating materials. Choate shared the following proverb on the overhead projector at the workshop:

The ancient Chinese Proverb says;

I hear,

and I forget.

I see,

and I remember.

I do,

and I understand.

The Socony Mobile Oil Company study (cited by Olney, 1989) reports the following results which support this ancient verse.

How People Learn:	
Taste	1%
Touch	1-1.5%
Smell	3-3.5%
Hearing	11%
Sight	83%

Children Retain:

10% - of what they read
20% - of what they hear
30% - of what they see
50% - of what they see and hear together
70% - of what they say
90% - of what they say and do at the same time

How Long Children Retain

	After 3 hours	After 3 days
Lecture only	70%	10%
Demonstration only	72%	20%
Demonstration and Lecture	85%	65%

This study substantiates the idea that what is said and done at the same time has the highest percentage of retention. Manipulative math builds on this premise.

The Francis Howell School District sponsored an inservice program entitled *Box It or Bag It Mathematics* in 1987 which is a takeoff from Mary Baratta-Lorton's program. The enthusiasm and carry-over that the K-2 teachers brought back was contagious.

CHAPTER TWO

REVIEW OF LITERATURE

Historical Perspective

Close scrutiny of the elementary math curriculum has been an ongoing process since the 1960's. The '60's decade saw the period of "new math". The 70's students learned their multiplication tables but could not apply math concepts to solve problems. Fennel (1981) says the growing uneasiness regarding the effectiveness of U.S. schools because of low standardized test scores and stronger demands for accountability in the early to mid 1970's caused schools to become the target of the "back-to-basics" movement. This resulted in changes in elementary math content and instructions technique. This decline in student achievement in the 1970's appears to have taken a slight upturn in the 1980's. However, this trend is little cause for celebration, as achievement of students at all levels still shows major deficiencies.

Carpenter (1988) stated:

> The most critical question is not whether students' performance has changed over time but whether they are learning what they should be learning. The results of the fourth NAEP mathematics assessment indicate that although most students are successfully learning a number of mathematical skills, they exhibit serious gaps in their knowledge and are learning a number of concepts and skills at a superficial level. (p. 40)

Carpenter concludes that "students apparently have not learned many of the more advanced skills, and they frequently do not apply the skills they have learned" (p. 41). Good and Grouws (1987) stated that "For a long time, mathematicians used mathematics as a way of representing true statements about nature. Mathematics was a way of knowing, a means of achieving certainty that relied on deductive proof from self-evident principles" (p. 778). Today most of the populace, including teachers still believe that "mathematicians produce irrefutable conclusions" (p. 778). In our world today there is no longer a claim by mathematicians to produce exact models, but instead most mathematics involves finding ways to approach problems. Yet many teachers still view math as the production of correct answers. Thus the prevalence of arithmetic drill and computation in the classroom with little attention to problem solving and estimation continues. The curriculum reforms proposed by the National Council of Teachers of Mathematics (1987) calls for "a reorientation of the school mathematics curriculum so that a greater emphasis is placed on helping students become mathematical problem solvers and be able to communicate and reason mathematically" (p. 41).

Good and Grouws (1987) discuss the fact that the mathematics used today is not the same used or needed a century ago because of calculators, yet observations in classrooms during math lessons

indicates that students still spend a great deal of their time on computational exercises. It is most likely that complex computations will be performed on calculators in real life experiences, but the importance of estimation skills is now needed to develop the believability of answers reached on calculators. Good and Grouws (1987) stated that, "Unfortunately, estimation was one of the areas in which students performed poorly on the NAEP assessment" (p. 778).

Good and Grouws (1987) also cite: "a number of studies that reveal critical weaknesses in the way teachers teach mathematics. Many of these problems are the result of teachers misunderstandings of basic mathematical concepts and their misconceptions about which skills deserve to be emphasized" (p. 778). Most math classes tend to be "text-book driven" and again with a strong emphasis on drill. Good, Grouws, and Ebmeier (1983) reported from their research that:

> Most development lessons focused on memorization rather than on understanding and visualizing concepts or on making generalizations. References to everyday situations that students could understand were infrequent and tended to be relatively trivial and uninteresting. Furthermore, teachers gave too much attention to procedural detail and too little to understanding mathematical concepts. There was little instruction in problem solving and estimation. Students were not encouraged to question, experiment, explore or suggest explanations. (p. 779)

One of the key topics addressed by the National Council of Teachers of Mathematics (NCTM) was inservice training. Good and Brophy (1987) found that the instruction/management strand needs to be based on teaching effectively with emphasis spent on the development and meaning of mathematical ideas, because "it is in the development portion of the lesson that students come to understand mathematical ideas and their value in real-world decision making" (p. 780). The widespread concern about achievement should also lead us to consider the evidence from research of Suydam and Higgins (1977) which indicates that, "1. Lessons using manipulative materials have a greater probability of increasing mathematics achievement; and 2. Achievement is enhanced across a variety of topics at every grade level (K-8) with the use of manipulatives" (p. 26). In research about manipulatives, most teachers indicate that they believe the value of a hands on approach yet Sirotnik (1983) states in his research "that most teachers use a narrow range of practices and that they expand their repertoire only when they are given substantial, carefully designed training" (p. 17).

Scott (1983) found similar results in his survey. A survey was made to gain information on the current use of concrete manipulative teaching materials in a large, urban school district. A list of twenty-five materials was compiled, consisting of a variety of items such as rulers, protractors, calculators, unifex cubes,

Cuisenaire Rods and small toy figures. Not all of the materials listed might be considered "concrete manipulatives" for example flash cards were also listed, but the developers and administrators of the survey felt that it was important to put in a wide range of materials so that more teachers could respond positively. The survey was also designed to get a feel for teacher inservice desires in the use of math materials. Copies of the survey were sent to area directors of elementary education and they were distributed to the 75 elementary schools in the district. There were responses from 88% of the schools and over 60% of the teachers. The results showed that only rulers and flash cards were used by over 50% of the teachers. Only 10 of the materials were reported used by at least one-third of all the teachers. Cuisenaire rods (46.4%), geoboards (46.6%) and sticks (42.2%) were the only materials regularly thought of as manipulative materials that received fairly widespread use.

Fennema (1981) has suggested that "while primary mathematics programs encourage the use of concrete and pictorial representations of mathematical ideas, by the time children are ten or eleven years old, symbolic representations are used almost exclusively" (p. 62). An analysis of the use of math materials by grade level seems to verify Fennema's conclusion. Seven of the manipulative materials; Cuisenaire rods, geoboards, popsicle sticks, abaci, bean sticks, base ten blocks and small toy figures which offer

children concrete experiences with math concepts, showed a steady decline in use in the survey. Whereas most measurement materials showed fairly equal use at all grade levels. Only compasses, protractors and calculators were used more in the upper grades. The survey showed a steady increase in the percent of teachers who indicated that they used none of the selected materials as the grade level increased, accompanied by a decrease in the total use. Average total use by first grade teachers was significantly higher than each of grades two through five (at the 0.0001 level). The fact that most teachers apparently used few materials might lead one to assume that there was a heavy reliance on textbooks. In fact, 86% reported using textbooks and those teachers who did not use texts did use significantly more materials (at the 0.0001 level).

The percentage of teachers indicating that they would like to have in-service on the use of math materials was 52.5%, with only the fifth grade level teachers being below 50%. It should also be noted that 17.7% of the teachers did not respond to this item about inservice on the survey. It was also found that teachers requesting inservice and more materials did tend to use more materials. Although teachers requesting more materials and in-service had less experience on the average, the differences were not statistically significant.

Conclusions from Scott's 1983 survey:

1. In general, most teachers use few materials other than textbooks in mathematics instruction.
2. Use of most materials tends to decrease as grade level increases.
3. Calculators are rarely used but use does tend to increase with grade level.
4. Most teachers used textbooks and those who do not use textbooks use significantly more manipulative materials.
5. There is a tendency for teachers with more recent training to use more materials.
6. Most teachers reported that they did not want more materials. (p. 67)

Current Trends and Concerns

The National Assessment of Educational Progress (NAEP) was established in 1969 to provide information on the educational performance of the United States youth and to measure changes in performance over time. Since its inception, the NAEP has gathered information about the performance of 9-year-olds, 13-year-olds, and 17-year-old students in mathematics, writing reading, science, social studies and other disciplines. The first three assessments in mathematics were conducted in 1973, 1978, 1982 and the fourth in 1987. Good and Grouws (1987) refer to the most recent results of the NAEP "showing that students demonstrate a high level of mastery of

computational skills, especially those involving whole numbers, but that in all other areas, however, many students are not learning some of the most basic mathematical concepts and skills" (p. 778). In efforts to study weaknesses and strengths of children's learning so they could increase teachers' understanding through inservice training, they found, "U.S. schools have taught only a small percentage of students how to analyze mathematical problems or apply mathematics to nonroutine situations. And, the majority of students perform especially poorly in the areas of problem solving and estimation" (p. 778). In their numerous studies, Good and Grouws revealed five critical weaknesses in the way teachers teach mathematics (p. 778):

1. *Lack of development.* Based on empirical evidence, Good and Grouws cite that 50% of class time should be spend on the development of the lesson. Yet in an 1987 observational study of 37 fourth-, fifth-, and sixth-grade classrooms in nine schools, only 14% of the class period was devoted to development or time spent introducing and explaining math concepts.

2. *Emphasis on procedural detail.* Schoenfeld (1987) "found that students in the class came to believe that the form of a mathematical answer was most important. In some class sessions, more time was spent discussing format than discussing the 'result's being proved" (p.779).

3. *Infrequent instruction in problem solving.* Burns and Lash (1984) research observed teachers teaching problem solving for one school week. They concluded that the problem-solving instruction was similar to regular instruction because the teachers had "a characteristic view

of mathematics instruction as oriented toward drill and review" (p.779).

Bourke (1984) found that higher-order questions and applications were extremely rare in these classrooms.

4. *Too little attention to estimation.* The increases in technological changes will necessitate the importance of estimation skills.

5. *Importance of understanding mathematics.* Linn (1986) provided evidence "that precollege work in mathematics and science fails to integrate low-level skills and high-level understanding, so that much of the material that is studied is neither remembered nor understood" (p.780).

Grouws (1987) states, "that how new knowledge is stored and related to previously acquired knowledge will determine how students use new knowledge to solve a problem" (p. 295).

Bruni (1982) suggests six very practical ways a primary-grade teacher can help promote the development of problem-solving skills:

1. Ask more open-ended questions.
2. Encourage the discovery of patterns.
3. Help children develop spatial imagery and spatial thinking.
4. Focus on mathematical language development.
5. Help children solve real-life problems.
6. Observe and reward the different ways children solve problems. (p. 10)

By the time children begin kindergarten, they have counted and sorted objects. They bring with them the natural curiosity that is essential for problem solving. The mathematics program that

children encounter in school must build on familiar experiences to extend the children's understanding and appreciation of mathematics.

Mary Baratta-Lorton's *Math Their Way* builds on the levels of abstractions from the concept, to connecting to the symbolic. (See Appendix F) The training this program emulates and the way the concepts are taught helps teachers to get away from drill and practice and develops skills that will be pertinent to all types of problem solving situations. Laura Choate, a second grade teacher in Fallbrook, California, and Lynda Holman, a kindergarten teacher in Westminister, California presented workshops to continue Mary Baratta-Lorton's *Math Their Way* manipulative-based math program. They advocated that the *Math Their Way* program is consistent with any math curriculum as the framework for all math textbooks cover the following content areas:

1. Numbers
2. Measurement
3. Geometry
4. Patterns & Functions
5. Statistics & Probability
6. Logic
7. Algebra

The National Council of Teachers of Mathematics goals for K-12 students are:

1. Become a mathematical problem solver.
2. Learn to communicate mathematically.

3. Learn to reason mathematically.
4. Learn to value mathematics.
5. Become confident in one's own ability.

These N.C.T.M. goals cited by Choate and Holman in the August 1988 workshop are also a basis of the *Math Their Way* program.

Fennel (1981) points out that Piaget's stages of cognitive development have been influential in popularizing the use of manipulative materials for developing mathematical concepts, as today's math programs include the use of manipulatives for concept development at the pre-operational (ages 2-7) and concrete operational stages (ages 7-11). Fennel also reported that the British Nuffield Project, a prototype of open education in the U.S. was one of the first to emphasize Piaget's ideas in the form of activity-centered mathematics laboratories. They were highly criticized as being purely "fun and games", lacked structure and that no consistent idea of what to expect from the students could be foreseen. Lewis (1985) suggested ways that teachers can avoid these gaps:

> First insure enough time with manipulatives to learn concepts at the concrete level. Teachers must also make a concerted effort to emphasize connecting representational level activities that clarify the connection between physical situations and mathematical computations. Teachers also, must return to the concrete level to introduce new concepts as they come up through the course of instruction. (p. 372)

Regarding "fun and games" aspect, after the free exploration with any new manipulative has taken place, children will usually be

discussing their ideas and problems with one another, which can also be very exciting!

Leonard M. Kennedy (1986) stated that:

> learning theories and evidence from research and classroom practice support the use of manipulative materials to help children learn and understand mathematics. Well-chosen and properly used manipulative materials enhance children's learning, generate interest, relieve boredom and promote problem-solving and computation skills. (p. 6)

Math Their Way and the *Box It or Bag It Mathematics* programs stress the value and importance of developing a clear understanding of mathematical concepts using manipulatives. Inservice training of these programs encourages teachers to participate in hands on learning, so they too can experience the situations their students would be exploring. While exploring in small groups, children exchange ideas on possible ways to solve problems, thus developing not only skills in mathematical problem solving, but also language skills in communicating with peers. These two manipulative math programs may not be the cure-all for our nation's math progress, but they could be a step in the right direction.

Methods of an Effective Presenter

In planning a workshop, numerous items and conditions must be taken into account to enable optimum effectiveness for the

participants. Garmston (1987) suggests the following list of guidelines for presenter:

1. Introduce self at the beginning of the meeting, define your goal.
2. Let the group know that you are there to serve them, and that you will check with them -- from time to time -- to see if you are pushing them too much, not enough, etc.
3. Do not talk too much.
4. Be an energizer. Use your body, your voice, your positive comments.
5. Set a positive tone. Smile. Focus on what can be done.
6. Use your good memory.
7. Accept incomplete ideas.
8. Protect the group from one person's domination. Protect individuals from attack.
9. Avoid constant repetition and prolonger comments.
10. Avoid interpersonal confrontation.
11. Respect silence, but give door openers.
12. Be keenly aware of the meeting's "space language". Arrange furniture before the meeting.
13. Do not get involved with the content.
14. Do not be defensive.
15. Educate the group.

Every effort was made to follow the guidelines that Garmston suggested.

CHAPTER THREE

WORKSHOP DESIGN

Theme of the Workshop

The theme of the workshop revolved around using manipulatives in math. Since this was the issue participants were led to experience what a child would feel by actually handling materials to solve a variety of problems. A range of two to three activities were presented in each of the following conceptual levels in math: (a) patterning, (b) measuring, (c) graphing, (d) place value, (e) estimating, (f) addition, (g) subtraction, (h) multiplication and (i) division.

All activities included "hands on" experience.

The workshop was designed to help teachers see the ease with which manipulatives could be accumulated, stored, used and evaluated. Ideas were also discussed on how to educate parents about the way their child was learning from such materials. A notebook with copies of all the activity worksheets, an index for the *Math Their Way* textbook, descriptions of various activities, several types of evaluation forms and a resource list of people and companies that handle various products were provided for each of the nineteen participants. Each participant was encouraged to realize the importance of manipulating materials to solve problems so they would see the enthusiasm and excitement that naturally flows from

handling such materials, the cooperation and diversity in solving problems, and the language skills that develop in order to express one's ideas and solutions to their peers. They will have had a better chance at remembering what they learned, because they not only heard it, saw it, but did it as well.

Justification

Since the way mathematics is used in the world today is much different than it was a century ago, teachers must adjust their curriculum and presentation to meet the current needs. Inexpensive calculators can accurately and quickly do long calculations, yet teachers continue paper-pencil drills on such problems. Good and Grouws (1987) cite classroom observations "that students spend a great deal of time working on just such exercises" (p. 778). Instead, teachers should be providing experiences that help students to analyze mathematical problems and apply mathematics to nonroutine situations, especially in the areas of problem solving and estimating.

The results of the fourth mathematical assessment of the National Assessment of Educational Progress (NAEP) of 1987, evoked continued concern regarding problem solving skills. Problem solving was also a concern in the previous assessments of 1973, 1978 and 1982. The National Council of Supervisors of Mathematics

suggested reforms in 1978 and the National Council of Teachers of Mathematics asked for reforms in 1980. In 1987 they called for "a reorientation of the school mathematics curriculum so that a greater emphasis is placed on helping students become mathematical problem solvers and be able to communicate and reason mathematically" (p.41).

The workshop was intended to support these groups by educating teachers about the need to develop problem solving skills and the variety of ways this could be implemented into the curriculum without a major expenditure of new text books.

Description of the Workshop

Focus and coverage

The focus of the workshop was on helping teachers of the Francis Howell School District feel comfortable using manipulatives with their students. A flyer (See Appendix A) was developed to entice participants to want to come to a hands-on workshop. Fliers were distributed to all K-2 teachers in the seven elementary schools in the Francis Howell School district during the last week of May 1989. Every effort was made to allow enough time for off-cycle teachers to have a chance to respond as well, because fliers were distributed just prior to one cycle leaving and another returning. Information on the flyer was devised as to reach teachers either at their schools or at

their homes. Information was needed in order to plan an appropriate agenda, therefore teachers were asked if they had previous exposure to these manipulative math programs.

The class was originally set up for 10 participants, but nine other individuals expressed an interest, so the class was enlarged to handle the other participants and funding was made available as well. Each participant received a confirmation letter (see Appendix B) to verify their attendance and to inform them of lunch arrangements and the type of activities they would be participating in so they could dress appropriately. The workshop was designed for a one day from 8:00 to 3:00, with informal breaks and a lunch period, because substitutes were needed to release some teachers. The subject matter of working with manipulatives and the amount of time needed to set them up was also more conducive to a one day meeting than several sessions.

Overall Objectives

1. Teachers will experience through simulated experiences how it feels to manipulate materials.

2. Teachers will experience different approaches to solving one problem.

3. Teachers will verbalize to their peers ways to solve a problem.

4. Teachers will discover a variety of materials that can be used as manipulatives.

5. Teachers will cooperate in working together to solve problems.

6. Teachers will experience the joy and excitement using their hands to solve a problem.

Approaches used and rationale.

After greetings and introductions of all the participants and the presenter were made, the agenda of concepts to be covered and a time frame were explained. Each participant knew from the flyer and the confirmation letter they received that the workshop would be a hands-on approach and that they would be actively involved in the activities. The classroom where the workshop took place was arranged so there could be ease of movement around the room and the desks were arranged to allow for as much table space as possible with a supply desk for manipulatives. Refreshments and drinks were available at a central spot so participants would have easy access to them throughout the day. A wide repertoire of materials and methods of presentations were utilized to make the experience interesting and stimulating. Charts, wall displays with provoking questions, the overhead using both transparencies and overhead pattern pieces to demonstrate various combinations, brief lectures, demonstrations and always follow-up activities by the participants for each of these methods were presented. A wide variety of methods and materials were used. Nuts, bolts, keys, shell macaroni, bread tabs, buttons, beads, lima beans, pattern blocks, unifex cubes,

geoboards, pieces of yarn, mirrors and milk cartons were used to stress various skills.

The presenter encouraged discussion on the activities so the participants could see the flexibility available with this method and to help teachers see a variety of solutions to problems. Math doesn't always have a concrete answer. The openness of the workshop allowed for questions, concerns and doubts to be aired as well as encouragement from participants who might have used similar techniques with their students. The workshop closed with ways to evaluate and record a child's progress. Each participant was asked to fill out the same survey form they had completed before the workshop to see if their viewpoints had changed and each was asked to fill out an evaluation form which allowed for written comments and suggestions.

The Morning Session

Welcome and introductions.

The Assistant Superintendent of Elementary Education and Special Services in the Francis Howell School District introduced the workshop and the presenter. The presenter then asked each participant to introduce themselves and tell what grade they taught and where.

Survey

Each participant was asked to fill out a survey about their viewpoints. (See Appendix C)

The Mathe Teakst Buk

The participants were asked to look at the first pages of their notebook and to answer the questions they posed. (See Appendix E) After several minutes and no written responses, the presenter placed a similar symbol line on the board and assigned numbers to the various symbols and asked the participants if they could solve the problems now. They were able to do so. The leader tied this experience into the feelings that a child may have when asked to solve problems in math. The child may not be ready for the symbolic level.

Levels of Abstractions

Participants were then asked to focus on the next page of the notebook and a description of each of the levels of abstraction were discussed. Time was allowed for each to read the small excerpt on visualization. (See Appendix F)

Overhead transparency of the Chinese Proverb

The presenter reviewed the ancient Chinese Proverb stressing the importance for teachers to use as many manipulatives in their lessons as possible.

Morning Opening

The following activities were explained and practiced so that the participants would have a selection of ideas that they might want to chose from to start their school day.

A. Calendars

Using a large calendar with two different pattern pieces to represent the days, the participants were asked if they could recognize a design or pattern that the pieces formed. Then, using a clap motion for the rectangular piece and a snap of the fingers for the star piece, the leader had the participants use these motions as she pointed to each piece of the calendar.

A variety of questions could be posed with a complete sentence response expected, such as what is today? What was last month? What will next month be? Is today an odd or even day?

B. How Many Days in the School Year?

This estimation question can last all year. On the first day of school supply a chart and ask the above question, let the children place their estimations on the chart and keep it posted somewhere in the room.

Using adding machine tape posted around the room, each day write the number of the day on it. Depending

on the grade level, choose a couple of numbers to look for their multiples throughout the year. For example, circle in green every third day and use an orange triangle around every fourth day. Again, look for patterns that develop from the daily markings.

A special celebration could take place on the 100th day of school. Do physical activities like jumping for 100 seconds or count out 100 cookies. The activities are endless.

C. Weather Chart and Temperature Graph

The leader suggested ways that the participants could have their students record daily the type of weather and the temperature. These charts and graphs could be saved and used to compare the change in temperatures and weather throughout the year. Reading a thermometer, recording the information and interpreting a line or bar graph are useful lifetime skills. (See Appendix G and H)

D. Tooth Chart

Participants saw how they could take advantage of the experience of their students loosing a tooth. By running twelve copies of the tooth pattern and labeling each one with a month of the year, the teacher then places

pictures of the students who lost teeth during that month on the tooth. Comparisons can be made as to which month the most teeth were lost and to which month the least amount of teeth were lost. (See Appendix I)

E. Measuring Through the Week Ideas

The leader shared the worksheet with a variety of other ideas on how to organize measuring concepts for throughout the week. (See Appendix J)

Ways to Collect Manipulatives

Everyday household items that are normally discarded are ideal for manipulatives in math. Recruiting students' help makes them feel apart of the program and gives them a feeling of ownership. Using a large sheet of paper make a graph with pictures of each child placed vertically and and display the actual item you wish to collect horizontally. Place an "X" in the category as students bring in the various items. The chart can also be used to discuss the results.

Pattern Activities

A. Using the overhead projector and overhead pattern pieces, the leader made a design. Each participant was then asked to make a design or picture with the wooden pattern blocks at their desks. Time was then allowed for each participant to visit the other desks and view other

possibilities. Participants were shown how they could have their students take their patterned designs to the connecting level using a template to draw their pattern on paper. Blackline masters are also available to run the patterned pieces on the various colors of construction paper. These paper-patterned-shapes can be glued down to also represent the designs the children can conceive.

B. The next activity involved taking one wooden pattern block and trying to enlarge it using only the blocks that were identical to its shape. All shapes were conducive to the procedure except for the hexagon. Participants volunteered to show their solutions on the overhead for all to see.

C. Using hinged mirrors and placing them around a design, the participants were asked if they could enlarge the pattern they viewed in the mirror.

Measuring Activities

A. Each participant was asked to cut off a piece of string from a spool that they thought would fit around their head. Then they were allowed to actually place it around their heads to see how close they were to their guess. On the wall was a chart and each participant placed it under the correct category for their guess. Categories

included too short, just right, and too big. Two way tape held the top of the strings and the results were discussed as to how accurate were the groups estimates.

B. Two pieces of construction paper, both the same size were rolled to form cylinders. One was rolled length wise, the other widthwise and were labeled "A" and "B". The participants were asked to see if they thought both would hold the same amount of rice or would one cylinder hold more than the other. Using a gallon milk container that was cut in half and turned up-side-down as a funnel to pour the rice through, one cylinder was filled to the top. Then the other cylinder was slipped over the top of the filled one and then pulled out releasing the rice into the new cylinder. It was observed that there was still room for more rice.

C. The leader then took a jar and asked participants how many scoops would it take to fill the jar. The scoop and jar were displayed and a decision was agreed upon before hand as to where the "top" would be on the jar. Participants were asked to count along as each scoop was placed in the jar. After two scoops were inside, the presenter placed a rubber band around the jar to mark the level and asked the participants if they wanted to

adjust their first estimate as to how many scoops? Another two scoops were placed in the jar and another rubber band was placed at this level, and again participants were asked if they wanted to adjust their estimates. This procedure continued until the jar was filled.

Graphing Activities

A. Each participant was given a portion cup of trail mix, a graph (see Appendix K) and asked to fill it in. By placing the pieces on the graph, participants were experiencing the conceptual level. When they counted each one, they moved to the connecting level and when they filled in the number for each item and wrote equations for the various combinations, they experienced the symbolic level.

B. Using a table cloth that was marked off in a grid, participants could see how real objects, such as shoes and fruit could be placed on it when it was on the floor. Using the bottoms of cardboard milk cartons and a picture of each student attached, a three-dimentional graph can be formed to complement the floor graph. The cartons can be attached to one another with paper clips. The floor graph might show the types of fruit the

students bring in for a friendship salad at Thanksgiving time. The milk carton graph can be constructed at the end of the fruit graph to show which fruit the students liked best, hence two graphs sharing two different ideas about fruit.

C. The workshop participants were given five lima beans that had been painted red on one side and yellow on the other, along with a lima bean worksheet. (See Appendix L) They were asked to roll the beans and record the results on their worksheets. Gathering in a circle on the floor, participants arranged their results to form a graph. Would we have the same results if we rolled another 10 times? 20? 50? ask the presenter.

Sorting and Classifying Activities

A. Using a variety of different sized bread tabs, the presenter made a pattern on the overhead, and asked if the participants could figure it out. Working with a partner, the participants used the "junk boxes" at their desks to also devise a pattern. They recorded the pattern on a 3 X 5 card and placed it face down. Next they visited the other tables to see if they could solve the patterns, checking the card to see if they were correct.

B. The presenter then called two participants to the front of the room and had them stand to the right and left of her. She then called on another participant to come forward and asked the group if they knew on which side she should stand. Finally, the presenter asked her to stand on the appropriate side, this continued until the group could decide the criteria for the classifying of some people on one side and some on the other. Wearing a watch, pockets on shirts, tie shoes or a bracelet are some of the types of criteria that could be used.

Apples in a Bag Problem

Participants were referred back to their handbooks to the Apples In a Bag Problem. (See Appendix M) After reading and discussing the problem participants decided on which paper bag at their desks they should open first. Time was spent exploring the different possibilities.

Handshake Problem

The presenter posed the handshake problem. (See Appendix N) and the participants acted out the problem as the presenter charted the results on the blackboard. Guesses were made along the way to see if they had discovered the pattern.

The Afternoon Session

Place Value

A. Using a place value board and unifex cubes, the participants modeled the presenter's lead on the overhead. The presenter led the group through regrouping in base three. Instead of referring to the number, a made-up word was used. For children using a name for a group of objects is less confusing than using a number.

B. After the participants regrouped using base three, the process was followed again only this time the participants recorded their results on a place value strip (See Appendix O) marking each step of the counting process. Then the presenter asked the participants to find all the possible combinations of patterns and she drew a loop around each one named, reminding the participants to look not only vertically but horizontally as well.

C. Participants were shown how their students could continue a plus-one game independently, writing the numbers on place value strips and taping them together, to form one continuous strip. These stripes can be rolled on a cardboard roll, attached with a clothespin with the

child's name on it to mark their level and then stored in a shoe box until the next working period.

Estimation Activities

A. Participants were shown a jar filled with lima beans and asked to make a recorded guess (See Appendix P). The suggestions was made to count about how many beans were going down the side of the jar and how many around the bottom of the jar, to aid in making an accurate estimate. After guesses have been made and recorded, the lima beans are then counted out in portion cups in groups of 10. The actual number is then recorded on the check side of the record sheet.

B. The numeral sequence worksheet was distributed (See Appendix Q) and directions were given on how to roll the dice and record the various number combinations and to place a tally mark on the combinations each time they appeared. Would the same results occur if 10 more rolls are made? 20? 50? Participants spent time exploring the various possibilities.

C. Using geoboards, bands and unifex cubes, participants were shown how to place a band around several of the pegs on the board. They then made an estimate as to whether they thought there were more pegs inside or

outside the bands and recorded the results (See Appendix R). The participants then placed one color of unifex cubes on all the pegs on the inside of the band and another color on the pegs outside of the band. The cubes are then counted to check each estimate and recorded. Ideas for expanding this idea included using two geoboards together or using one geoband and encompassing as many pegs as possible.

Addition and Subtraction Activities

A. Each participant was given a story card and unifex cubes or other appropriate manipulate and asked to make a story problem using either addition or subtraction concepts. Time was spent to share some of the various stories.

B. The presenter shared a subtraction flips packet and demonstrated how it could be used to visually see the various combinations of the number eight by flipping cards. The flip cards could be used to drill addition facts as well.

Multiplication and Division activities

A. Participants were asked to name items that come in two's, three's and four's to introduce the concept of multiples.

B. Using a set of jewels that had been cut apart in various groups of two, three and four, the participants could see how they could make a set of three strings with four beads on each string and then count the total number of beads.

C. Using the string of jewels again, participants could separate strings to see the resulting amount.

Other Manipulative Activities

The presenter introduced a half a dozen more activities related to a variety of concepts and the participants rotated from one pod to the next exploring each one.

A. Using a number grid worksheet (See Appendix S), the task cards (See Appendix T), and unifex cubes; place the color cubes on the stated number square to see the pictures that will develop.

B. Using the worksheet (See Appendix U), unifex cubes and the line cards; guess how many unifex cubes will fit around each shape. Then fill in the discovered answer.

C. Using the worksheet (See Appendix U), unifex cubes and various construction paper shapes; estimate how many cubes it will take to cover the shape. Record actual answer when it is discovered.

D. Using various lengths of yarn, worksheet (See Appendix U) and unifex cubes; guess how many cubes-long the string is and record results.

E. Cards that have a variety of patterned designs on them are to be continued using unifex cubes.

F. Cards that have a variety of shapes on them, worksheet (See Appendix U) and lengths of yarn that have been tabbed with alphabet letters are matched together for actual lengths.

Ways to Evaluate

A. The student composite sheet (See Appendix V) was referred to as one possible means to record a teacher's observation of their students as they work in the classroom. Several other suggestions and ways were presented on how to evaluate and record throughout each lesson.

B. The number range card (See Appendix W) is a device to plan the numbers that a child needs to work with at the stations or activities that are set up in the room.

C. Parent Involvement

A set of letters that asked for parent contribution of materials, suggestions on ways to help their child, and the concepts that the teacher would be working on were

presented to the participants. Workshops or demonstrations at open house nights or P.T.A. meetings were suggested to show parents what the teacher was endeavoring to do in math lessons. It was suggested that parents help to make some of the needed materials at an afternoon or evening get-together.

Questions and Answers

A period of time was allotted to answer questions and give suggestions that would be helpful to implement a manipulative math problems.

Favorite Resources

A list of people, books and companies (See Appendix X) was provided for each participant in their notebooks.

Survey and Evaluation Forms

The survey (See Appendix Y) and an evaluation form (See Appendix Z) were distributed to culminate the days events.

CHAPTER FOUR

EVALUATION - OBSERVATION

Nineteen teachers from grade levels K-4 attended the one day workshop. Each completed a survey (See Appendix C) prior to the workshop presentation and each completed a survey (See Appendix V) after the presentation along with an evaluation form (See Appendix W) at the end of the workshop. The results of the surveys are summarized in Table 1. It should be noted that seven out of the nineteen had had a previous workshop or encounters with manipulative math materials. This information was obtained from one of the questions on the teachers' enrollment form. Previous exposure to manipulative materials and teachers' belief in their value could explain why the results of the survey were so high.

The question on what was the most important thing you learned from the presentation netted a 100% response that using manipulatives are valuable to students and they could stimulate thinking. In 13 of the 14 survey questions, the participants all believed more strongly in the statements than they did before the workshop. Item #12 showed some teachers did not feel comfortable with a noisy atmosphere. When asked what suggestions would improve this presentation, 26.3% of the participants suggested include more elbow room at the tables; more workshops to continue teacher support and interest; add a supply charge to make items to

take back to the classroom; and explain further the theory of the concept, connecting and symbolic levels.

Positive comments included the actual "hands on" nature of the workshop, the way the presenter moved around to comment and help, the pacing of the activities, the organization, and the enjoyable and friendly atmosphere.

Observations

As the presenter, I saw the interaction of the teachers at the various tables and the variety of ways they solved problems and discussed them with each other. Teachers shared other ways to present the same idea that had worked for them.

The teachers seemed relaxed and anxious to learn.

Suggestions

1. Future presentations, should provide more table surface per participant to take notes and manipulate materials.

2. Participants should review the three levels of how children learn with an activity showing the concept, connecting and symbolic levels.

3. Teachers need information presented more than once. Following workshops are needed to refresh memories and motivate teachers to renewed efforts.

TABLE 1

Mean Teacher Responses Prior to and After the Workshop

1 = Strongly Agree
2 = Agree
3 = No opinion
4 = Disagree
5 = Strongly disagree

1.	Manipulatives stimulate thinking	1.2	1.0
2.	Manipulatives can be used with all ability levels	1.4	1.1
3.	Learning effectiveness of manipulatives can be evaluated	1.7	1.4
4.	Manipulatives are easy to obtain	2.0	1.6
5.	Manipulatives are inexpensive	2.2	1.7
6.	Manipulatives help develop social interaction skills	1.3	1.1
7.	Manipulative activities can be organized and structured to operate smoothly in the classroom	1.6	1.4
8.	Manipulatives are important in the learning process	1.2	1.0
9.	Manipulative activities encourage language experiences	1.6	1.1
10.	Manipulative activities encourage an exchange of different viewpoints on how to solve problems.	1.4	1.1
11.	Parents understand the importance of manipulatives in the classroom.	2.7	2.3
12.	I enjoy the atmosphere of students talking and using manipulative materials in the classroom.	1.6	1.7
13.	It's important to involve parents in how manipulatives are being used in the classroom	1.5	1.4
14.	Manipulatives encourage scientific methods like predicting, experimenting, comparing and graphing data.	1.3	1.0

Appendix A

Flyer

MANIPULATIVE MATH WORKSHOP

July 18, 1989
Daniel Boone School
8:00 AM - 3:00 PM
Presenter: Connie Burbes

Ideas will be presented from *Math Their Way* and the *Box It Bag It* programs. K-2 teachers are invited to come and learn how to use manipulatives in your math program. Learn how to take children from the concrete stages with manipulatives to the abstract levels in math. Hands on experiences in exploring, developing patterns, measuring, graphing, sorting, classifying, estimating and how to introduce the four basic operations while using a wide variety of inexpensive materials will be presented.

Class enrollment is limited to ten participants and will be filled on a first come basis. A substitute or stipend will be provided if you teach in the Francis Howell School District. Please complete the form below and return to Connie Burbes, 11 New Melle Woods Dr., Wentzville, MO 63385 by June 16. Your participation will be verified.

Name_____School_____

Home Address_____

Home Phone_____School Phone_____

Grade you will teach for the 89/90 school year_____Cycle_____

Have you ever attended a workshop on the *Math Their Way* or the *Box It or Bag It* programs?_____

Appendix B

Confirmation Letter

July 5, 1989

Dear

This is to confirm your attendance for the Manipulative Math Workshop on July 18 from 8:00 to 3:00 at Daniel Boone School. We will be meeting in room 114 which is right across from the office.

Should you need a substitute for that day, please make your own arrangements, and please remember to indicate that this is coming out of Title II funds when you make your call and when you fill out your half sheet.

You may bring your lunch or eat out at one of the nearby restaurants if you wish, during our hour lunch break. Please dress casually as some activities will take place on the floor.

I'm looking forward to our activity filled day. If you have further questions concerning this workshop, please feel free to contact me. See you on the 18th.

Sincerely,

Connie Burbes

Appendix C

Handout #One

SURVEY PRIOR TO WORKSHOP PRESENTATION

Indicate how strongly you agree or disagree with each of the following statements. Use the following scale:

1 = Strongly agree
2 = Agree
3 = No opinion
4 = Disagree
5 = Strongly disagree

1. _____ Manipulatives stimulate thinking.
2. _____ Manipulatives can be used with all ability levels.
3. _____ Learning effectiveness of manipulatives can be evaluated.
4. _____ Manipulatives are easy to obtain.
5. _____ Manipulatives are inexpensive.
6 _____ Manipulatives help develop social interaction skills.
7. _____ Manipulative activities can be organized and structured to operate smoothly in the classroom.
8. _____ Manipulatives are important in the learning process.
9. _____ Manipulative activities encourage language experiences.
10. _____ Manipulative activities encourage an exchange of different viewpoints on how to solve problems.
11. _____ Parents understand the importance of manipulatives in the classroom.
12. _____ I enjoy the atmosphere of students talking and using manipulative materials in the classroom.
13. _____ It's important to involve parents in how manipulatives are being used in the classroom.
14. _____ Manipulatives encourage scientific methods like predicting, experimenting, comparing and graphing data.

Appendix D

Handout #Two

MANIPULATIVE

MATH

WORKSHOP

JULY 18, 1989

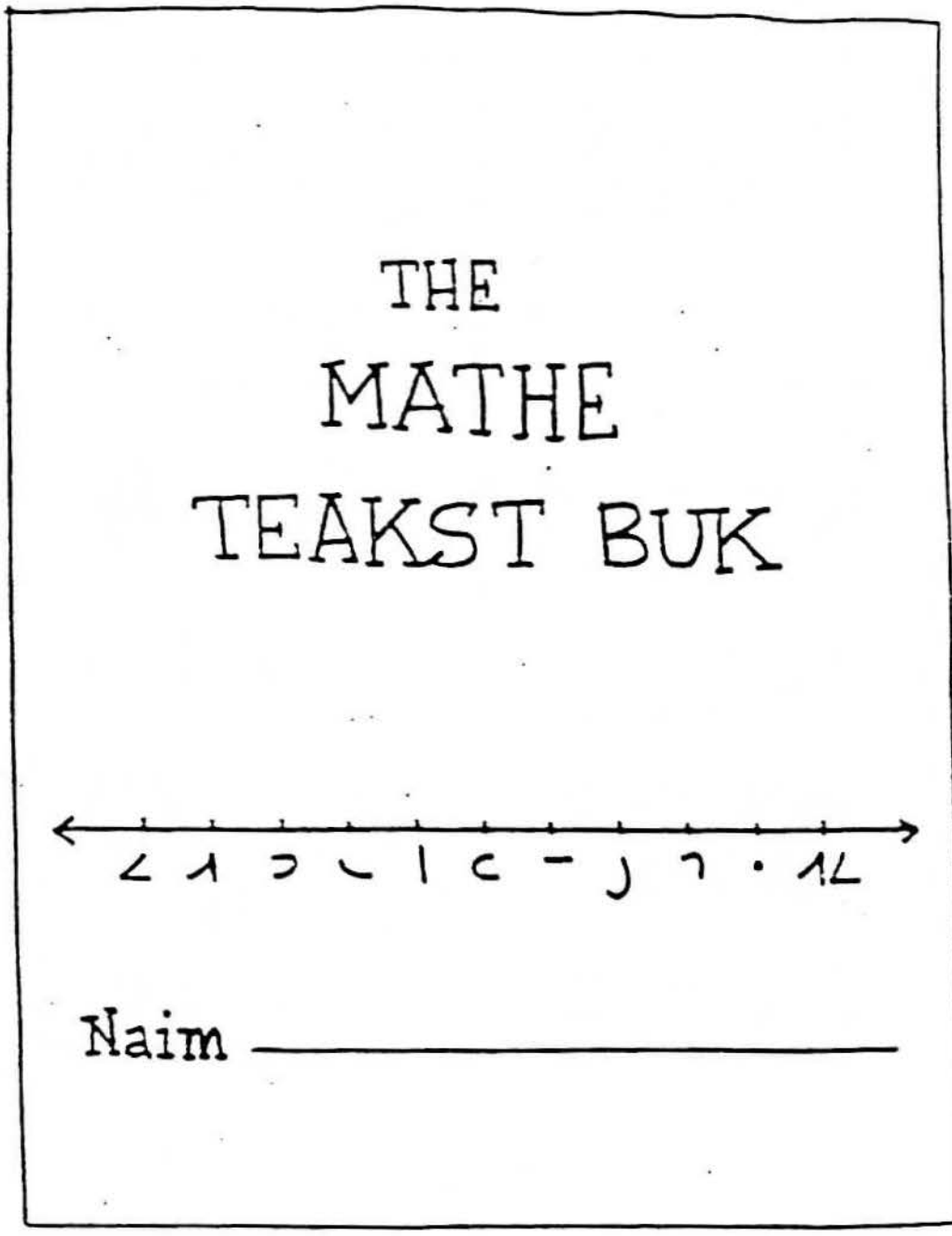
DANIEL BOONE SCHOOL

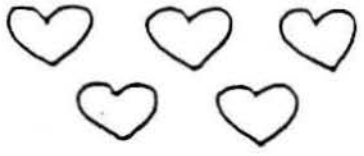
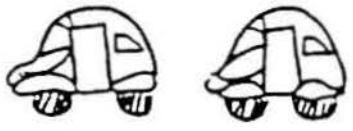
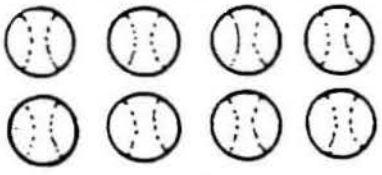
8:00 A.M. - 3:00 P.M.

PRESENTER: CONNIE BURBES

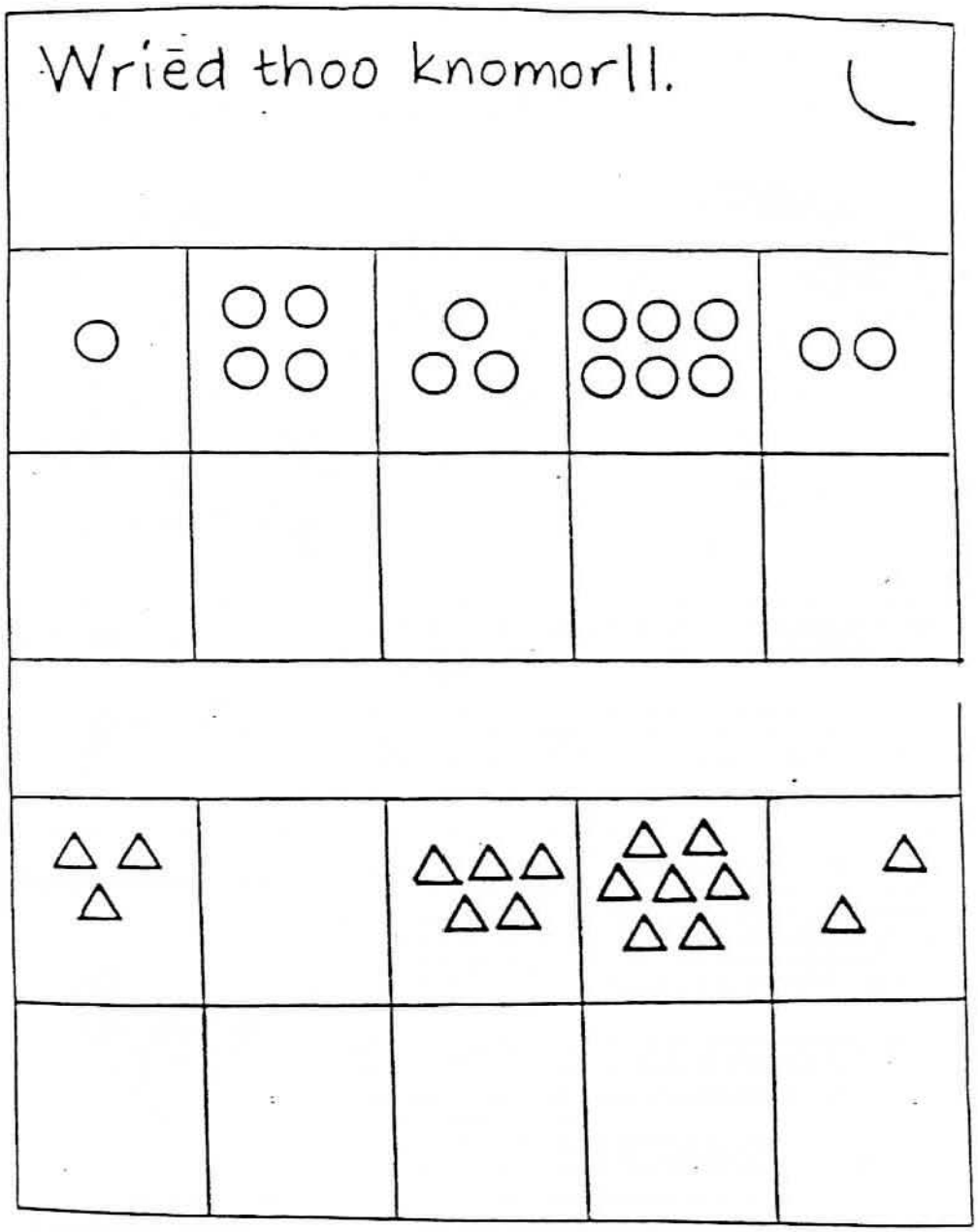
Appendix E

Handout # Three



Whow mene ċ	1
	
	
	

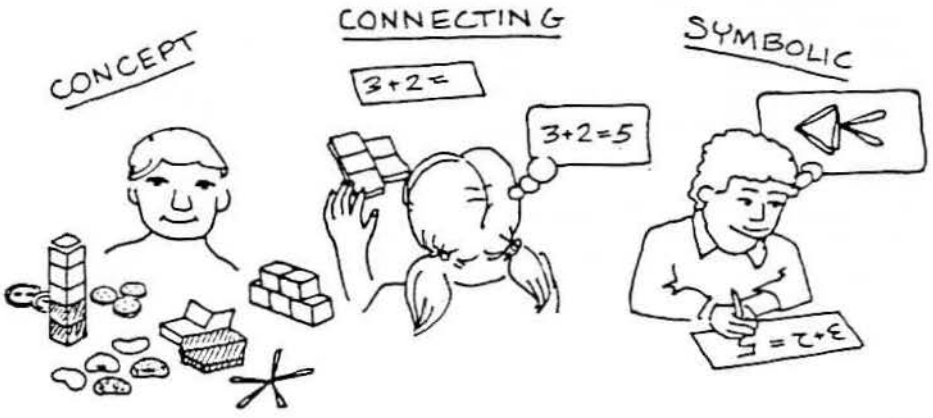
Soe thoo cete.	⊃
⌊	⊃
1	C
⅄	•


Wriēd thoo knomorll.

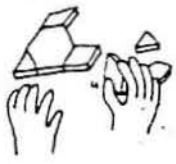
Appendix F

Handout # Four

LEVELS OF ABSTRACTIONS



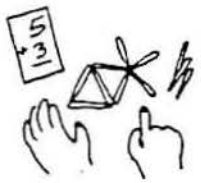
Concept Level



Children develop an understanding of mathematical language as they explore number patterns under ten in context of real events and/or concrete materials.

Children demonstrate their understanding at this level by:
- building concrete models with a variety of manipulatives.
- describing what they have created using mathematical language.

Connecting Level

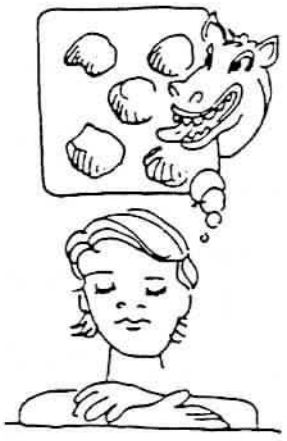


Children usually have not encountered mathematical symbols in the context of their natural environment. Often, their first experiences occur in school-related activities. Numerical and mathematical symbols are introduced at the connecting level. Mathematical symbols (e.g., +, –, =) are even more abstract than the mathematical language they represent. Both vertical and horizontal equations should be experienced.

Children visualize symbols as they solve number problems using manipulatives. They show their understanding at this level by:
- building concrete models with various types of manipulatives to match written equations.
- relating equations to manipulatives or to a word problem they have created. The teacher records the mathematical symbols.

Symbolic Level

At this stage, children record equations to represent concrete number patterns. Eventually they will develop an ability to record equations by visualizing concrete experiences. Children record on their own when they are comfortable writing numerals. This occurs when they have developed the necessary fine motor skills. Many kindergartners are not ready for this level.



Visualization

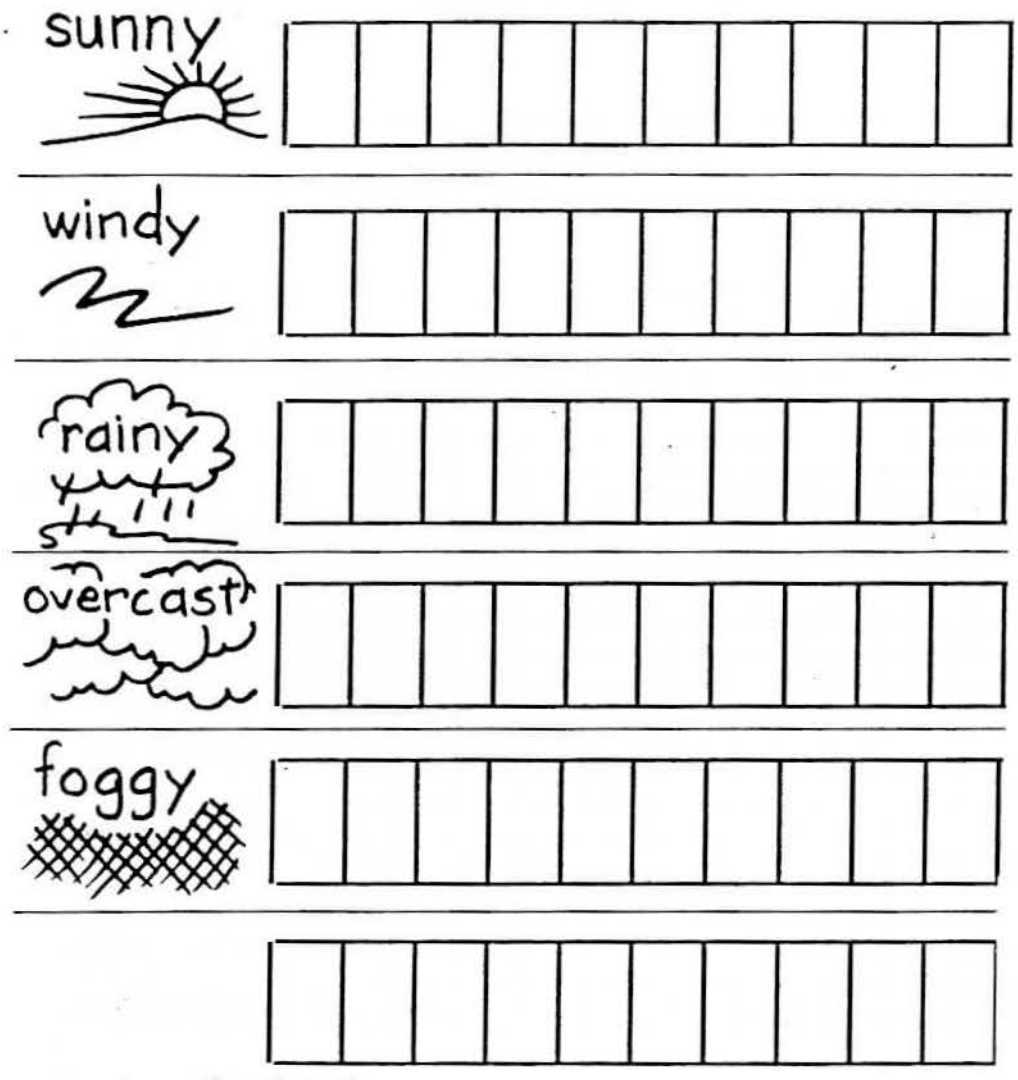
One way children transfer the number concepts learned at each level is through visualization. Once children have internalized an abundance of concrete number experiences, the teacher may tell number stories that encourage children to visualize the process.

A first grade teacher explained how her children visualized word problems and wrote them on their individual chalk boards. She told the following number story: "Close your eyes and think of 5 rocks in the domino pattern. Put them right into your head so you can see them. Pretend that the Big Red Rockeater came and stole one rock out of the center of the pattern. Now write a story of what happened on your blackboard."

The next day, the mother of one of her students came to her and said, "You'll never believe what my daughter said last night. She asked me what (5 – 1) was and I said 4! Then she said, "Mom, do you have rocks in your head?"

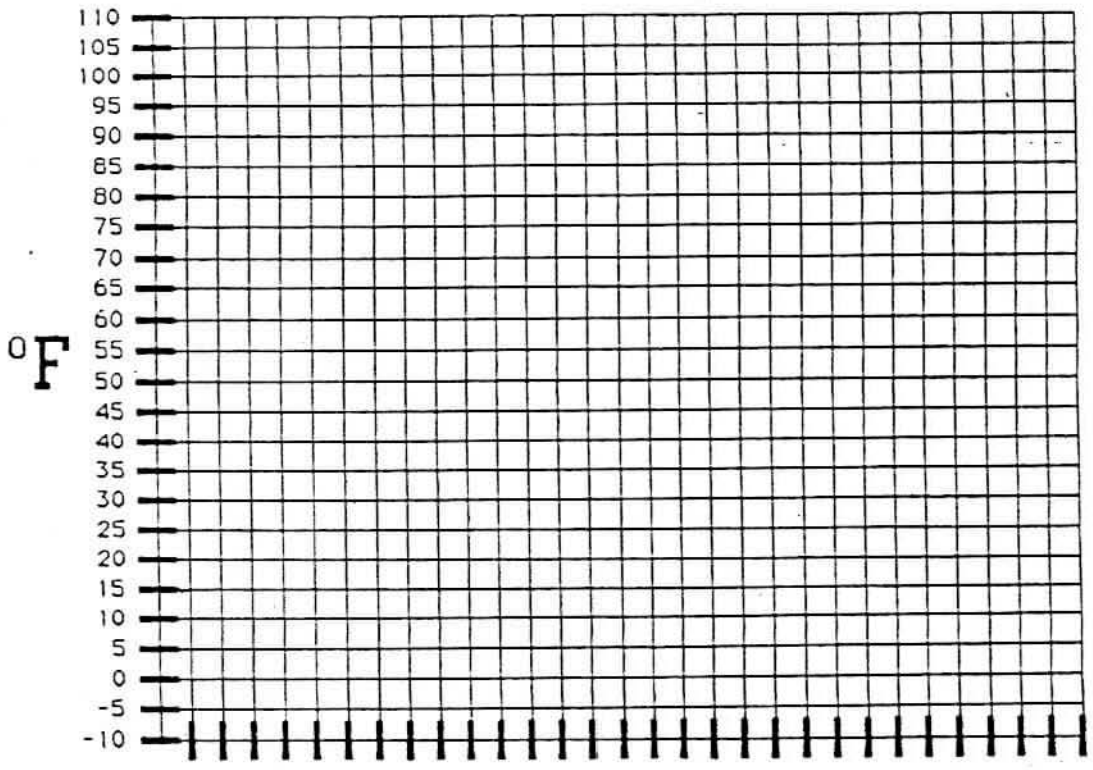
Appendix G

Handout # Five



Appendix H

Handout # Six

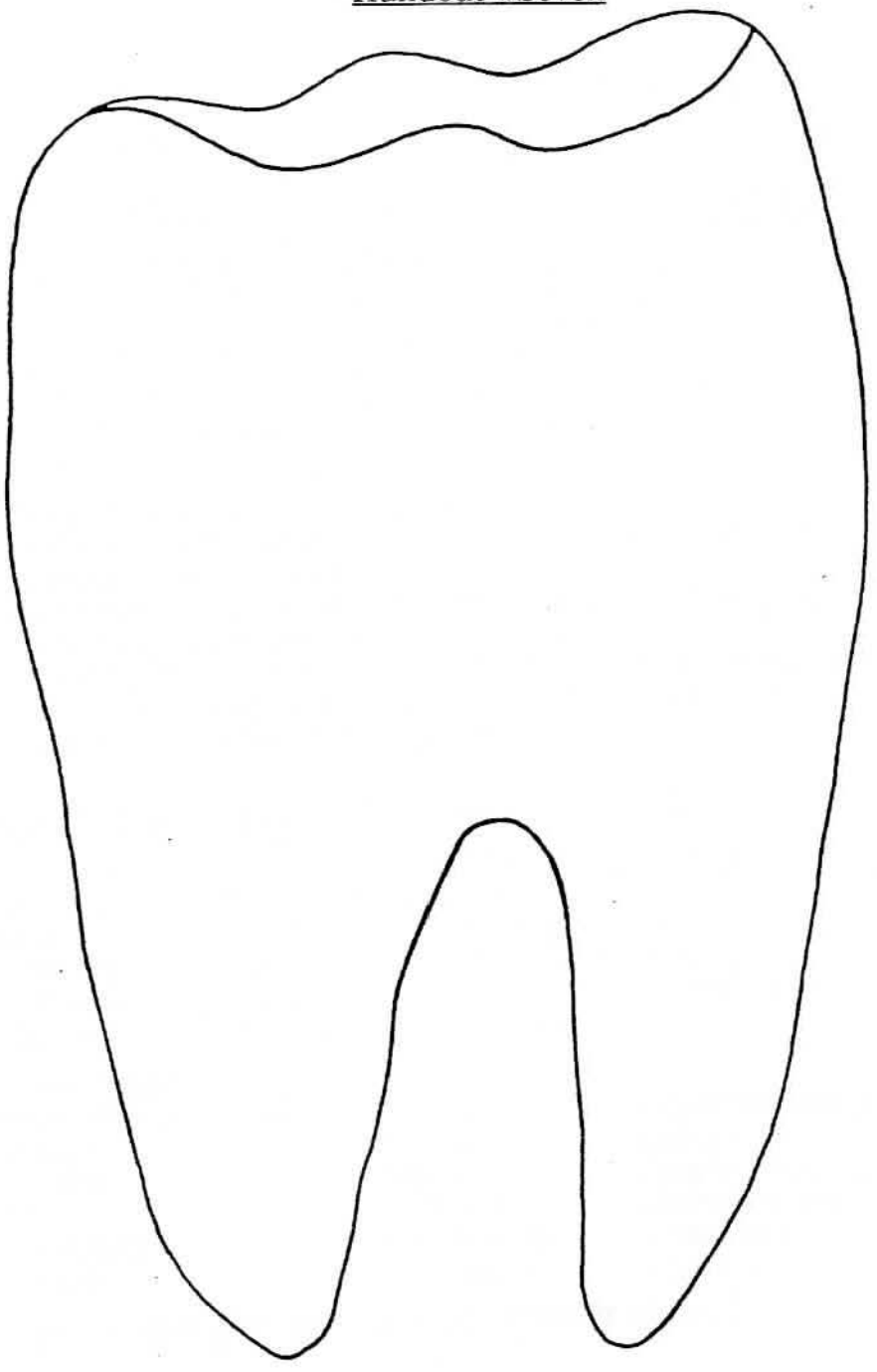


DATE

Graph by Paula Minski

Appendix I

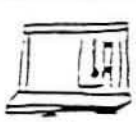
Handout #Seven



Appendix J

Handout # Eight

Measure Through the Week...

Monday	Tuesday	Wednesday	Thursday	Friday
				
Outside Temperature	Sweet Potato	Water Jar	Hamster	Counting

Begin the weekly measurement series at the comparison stage. During the year, graduate to non-standard units, and, if appropriate, go on to standard units of measure.

Stages in the measurement process:

- COMPARISON – a measurement process at the concrete level utilizing sensory input from sight and touch.
- NON-STANDARD UNITS – an indirect process of measurement using an arbitrarily chosen object as a unit of measurement.
- STANDARD UNITS – an indirect process of measurement using traditional, well-defined units, such as the United States customary system and the international metric system.

Friday Estimating and Counting

Show container of objects on Monday. Students estimate the number of objects and write their predictions on paper during the week. On Friday, count the objects, placing each group of ten in a counting cup. Repeat this activity each week, and see your students' skill in estimating improve as they have more experience.

ESTIMATING IDEAS~

- contents from junk boxes
- paper clips
- apple seeds
- pasta
- popcorn kernels
- jelly beans
- peanuts
- red hots
- pennies
- jingle bells
- pumpkin seeds
- marshmallows
- styrofoam packing
- peanuts
- goldfish crackers
- animal crackers
- candy corn
- bottlecaps

Make use of everyday <u>and</u> seasonal counting objects!

Appendix K

Handout # Nine

Name _____

Raisin												
Peanut												
M & M (m)												

I had ___ raisins.

I had ___ peanuts.

I had ___ m and ms.

I had the most _____.

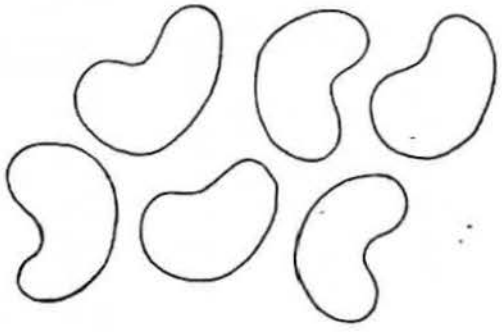
I had the least _____.

I had ___ altogether.

Equations

Appendix L

Handout # Ten



Appendix M

Handout # Eleven

Apples in a Bag

There are two apples in each bag.

One bag contains 2 green apples.
One bag holds 2 red apples.
Another has 1 green apple and 1 red.

* You know all bags are mismarked.

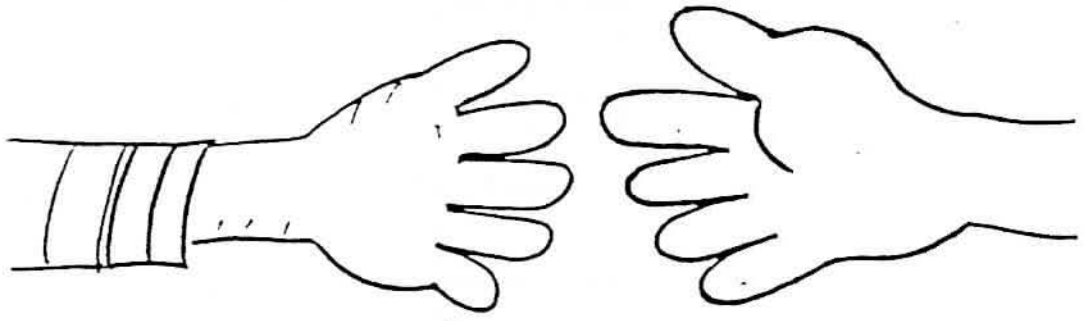
* You may pick only one apple from any of the bags.

* Which bag will you take from to determine the contents of each bag?

Appendix N

Handout # Twelve

Handshake Problem



When two people shake hands there is one handshake.

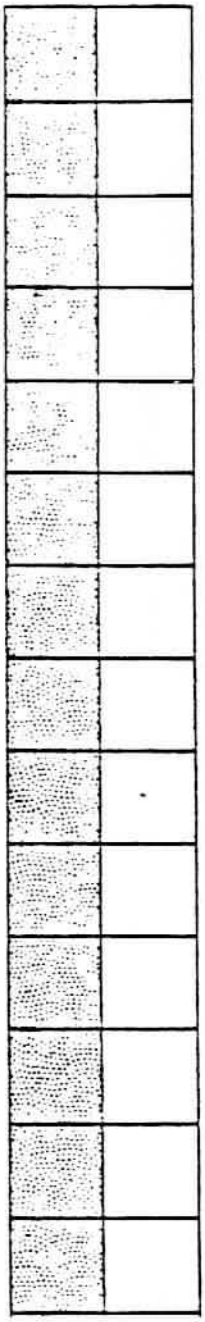
When three people shake hands with one another there are three handshakes.

* How many with 4 people?
  5 people?
  10? 50?

* Can you find a pattern?

Appendix O

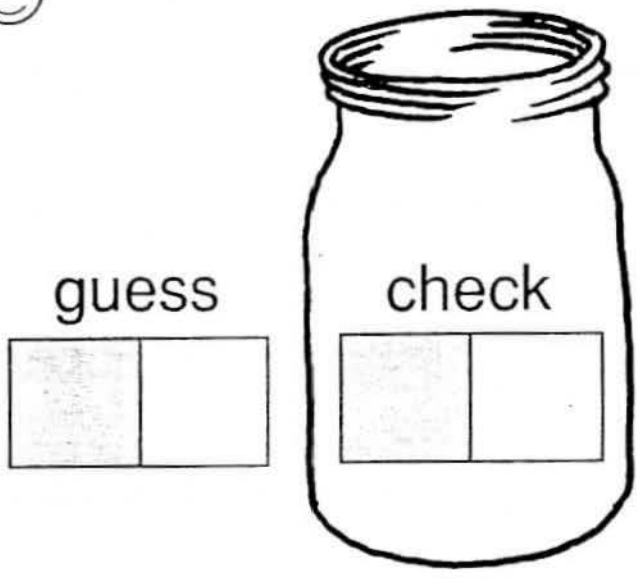
Handout # Thirteen



Appendix P

Handout # Fourteen

Estimating Objects in Containers



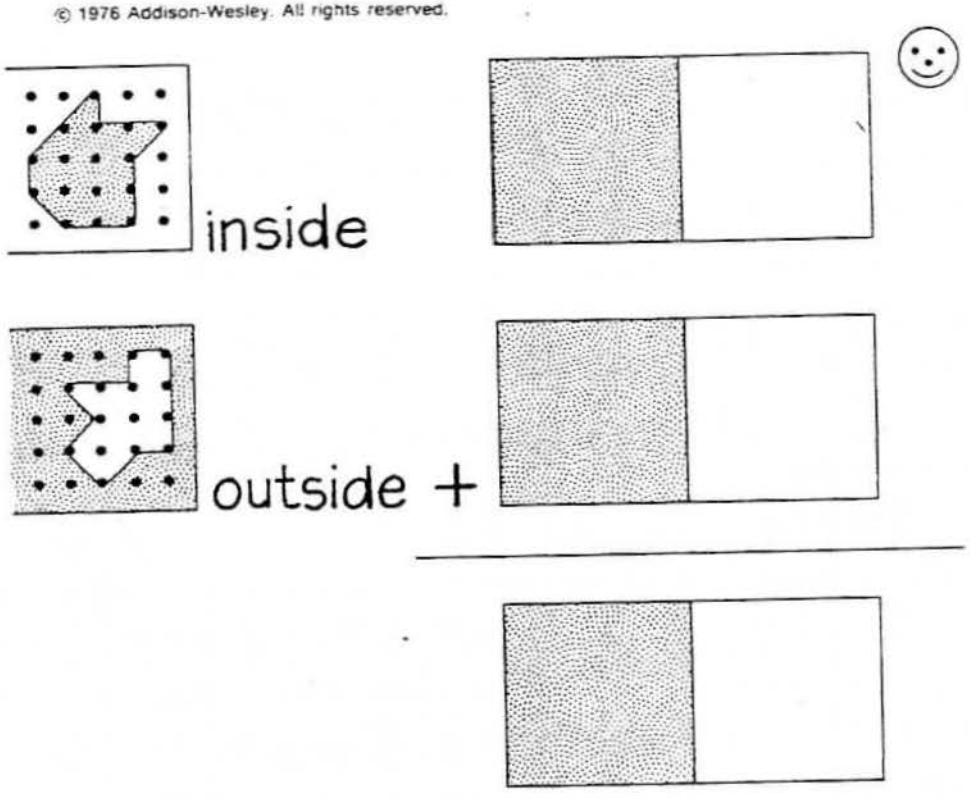
Appendix Q

Handout # Fifteen

Numeral Sequences	Tallies

Appendix R

Handout # Sixteen



Appendix S

Handout # Seventeen

00	01	02	03	04	05	06	07	08	09
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Appendix T

Handout # Eighteen

Task Card 1

Red: 4, 13, 14, 15, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 58

Black: 64, 74, 84, 94, 82, 92, 93,

TASK CARD 2

Green: 4, 5, 13, 14, 15, 16, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68,

Brown: 74, 75, 84, 85, 94, 95

TASK CARD 3

Black: 11, 12, 20, 21, 22, 30, 31, 32, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 59, 63, 64, 65, 60, 67, 72, 73, 77, 78, 81, 82, 83, 89, 90, 91,

TASK CARD 4

Green: 35, 36, 44, 45, 46, 47, 53, 54, 55, 56, 57, 58, 40, 41, 50, 51, 62, 63, 64, 65, 66, 67, 68, 69, 72, 79

TASK CARD 5

Orange: 11, 12, 21, 22, 16, 17, 26, 27, 51, 52, 61, 62, 56, 57, 66, 67

Blue: 33, 34, 35, 43, 44, 45

Green: 54, 64, 74, 84, 94, 85, 76, 93, 82

TASK CARD 6

Yellow: 0

Brown: 10, 20, 30, 40, 50, 60, 70, 80, 90

Blue: 11, 12, 21, 22

Red: 13, 14, 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 36, 37, 38, 39, 51, 52, 53, 54, 55, 56, 57, 58, 59,

White: 23, 24, 25, 26, 27, 28, 29, 41, 42, 43, 44, 45, 46, 47, 48, 49

TASK CARD 7

Blue: 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90
91, 92, 93, 94, 95, 96, 97, 98, 99

Brown: 40, 41, 42, 47, 48, 49, 51, 52, 53, 54, 55,
56, 57, 58, 62, 63, 64, 65, 66, 67, 73, 74,
75, 76

Red: 4, 14, 24, 34, 44

Orange: 5, 6, 7, 8, 15, 16, 17, 18

TASK CARD 8

Red: 10, 20, 30, 40, 50, 60, 70, 80, 21, 31, 41,
51, 61, 71, 32, 42, 52, 62, 43, 53, 18, 28,
38, 48, 58, 68, 78, 88, 27, 37, 47, 57, 67,
77, 36, 46, 56, 66, 45, 55

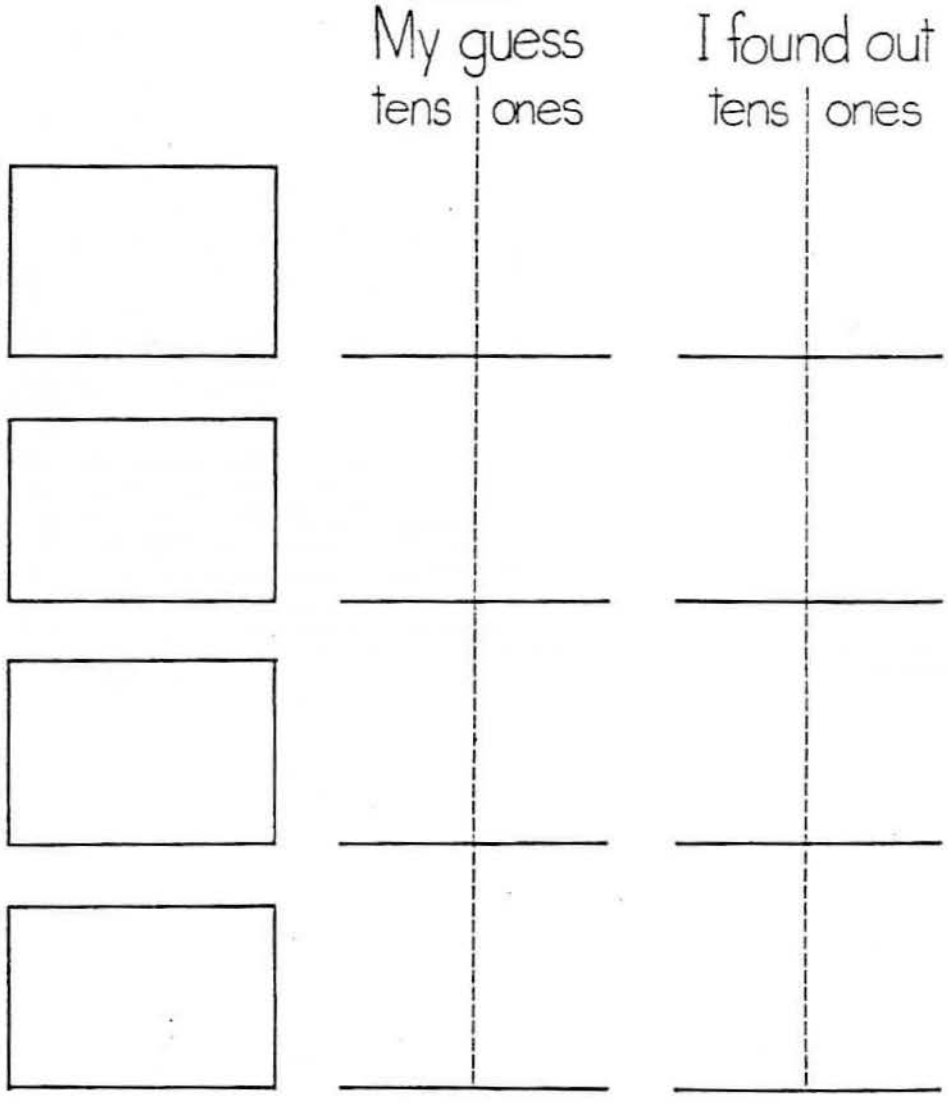
Blue: 2, 13, 15, 6, 24, 34, 44, 54, 64, 74, 84, 94

Appendix U

Handout # Nineteen

Name _____

I worked with _____



Appendix V

Handout # Twenty

Student Composite Record

Child's Name: _____ Grade Level: _____ Age: _____ Teacher's Name: _____

Assessment Dates: _____ _____ _____ _____ _____ _____

1. **Rote Counting**
   By 1's: ___ ___ ___ ___ ___
   By 2's: ___ ___ ___ ___ ___
   By 5's: ___ ___ ___ ___ ___
   By 10's: ___ ___ ___ ___ ___

2. **1:1 Correspondence:** 4 ___ 8 ___ 12 ___ All ___

3. **Instant Recognition:** 2 3 4 5

4. **Conservation of Number:** *Yes* ___ *No* ___

5. **Counting Backwards:** 7 8 9 10 11 12 13 14 15 16...

6. **Estimation**
   Bottle A (25 objects): ___ ___ ___ ___ ___
   Bottle B (50 objects): ___ ___ ___ ___ ___
   Bottle C (100 objects): ___ ___ ___ ___ ___

7. **Numeral Recognition:**
   Numerals 0-10: 0 1 2 3 4 5 6 7 8 9 10
   Numerals 11-20: 11 12 13 14 15 16 17 18 19 20

8. **Numeral Forms:** 0 1 2 3 4 5 6 7 8 9

9. **Number:**
   Concept: 3 4 5 6 7 8 9
   Connecting: *Addition* _____ *Subtraction* _____
   Symbolic: *Addition* _____ *Subtraction* _____
   Visualization: *Addition* _____ *Subtraction* _____

10. **Place Value:** *Building a number* *Addition* *Subtraction*
    Concept: _____ _____ _____
    Connecting: _____ _____ _____
    Symbolic: _____ _____ _____

Appendix W

Handout # Twenty-One

Number Station Range Cards

Name:			Name:		
Name:			Name:		

Appendix X

Handout # Twenty-two

FAVORITE RESOURCES

Addison Weslay
1643 Hicks Road
Rolling Meadows, Illinois 60008
1-800-535-4391

Mary Baratta-Lorton
Mathematics Their Way
Workjobs
Workjobs II
Workjobs ... For Parents

Center for Innovation in Education
19225 Vineyard Lane
Saratoga, CA 95070

Laura Duncan Choate
Fallbrook Union School District
P.O. Box 698
Fallbrook, CA 92028

Lynda Holman
16270 Jupiter Circle
Westminister, CA 92683

The Learning Center
4504 Westminister Place
St. Louis, MO 63108
(314) 361-1908

Kathy Richardson:
Developing Number Concepts Using Unifix Cubes

Appendix Y

Handout # Twenty-three

SURVEY AFTER WORKSHOP PRESENTATION

Indicate how strongly you agree or disagree with each of the following statements. Use the following scale:

1 = Strongly agree
2 = Agree
3 = No opinion
4 = Disagree
5 = Strongly disagree

1. _____ Manipulatives stimulate thinking.
2. _____ Manipulatives can be used with all ability levels.
3. _____ Learning effectiveness of manipulatives can be evaluated.
4. _____ Manipulatives are easy to obtain.
5. _____ Manipulatives are inexpensive.
6 _____ Manipulatives help develop social interaction skills.
7. _____ Manipulative activities can be organized and structured to operate smoothly in the classroom.
8. _____ Manipulatives are important in the learning process.
9. _____ Manipulative activities encourage language experiences.
10. _____ Manipulative activities encourage an exchange of different viewpoints on how to solve problems.
11. _____ Parents understand the importance of manipulatives in the classroom.
12. _____ I enjoy the atmosphere of students talking and using manipulative materials in the classroom.
13. _____ It's important to involve parents in how manipulatives are being used in the classroom.
14. _____ Manipulatives encourage scientific methods like predicting, experimenting, comparing and graphing data.

Appendix Z

Handout #Twenty-four

EVALUATION

Manipulative Math Workshop
July 18, 1989
Presenter: Connie Burbes

1. Did the Presenter Fulfill your expectations? (Circle)
   A. Yes
   B. No - Why?
2. Did the presenter seem knowledgeable of the subject matter?
   A. Yes
   B. No - Why?
3. Was the material presented in an easy to understand manner?
   A. Yes
   B. No - Why?
4. What was the most important thing you learned from this presentation?

5. What suggestions would you make to improve this presentation?

6. Additional comments or suggestions.

Bibliography

Ashlock, R.R. (1978). Research and development related to learning about whole numbers: Implication for classroom/resource room/clinic. In Topics related to diagnosis in mathematics for classroom teachers, edited by M.E. Hynes. Kent, Ohio: Research Council on Diagnostic and Prescriptive Mathematics. (ERIC Document Reproduction Service No. ED 243 694).

Baratta-Lorton, M. (1972). Workjobs. Menlo Park, CA: Addison-Wesley.

Baratta-Lorton, M. (1976). Math their way. Menlo Park, CA: Addison-Wesley.

Baratta-Lorton, R. (1977). Mathematics...a way of thinking. Menlo Park, CA: Addison-Wesley.

Baroody, A. J., Gannon, K.E., Berent, R., and Ginsburg, H.P. (1983). The development of basic formal math abilities. Paper presented at the meeting of the Society for Research in Child Development, Detroit, April 1983. (ERIC Document Reproduction Service No. ED 229 153).

Barr, D. C. (1978). A comparison of three methods of introducing two-digit numeration. Journal for Research in Mathematics Education, 9 (1), 33-43

Bednarz, N. & Janvier, B. (1982, February). The understanding of numeration in primary school. Educational Studies in Mathematics, 33-57.

Bruni, J. V. (1982). Problem solving for the primary grades. Arithmetic Teacher, 29 (6), 10-15.

Burk, D., Snider, A., Symonds, P. (1981). Box it or bag it mathematics. Salem, OR: Math Learning Center.

California State Department of Education. (1985). Mathematics framework for California public schools: Kindergarten through grade twelve. Sacramento, CA: The Department.

Carpenter, T. P., Lindquist, M.M., Brown, C.A., Kouba, V.L., Silver, E.A. & Swafford, J.O. (1988). Results of the fourth NAEP assessment of mathematics: Trends and conclusions. Arithmetic Teacher, 36(4), 38-41.

Choate, L. & Holman, L. (1988). Math through a child's eyes and math their way. Learning Center, workshop.

Cobb, P. & Merkel, G. (1989). Thinking strategies as an example of teaching arithmetic through problem solving. In Elementary School Mathematics: Issues and directions, 1989 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: The Council, In press.

Cobb, P. & Wheatley, G. (1987, April). Children's initial understanding of ten. Paper presented at the annual meeting of the American Educational Research Association, Washington, D.C.

Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics. (1987). Curriculum and evaluation standards for school mathematics. Working Draft. Reston, VA: The Council.

Committee of Inuiry into the Teaching of Mathematics in the Schools. (1982). Mathematics counts (The Cockroft Report). London: Her Majesty's Stationery Office.

Dossey, J. A., Mullis, I.V.S., Lindquist, M.M., and Chambers, D.L. (1988). The mathematics report card: Are we measuring up? Trends and achievement based on the 1986 National Assessment. Princeton, NJ: Educational Testing Service.

Fennel, F. M. (1981). Elementary mathematics: Priorities for the 1980's. Bloomington, IN: The Phi Delta Kappa Educational Foundation. Fastback #157.

Fennema, E. (1981). The sex factor, mathematics education research: implications for the 80's. Alexandria, VA: Association for Supervision and Curriculum Development.

Fenstermacher, G. & Berliner, D. (1985). Determining the value of staff development. Elementary School Journal, 85, 281-314.

Fuson, K. (1986). Roles of representation and verbalization in the teaching of multi-digit addition and subtraction. European Journal of Psychology of Education, 1, 35-56.

Garland, C. (1988). Mathematics their way summary newsletter. Saratoga, CA: Center for Innovation in Education.

Garmston, R. (1987). Effective meeting management workshop: The Network for Educational Development, 6800 Wydon, St. Louis, Missouri 63105.

Good, T. L. & Brophy, J. (1987). Looking in classrooms (4th ed.). New York: Harper and Row.

Good, T. L. & Grouws, D.A. (1987). Increasing teachers' understanding of mathematical ideas through inservice training. Phi Delta Kappa, 68 (10) 778-783.

Good, T. L., Grouws, D.A., & Ebmeier, H. (1983). Active mathematics teaching. New York: Longman.

Goodlad, J. & Klein, F. (1970). Looking behind the classroom door. Worthington, OH: Charles A. Jones.

Griffin, G. (1985). The school as a workplace and the master teacher concept. Elementary School Journal, 86, 1-16.

Grossnickle, F. E. et al. (1983). Discovering meanings in elementary mathematics. New York: Holt, Rinehart & Winston.

Howden, H. (1989). Teaching number sense. Arithmetic Teacher, 36(6), 6-11.

Isenberg, J. P. & Altizer-Tuning, C.J. (1984). The mathematics education of primary-grade teachers. Arithmetic Teacher, 31, 23-27.

Jackson, R. L. & Phillips, G. (1973). Manipulative devices in elementary school mathematics. In Instructional Aids in Mathematics, 1973 Yearbook of the National Council of Teachers of Mathematics, edited by E.J. Berger, 299-344. Reston, VA: The Council.

Kamii, C. (1986). Place value: An explanation of its difficulty and implications for the primary grades. Journal for Research in Childhood Education, 1, 75-86.

Kamii, C. & Joseph, L. (1988). The teaching of place value and couble-column addition. Arithmetic Teacher, 35, 48-52.

Kennedy, L. M. (1986). A rationale. Arithmetic Teacher, 33, 6-7.

Kline, M. (1980). Mathematics: The loss of certainty. New York: Oxford University Press.

Labinowicz, E. (1985). Learning from children. Menlo Park, CA: Addison-Wesley.

Langbort, C. R. (1988). Jar lids - an unusual math manipulative. Arithmetic Teacher, 36 (3), 22-25.

Lewis, K. E. (1985). From manipulatives to computation: Making the mathematical connection. Childhood Education, 61 (5) 371-374.

Medley, D. (1977). Teacher competence and teacher effectiveness. Washington, D.C.: American Association of Colleges of Teacher Education

Miura, I. T. (1987). Mathematics achievement as a function of language. Journal of Educational Psychology, 79, 79-82.

The National Advisory Committee on Mathematics Education. (1975). Overview and analysis of school mathematics grades K-12. Washington, D.C.: Conference Board of the Mathematical Sciences.

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980's. Reston, VA: The Council.

Olney, C. W. (1989). Where there's a will there's an...A. Parent Manual. Paoli, PA: Chesterbrook Educational Publishers, Inc.

Richardson, K. (1984). Developing number concepts using unifex cubes. Menlo Park, CA: Addison-Wesley.

Ross, S. H. (1986). The development of children's place-value numeration concepts in grades two through five. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 278 482).

Ross, S. H. (1989). Parts, wholes, and place value: A developmental view. Arithmetic Teacher, 36 (6), 47-51.

Scott, P. (1983). A survey of perceived use of mathematics materials by elementary teachers in a large urban school district. School Science and Mathematics, 83(1), 61-68.

Sirotnik, K. A. (1983). What you see is what you get - consistency, persistency and mediocrity in classrooms. Harvard Educational Review, 53 16-31. (ERIC Document Reporduction Service No. EJ 275 754).

Suydam, M. N. & Higgins, J.L. (1977). Activity-based learning in elementary school: Recommendations from research. Columbus, OH: ERIC Center for Science, Mathematics and Environmental Education.

Thornton, C. A. (1978). Emphasizing thinking strategies in basic fact instruction. Journal for Research in Mathematics Education, 9 214-227.

Van de Walle, J. & Thompson, C.S. (1985). Let's do it: Promoting mathematical thinking. Arithmetic Teacher, 32, 7-13.

Weaver, J. F. & Suydam, M.N. (1972). Meaningful instruction in mathematics education. Columbus, OH: ERIC Center for Science, Mathematics and Environmental Education.

Weibel, J. H. (1982). The use of manipulative materials in first grade mathematics: A preliminary investigation. School Science and Mathematics, 81, 388-390.

Whitin, D. J. (1989). Bring on the buttons. Arithmetic Teacher, 36(5), 4-6

Wirtz, R. (1974). Mathematics for everyone. Washington, D.C.: Curriculum Development Associates.