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The Relationship between Multiplication Fact Speed-Recall and Fluency and Higher Level Mathematics Learning with Eighth Grade Middle School Students

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by

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Declaration of Originality

I do hereby declare and attest to the fact that this is an original study based solely upon my own scholarly work here at Lindenwood University and that I have not submitted it for any other college or university course or degree here or elsewhere.

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Abstract

This quantitative study investigated relationships between higher level mathematics learning and multiplication fact fluency, multiplication fact speed-recall, and reading grade equivalency of eighth grade students in Algebra I and Pre-Algebra. Higher level mathematics learning was indicated by an average score of 80% or higher on first and second semester mathematics assessments and proficient or advanced descriptor on the mathematics Missouri Assessment Program tests. Timed multiplication fact quizzes were administered to eighth grade students. Speed-recall scores were measured by the number of accurate answers in a 45-second time frame. Fluency was obtained by a student score of 35 accurate answers in a time frame of one minute and 48 seconds. Reading level grade equivalency was measured by the Gates-MacGinitie Reading Test.

A *z* test for difference in proportions analyzed differences in proportions of students who exhibited higher level mathematics achievement, proficiency on mathematics MAP, and a reading level of eighth grade or above. A *t* test for difference in means compared multiplication fact speed-recall scores and fluency scores generated by algebra students to those generated by pre-algebra students. A Pearson Product Moment Correlation Coefficient was calculated to analyze relationships between higher level mathematics achievement, multiplication fact speed-recall, and reading grade level equivalency. No relationship was found between higher level mathematics, multiplication fact speed-recall, and reading grade equivalency for students in Algebra I and Pre-Algebra. Data supported measureable differences in comparisons of multiplication fact speed-recall scores and fluency scores generated by algebra students to those generated by pre-algebra students. Measureable differences were found for pre-algebra students between the proportion with an average of 80% or above on first and second semester mathematics assessments and the proportion with multiplication fact fluency. The proportion of students with fact fluency was significantly higher than that of students who scored 80% or higher on mathematics assessments. No other differences were identified.

Data from this study did not support a major contribution from multiplication fact speed-recall and fluency to higher level mathematics achievement. However, further study involving other grade levels and longitudinal timelines is indicated to define the influence of multiplication fact knowledge on higher level mathematics.

Table of Contents

List of Tables

Table Page

Table Page

List of Figures

List of Acronyms

AYP	Adequate Yearly Progress
CCSS	Common Core State Standards
EOC	End-of-Course
GMRT	Gates-MacGinitie Reading Test
IEP	Individualized Education Plan
LEP	Limited English Proficiency
MAA	Mathematical Association of America
MAP	Missouri Assessment Program
MO DESE	Missouri Department of Elementary and Secondary Education
NAEP	National Assessment of Education Progress
NCEE	National Commission on Excellence in Education
NCES	National Center for Education Statistics
NCLB	No Child Left Behind
NCTM	National Council of Teachers of Mathematics
NYC HOLD	New York City Honest Open Logical Decisions
PSSM	Principles and Standards for School Mathematics
TIMSS	Trends in International Mathematics and Science Study
US DOE	United States Department of Education

Chapter 1 - Overview of the Study

The four basic math computations of addition, subtraction, multiplication, and division are fundamental and foundational operations for continuation of higher mathematics learning (Johnson, 2001). According to Loveless (2003), the teaching of basic computation skills in the mathematics classroom diminished during the 1990s. “The research evidence consistently suggests that by the end of middle childhood…their factual, procedural, and conceptual knowledge of multiplication and division still requires further development” (Robinson, 2009). Without these basic four computation skills, students will undergo difficulty with the process to comprehend or engage with higher level mathematical thinking and concepts (Loveless, 2003; Johnson, 2001). With memorization of multiplication facts characterized as a basic math computation, Wong and Evans (2007) considered quick recall of the multiplication facts as an essential foundational factor for mathematics achievement.

Reed (2011) realized through her experience as a former science and math teacher that multiplication fact mastery is an influential skill for math success:

> Your child must know each fact as well as he knows his own name. If you wake him up from a deep sleep and ask what 7 times 4 is, he will mumble ’28.’ That is mastery, and if you do not work with him until he reaches this point with every fact, he will forever have difficulty with math. (p. 136)

Loveless and Coughlan (2004) also reported computation skills were necessary to advance in the study of mathematics.

From the beginning of the Industrial Age to the present, the mathematics curriculum underwent historic changes that matched and correlated to the growing demands of the United States (U.S.). From the later part of the 19th century into the beginning of the 20th century, students primarily studied algebra and geometry in high school while students who enrolled in college studied trigonometry and calculus (Klein, 2003). Both World Wars, along with the push for technology, caused the need for a stronger mathematic curriculum to produce more scientific and mathematically inclined students upon graduation from schools (Klein, 2003). “The 20th century can be viewed as the century of democratization of schooling in the United States” (Schoenfeld, 2004, p. 256). The democratization of schooling was especially evident during the 1960’s social movement. All students had the right to a quality education: the quality of instruction and the curriculum became a factor of emphasis to hold public schools accountable for an appropriate education.

The ‘back-to-basics’ curricula of the 1970s and 1980s focused heavily on algorithms (basic arithmetic skills), in which a large part of the instructional methods relied more on drill and procedural methods rather than problem solving (Schoenfeld, 2004; Perso, 2007). Rote memorization of the basic arithmetic skills became the accepted instructional practice during the students’ elementary years as preparation for middle and high school mathematics. In support of numerous mathematical studies performed by the National Science Foundation, the National Council of Teachers of Mathematics (NCTM) created in April of 1980 an *Agenda for Action* that strengthened the recognition for mathematical curriculum and instructional change.

NCTM recommended eight changes for the mathematics curriculum. One of those changes involved basic mathematic computations. NCTM wanted a decreased emphasis on isolated drill exercises and more basic operation of numbers within problem contexts. NCTM discussed more time on problem analysis and interpretation in which students not only identified the necessary mathematic operations to use, but also how the mathematics computations are integrated together to solve mathematics problems accurately (NCTM, 1980). It was not until the publication of *A Nation at Risk* in April of 1983 that public awareness of the overall educational achievements of the U.S. schools increased.

Under President Ronald Reagan, the National Commission on Excellence in Education (NCES) conducted in 1983 a report, *A Nation at Risk: The Imperative for Educational Reform,* (U.S. Department of Education [US DOE], 1983) that questioned the current education practices and raised a number of concerns for education improvement. Through *A Nation at Risk: The Imperative for Educational Reform*, NCES educationally and politically campaigned for educational change: a reformation of the 'back-to-basics' instructional practice to more of an emphasis with a conceptual instructional practice. The document emphasized higher curriculum standards and instruction that involved more critical-thinking skills. Schools needed to adopt new curriculum, instructional and evaluation standards that involved more conceptual, problematic, and technological approaches toward learning. A conceptual, instructional approach focused on a more interactive classroom environment where the students developed meaning and understanding of key mathematical concepts through problems and math applications with discovery-oriented activities.

Problem solving provided a context and a new approach for students to think, understand, learn, and communicate mathematics concepts. Competent problem solvers not only pursued solutions relentlessly, but they also effectively communicated the results of their mathematical work, both orally and in writing (Schoenfeld, 2004). The "…goals for mathematics instruction had to be much broader than mere content mastery" (Schoenfeld, 2004, p. 263). Both publications, *An Agenda for Action* and *A Nation at Risk*, imposed a needed change with mathematics instruction: less memorization of facts and more understanding behind the meaning of the concepts.

With society changing from an industrial to informational age due to the advancement of technology, a paradigm shift in mathematics learning and instruction occurred with NCTM. NCTM supported the following mathematics instructional practice: "All mathematics should be studied in contexts that give the ideas and concepts meaning…Instructional approaches should engage students in the process of learning rather than transmit information for them to receive" (NCTM, 1989b, p. 2). How were schools going to mathematically educate and prepare the future children to fulfill the ever-changing demands and needs of a technological society, where the ability to socially work as a group or team to successfully solve mathematical problems, has become a norm (NCTM, 1989a)? The answer to this question was proposed through a series of new curriculum and evaluation standards commonly known as the "Curriculum and Evaluation Standards for School Mathematics" by NCTM in 1989 (1989a).

The1989 Standards focused more on constructive and critical thinking instructional practices with an emphasis on problem solving, along with the implementation of technology, calculators and computers in the classroom. "Mathematics must become, for all children, a basic right afforded to all in a manner that provides each child with the power required to face mathematics situations with confidence and visions of success" (Dossey, 1989). NCTM wanted all students to value mathematics, become confident in their mathematical ability and problem solving, to effectively communicate mathematical ideas as well as reason, generate, and apply mathematical ideas and strategies in mathematical problematic areas (NCTM, 1989a). These five goals attempted to ensure a democratic equality by all students rather than a select few, but also to allow students to experience math as more of a “doing” process rather than just a “knowing that” process (NCTM, 1989a). The 1989 Standards looked to create for everyone a better and more equal opportunity to compete in the nation’s job market. “If all students do not have the opportunity to learn this mathematics, we face the danger of creating an intellectual elite and a polarized society” (NCTM, 1989a, p. 6). Both Anderson (2010) and Ehlers (2007) reiterated the importance of high quantitative literacy skills for the 21st century due to the rise of science and technology.

Two organizations, the National Assessment of Education Progress (NAEP) and the Trends in International Mathematics and Science Study (TIMSS) provided the nation’s mathematics academic standing with in the United States and with other countries. Although school districts across the United States adopted the 1989 Standards within the mathematics curriculum, low scores revealed by the NAEP resulted in little mathematics improvement. In fact, during the implementation of the 1989 Standards throughout the 1990s, fourth graders revealed a decline with most of the computation skills – addition, subtraction, multiplication, and division of whole numbers (Loveless, 2003). The TIMSS, in the early part of the 1990s, revealed that the European and Asian countries had stronger mathematics curricula and instructional practices when compared to the United States. The 1989 Standards brought a mathematics curricular change that emphasized a stronger conceptual instructional practice with more problem solving. As a result of the new instructional practices and a de-emphasis on basic skills, the students' mathematics assessment scores declined. Other nations around the world achieved higher mathematics assessment scores. Loveless (2003) concluded a need to go forward, not backward, with basic math skills as part of the mathematics curriculum rather than just an emphasis on a conceptual instructional practice through the use of the 1989 Standards. "…the Standards [1989] aim was not to downplay the importance of basic skills. It was hoped…students would be motivated to understand the mathematical concepts as well as master the skills" (Hekimoglu & Sloan, 2005). NCTM revised the 1989 Standards in 2000 with the publication of the "Principles and Standards for School Mathematics" (PSSM), commonly referred as the 2000 Standards.

The 2000 Standards understood the importance of computational fluency with whole numbers in Grades 3 to 5. The 2000 Standards defined fluency as the ability to have "…efficient, accurate, and generalizable methods (algorithms) for computing that are based on well-understood properties and number relationships" (NCTM, 2000a, p. 1). "Learning the 'basics' is important; however, students who memorize facts or procedures without understanding often are not sure when or how to use what they know" (NCTM, 2000b, p. 1). NCTM emphasized conceptual learning and "thinking strategies," rather than memorization, as the new standard of mathematics instruction (Quirk, 2000b). The 2000 Standards concluded with fourth grade students who struggled with multiplication and division fluency that "…they must either develop strategies so that they are fluent with these combinations or memorize the remaining 'harder' combinations" (NCTM 2000c, p. 5). NCTM emphasized "thinking strategies" through a variety of instructional models and methods for the attainment of multiplication fact fluency.

Multiplication Fact Fluency, Recall, and Automaticity

The meanings of multiplication fact fluency, recall, and automaticity have similarities (Dougherty & Johnston, 1996) and differences. This researcher defined accuracy to be the similarity to multiplication fact fluency, recall, and automaticity; from *The American Heritage Dictionary of the English Language*, Morris (1981) edited accuracy to be the measure of "…exactness or correctness" (p. 9). The students' answers to the basic math computations must be without error. This researcher used time as the underlying difference with multiplication fact fluency, recall, and automaticity. Although recall and automaticity both have "time" as a similarity, the "time" for multiplication recall may vary, while only a very small unit of time, less than three seconds per problem, characterized students with multiplication fact fluency or automaticity. Crawford (2003) defined automaticity as the students' ability to provide an answer quickly without much conscious effort. For the purpose of this study, this researcher used only multiplication fact recall and fluency.

This researcher adapted a definition and contextualized multiplication fact fluency as the ability to recall the product for single-digit multiplication accurately in three seconds or less (Michalczuk, 2007). The 2000 standards did not equate time with computational fluency, but rather equated computational fluency under three guided principles: efficiency, accuracy, and flexibility (Russell, 2000). The writers of the 2000 Standards understood computational fluency required understanding and meaning, rather than just a memorization of numbers and operations. The understanding, process, and approach of the correct answers far outweighed the student's ability to provide quick correct answers; understanding rather than quick recall or automaticity defined computational fluency with the 2000 Standards (Russell, 2000).

Possible Effects of Lacking Basic Mathematics Computations

Research showed students who have not mastered the multiplication tables with ease or confidence fell behind in their math skills, lost confidence, and developed a resistance toward learning higher level math skills (Caron, 2007; Greenwald, n.d.; Jarema, 2010). Michalczuk (2007) suggested the lack of basic math skills caused students to formulate a genuine dislike for math. Ashcraft (2002) saw serious negative consequences, even to the point of math anxiety, for those students who continued to show an unwillingness to learn math.

Mathematics anxiety has been a significant barrier that prevented mathematics achievement for students (Kesici & ErdoGan, 2010). "Math anxiety is commonly defined as a feeling of tension, apprehension, or fear that interferes with math performance" (Ashcraft, 2002, p. 181). Kesici and ErdoGan's (2010) study revealed students who possessed both a high achievement motivation and a negative or low self-esteem created mathematics anxiety. "Someone with math anxiety feels negative emotions when engaging in an activity that requires numerical or math skills" (Sparks, 2011, p. 1). Mathematics competence positively correlated with the students' computation and problem solving skill abilities; students who showed a higher computation and problem solving skill ability also showed a higher mathematics competence. Kesici and ErdoGan (2010) reported "…self-efficacy beliefs are identified as most highly related with performance in mathematics and percentages" (p. 61). Students who utilize math strategies more often would not only increase their success and self-efficacy, but also decrease mathematics anxiety toward math (Ramdass & Zimmerman, 2008; Kesici & ErdoGan, 2010). Cates and Rhymer (2003) suggested fluency or a quick accurate recall, rather than just an accuracy of the facts. “Students with higher anxiety levels were not any less accurate, but they were less fluent” (Cates & Rhymer, 2003, p. 31). Students who exhibited fluency or an automaticity of the facts as opposed to accuracy appeared less likely to exhibit higher levels of mathematics anxiety (Cates & Rhymer, 2003).

Students who had not established mastery of the four basic math computation skills were most likely to struggle with the higher level math concepts; mathematics is a tiered and incremental process for higher level mathematics learning (Johnson, 2001; Michalczuk, 2007). Students who struggled with quick and accurate recall of multiplication facts struggled with higher level mathematics learning such as problem solving and subsequent math courses like algebra and/or geometry related math skills (Loveless & Coughlan, 2004). Loveless and Coughlan (2004) stated “eighth graders who cannot do basic arithmetic with ease, who cannot find the right answer quickly and confidently without a calculator, will be hampered in their efforts to learn algebra and geometry in high school” (p.56). Jarema (2010) identified students who had not mastered the multiplication facts with fluency could easily fall behind in math and lose complete confidence. Just as researchers considered computational fluency with whole number operations a critical factor for higher level mathematics learning (Wu, 1999; Wong & Evans, 2007), researchers also identified reading ability as an important factor in mathematics problem solving (Fite, 2002; Capraro & Joffrion, 2006).

Reading and Mathematics Achievement

Anderson (2010) linked both reading and math fluency as similar principles, where both required a functional skill when exercised over time, led to automaticity to solve problems. Both Fite (2002) and Capraro and Joffrion (2006) recognized that a mathematical reading ability produced a better chance of success with mathematic problem solving. Furthermore, Fite (2002) and Capraro and Joffrion (2006) recognized a difference between reading math material and reading running text. Fite (2002) emphasized "the syntax of math and the syntax of running narrative are different and require different strategies for instruction and learning" (p. 9). Reading math material required students to not only know how to use procedures and algorithms, but also when to use them (Fite, 2002). Capraro and Joffrion (2006) considered a conceptual rather than only an algorithmic understanding provided students with a better understanding of mathematics word problems. A conceptual understanding of mathematics coupled with reading comprehension skills allowed students to make the necessary translation of words that are involved in mathematics word problems into mathematics symbols (Capraro & Joffrion, 2006). Both the ability to read and understand text along with a conceptual mathematics understanding and computation skills affected the students' math performance, especially with mathematics assessments that involved solving word problems.

Problem Statement

The percentage of students meeting mathematics proficiency within Asian countries among other countries, which included the Russian Federation, displayed mathematics superiority over the fourth and eighth grade U.S. students (US DOE, 2009a).

Kilpatrick, Swafford and Findell (2001) stated the following introductory remarks about the U.S. school of mathematics:

> State, national, and international assessments conducted over the past 30 years indicate that, although U.S. students may not fare badly when asked to perform straightforward computational procedures, they tend to have a limited understanding of basic mathematical concepts. They are also notably deficient in their ability to apply mathematical skills to solve even simple problems. (p. 4)

The 1989 and 2000 standards, along with the 2010 Common Core State Standards (CCSS), emphasized the importance for students to develop a mathematics understanding for success. The developers of the Standards recognized mathematics to be an essential and vital area of knowledge for individuals to have a productive and meaningful life. The expansion of employment positions throughout society has moved from an industrial to more of an informational emphasis, to the ability to use and create with technology. Both the 1989 and 2000 Standards recognized math literacy, the ability to set up and individually or collaboratively solve problems, and to be an essential skill and preparation for the future.

The enactment of the regulations developed in association with the No Child Left Behind Act (NCLB) in 2002 caused mathematics to become one of two focused subject areas for academic improvement. As a continual act to meet the demands for all students to meet or exceed the requirements for mathematics proficiency, (a descriptive word used internationally and nationally as the minimum standard for acceptable academic performance), the U.S., except for Alaska and Texas, adopted the 2010 CCSS as of June 15, 2010. For students to become mathematically proficient, CCSS endorsed basic math computational fluency as an essential component for earlier elementary grades (Common Core State Standards Initiative [CCSSI], n.d.).

Significance of the Study

This study has prompted great concern regarding the requirement of NCTM for computational fluency of whole numbers as a means to help students gain a mathematics competence to learn more advanced or higher level mathematics. This quantitative study not only investigated whether a relationship existed between multiplication fact speed-recall and higher level mathematics learning, but also whether or not a difference in proportion exists between multiplication fact fluency and higher level mathematics learning for eighth grade middle school students in Algebra I or Pre-Algebra.

Both the 1989 and 2000 standards emphasized the importance of mathematics computations; the 2000 standards used computation fluency rather than computation automaticity with whole number operations. Basic facts are not only the key to a student’s success in math, but also essential skills that are required and applied for every concept in math (Michalczuk, 2007). Research has revealed quick single-digit multiplication fact recall not only acts as an important computational tool, but it also frees up the necessary cognitive capacity and resources to solve more complex or higher level math problems (Caron, 2007; Cavanagh, 2008; Wong & Evans , 2007; Jarema, 2010; Loveless & Coughlan, 2004; Wu, 1999). Although a number of research articles and studies highlighted the importance of computational automaticity through rationale, or specific instructional methods, this specific study searched to determine whether a possible relationship existed between multiplication fact fluency and higher level mathematics learning with eighth grade middle school students in Algebra I or Pre-

Algebra. If such a relationship is found to exist between multiplication fact fluency and higher level mathematics learning, the results of this study may reiterate the importance for the development of multiplication fact fluency as recommended by the 2010 CCSS. Could multiplication fact fluency act as a variable of significance for higher level mathematical achievement with eighth grade students enrolled in Pre-Algebra or Algebra I?

In addition to the effects of math achievement, this study also investigated whether a relationship existed between multiplication fact speed-recall and the student's GMRT equivalency grade. Anderson (2010) defined fluency for both reading and math as an acquirement of a functional skill: "…in reading, fluency requires decoding skills and is related to comprehension of the text, math fluency requires algorithmic skills and is related to comprehension of the underlying properties" (p. 1). "Both reading fluency and math fluency are significantly associated with automaticity - the capacity to simply recall the answers to facts without resorting to anything other than direct retrieval of the answer" (Crawford, 2003, p. 7.), which may free up the necessary cognitive resources to read, think and understand the mathematics text within word problems. "Being able to think mathematically is reflected by the ability to read and comprehend mathematical symbolism in much the same way that we read words" (Fite, 2002, p. 9). The development of reading comprehension and thinking skills are necessary for problem solving.

This study also investigated the percentage of students who achieved multiplication fact fluency was significantly different from the percentage of students who achieved a (GMRT) grade equivalency score at eighth grade or higher. The performance of mathematics assessments required both math and non-math vocabulary to read and interpret mathematical text for solving word problems (Fite, 2002). Computation and reading automaticity in both of these skills provided students the working memory to learn the necessary instructional strategies to think mathematically with word problems (Anderson, 2010; Fite, 2002).

Independent Variables

In this study, this researcher used two independent variables: single-digit multiplication fact speed-recall, measured by the speed-recall quiz score (the number of correct problems a student is able to accomplish in 45 seconds), and the single-digit multiplication fact fluency, measured by the fluency quiz score (a score of 35 or 36 accurate answers performed out of 36 total problems in no more than 1 minute and 48 seconds: Appendix B). The speed accuracy quiz (Appendix A) was renamed for the purpose of this study as the speed-recall quiz.

Dependent Variables

In this study, this researcher used three dependent variables for eighth grade students enrolled in Pre-Algebra and Algebra I students: mathematics achievement as measured by the 2010-2011 combined average score of first and second semester assessment scores, mathematics achievement as measured by the 2011 mathematics MAP test scale score, and reading achievement as measured by the Gates-MacGinitie Reading Test (GMRT) grade equivalency. This researcher also included a fourth dependent variable, Algebra I EOC raw score, in early May, 2011 for the Algebra I students who took this assessment.

Hypotheses

Hypothesis # 1. There will be a relationship between the speed-recall score and 2010 – 11 combined average score of first and second semester mathematics assessment score, 2011 mathematics MAP test scale score, 2011 Algebra I EOC raw score, and GMRT grade equivalency.

Hypothesis # 2. There will be a difference in fluency and speed-recall scores when comparing Algebra I student multiplication fact quizzes to Pre-Algebra student multiplication fact quizzes.

Hypothesis # 3. There will be a difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved 80% or higher on the average of the first and second semester mathematics assessments.

Hypothesis # 4. There will be a difference in the proportion of students with multiplication fact fluency, and the proportion of students who achieved proficient or advanced on the mathematics MAP test.

Hypothesis # 5. There will be a difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved a GMRT grade equivalency at eighth grade or above.

Hypothesis # 6. There will be a difference in the proportion of students without multiplication fact fluency, and the proportion of students who did not achieve proficient or advanced on the mathematics MAP test.

Hypothesis # 7. There will be a difference in the proportion of students without multiplication fact fluency, and the proportion of students who did not achieve a GMRT grade equivalency at eighth grade or above.

Hypothesis # 8. There will be a difference in the proportion of students with multiplication fact fluency, and the proportion of students who achieved proficient or advanced on the Algebra I EOC test.

Limitations of Study

Specific Setting. The setting of this study involved one particular Missouri middle school that reflects a very small percentage of possible participants across the nation.

Assessments. The assessments reflected a percentage of the students who took the same assessments. The first and second semester assessments were limited to the participant's specific school setting. The mathematics MAP and Algebra I EOC tests were limited only to the students of Missouri.

Assessment Scores. The assessment percentage scores for each semester resulted from assessments used specifically to the school of study and the mathematics MAP and Algebra I EOC tests scores resulted specifically for the state of Missouri.

Participants. The total population sample of this study included only a portion of all the eighth grade students enrolled in Pre-Algebra and Algebra I classes for this one middle school within one school district in Missouri. This population only reflects a small percentage of students across the nation with similar demographics. The results may not be accurate when applied to other middle schools in other districts with similar or different demographics.

Time Frame of Study. This study only utilized an analysis of one year of data. In order to determine the impact of multiplication fact fluency as a viable variable in mathematics competence to advance students in higher level mathematics learning, additional years of data with different samples of students are necessary.

Definition of Terms:

Advanced: students utilize "a wide range of strategies to solve problems and demonstrate a thorough understanding of important mathematical content and concepts" (Missouri Department of Elementary and Secondary Education [MO DESE], 2011a, p. 9)

Algorithm: "a procedure involving prescribed steps that lead to a specific outcome, which is often the calculation of something" (Ross, 1997, p. 1).

Assessment: "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (NCTM, 1992, p. 2).

Automaticity: "the capacity to simply recall the answers to facts without resorting to anything other than direct retrieval of the answer" (Crawford, 2003, p. 6).

Basic Math Facts: "computations involving the four basic math operations: addition, subtraction, multiplication, and division; using the single-digit numbers, 0 – 9" (Basic math facts: A sequence of learning, 2007, p. 1)

Commutative Property of Multiplication: "The property that states that two or more numbers can be multiplied in any order without changing the product" (Bennett, Chard, Jackson, Scheer, & Waits, 2008, p. A35)

Computational Fluency: an "efficient, and accurate method for computing that are based on well-understood properties and number relationships" (NCTM, 2000a, p. 1).

Conceptual Learning: students who undergo a constructivist instructional approach to learn concepts where "the learner is the constructor, or elaborator, of mathematical concepts, and the instruction is designed to correspond to the mathematical thinking of the learner" (Suydam & Kasten, 1988, p. 7).

Constructivism: an instructional style where, "learning is an active process in which learners are active sense makers who seek to build coherent and organized knowledge" (Mayer, 2004, p. 14).

Constructivist Instructional Practices: instructional approach where "the learners be provided with the autonomy to select activities that blend with their interests and prior experiences to build mathematical connections through active learning using concrete materials" (Chung, n.d., p. 272).

End-of-Course (EOC): MO DESE course-level assessments created for middle school eighth-grade subject area (Algebra I) and secondary students enrolled in one of the following core subject areas: Algebra I, Algebra II, Geometry, Biology, English I, English II, American History, and Government (MO DESE, 2011a).

Evaluation: "the process of determining the worth of, or assigning a value to, something on the basis of careful examination and judgment" (NCTM, 1992, p. 3).

Gates-MacGinitie Reading Test (GMRT): a reading assessment "that is useful for teachers and schools to know the general level of reading achievement of individual students throughout their entire school careers" (Riverside Publishing, 1999, p. 1).

Mathematics Achievement: "Level of attainment in any or all mathematics skills, usually estimated by performance on a test" (Education, 2012a). For the purpose of this study, students who have received an average semester grade of 80% or higher or a proficient or advance score on the mathematics MAP test and/or on the Algebra I EOC test.

Missouri Assessment Program (MAP): An assessment that is designed to measure how well students acquire the skills and knowledge described in the Missouri's Grade-Level Expectations for communication arts, mathematics, and science (MO DESE, 2010).

Mathematics Curriculum: "an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur" (NCTM, 1989a, p. 1).

Mathematical Power: "an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems" (NCTM, 1989a, p. 3).

Multiplication Fact Fluency: aligned with Michalczuk's (2007) understanding for the purpose of this study, the student's ability to multiply two single-digit factors, 2 through 9 in 3 seconds or less quickly and accurately.

Multiplication Fact Speed-Recall: for the purpose of this study, the number of two single-digit factors, 2 through 9, which students are able to multiply accurately in 45 seconds.

No Child Left Behind Act (NCLB):

> "a federal legislation that enacts the theories of standards-based education reform….ensures that all children have a fair, equal, and significant opportunity to obtain a high-quality education and reach, at a minimum, proficiency on challenging state academic achievement standards and state academic assessments." (USLegal, 2012, p. 1)

Product: "the result when two or more numbers are multiplied" (Bennett et al., 2008, p. A56).

Proficient or Proficiency: "Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter" (US DOE, 2011c, p. 21).

Raw Score: "the original score, as of a test, before it is statistically adjusted" (Raw Score, 2012, p.1)

Rote Memorization: a form or way to know and remember information through a process of repetition - going over something again and again to secure information from short-term memory to long-term memory; "a term for fixing information to your memory through sheer repetition" (Fleming, 2012, p. 1).

Scale Score: "Conversion of student's raw score on a test or version of test to a common scale that allows for numerical comparison between students" (Education, 2012a, p.1).

Standard: "is a statement that can be used to judge the quality of a mathematics curriculum or methods of evaluation. Thus, standards are statements about what is valued" (NCTM, 1989a, p. 1).

Traditional Instructional Practice: as it pertains to this study, a "practice in manipulating expressions and practicing algorithms as a precursor to solving problems" (NCTM, 1989a, p. 6).

Conclusion

Since the enactment of NCLB in 2002, 100% of all students are required to be mathematics proficient within their grade level by 2014. Educators across the nation have implemented a wide array of mathematics instructional strategies to learn and understand various mathematics concepts. During the elementary school years, students primarily learn basic whole number computations with addition, subtraction, multiplication, and division. Researchers have regarded the properties, understanding, and fluency of basic whole number computations as an essential foundation for higher level mathematics learning.

This quantitative study not only investigated a possible relationship between the multiplication fact speed-score and higher level mathematics learning, but also investigated the GMRT grade equivalency score of eighth grade students in Algebra I and Pre-Algebra. This quantitative study investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and a GMRT equivalency grade at eighth grade or higher.

Through four out of the seven hypotheses of this study, this researcher will observe whether a difference occurred between the proportion of students who achieved multiplication fact fluency and higher level mathematics learning, along with the proportion of students who achieved multiplication fact fluency and a reading grade equivalency at eighth grade or higher. Through two of the seven hypotheses, this researcher will observe whether a difference occurred between the proportion of students who did not achieve multiplication fact fluency and higher level mathematics learning, along with the proportion of students who have not achieved multiplication fact fluency and a reading grade equivalency at eighth grade or higher. The remaining hypothesis will determine whether a difference occurred between multiplication fact fluency and recall-speed scores of students in Algebra I and Pre-Algebra.

This researcher explored related literature, studies, and the historical changes of the mathematical curriculum within Chapter 2. This researcher also included in parts of Chapter 2 literature review the origins and basis of the "Math Wars" as a fundamental protest of the de-emphasis of mathematics procedures and algorithmic instructional practices from both the 1989 and 2000 mathematics Standards. Studies summarized within Chapter 2 also revealed the effects and importance of multiplication fact fluency on working memory and higher level mathematics learning. Other studies discussed within Chapter 2 provided instructional methods for multiplication fact recall improvement. Furthermore, this researcher addressed in Chapter 2 the relationship and the effects of reading and mathematics fluency with working memory and higher level mathematics learning.

Chapter 2 - Literature Review

Introduction

Michalczuk's (2007) understood basic math facts to be an important skill for students to succeed in math. Michalczuk (2007) generalized students who could answer single-digit, 0 – 9, multiplication problems (2 times 3, or 5 times 7, etc.) "within three seconds will do well at math and those that answer in less than one second will do excellent at math" (p. 1). Perso (2007) has defined the "looking back-to-basics" as a need to teach and focus on the basic arithmetic skills as a needed skill for higher level mathematics learning.

Some educational experts considered NCTM efforts for the past 20 or more years as more of a change to meet cultural demands rather than the academic demands for stronger mathematics instruction and learning. Some researches believed NCTM produced standards to change the instructional practices in lieu of the student's past experiences and cultural backgrounds rather than the student changing in lieu of the subject or curriculum demands (Allen, n.d.).

Hersh (2009) stated that skills in general are more important than ever because they allow students to further their learning as well as make judgments about the meaning, adequacy, and accuracy of the overall content. Clavel (2003) stated if students wanted to engage and be successful in higher-order mathematical thinking skills, mastery of the multiplication table was critical. Henry and Brown (2008) reported "students who learn to use derived-fact strategies in concert with memorization are more likely to develop mathematical proficiency than those students who have memorized the facts without supplementary strategies" (p. 172). Wallace and Gurganus (2005) identified students who mastered the multiplication facts, not only acquired a more positive attitude to mathematics learning, but they also developed an overall positive mathematics experience. Basic math fact fluency provided the necessary foundation to succeed in higher level mathematics learning.

Both documents of the NCTM Standards, 1989 and 2000, and the 2010 Common Core Standards confirmed the importance of the knowledge and understanding of the basic math facts as essential mathematics skills and tools for higher level mathematics learning. Although there is an agreement for basic math fact fluency, the instructional approaches to foster basic math fact fluency have been highly debated. The writers for the 1989 Standards rejected the memorization of mathematics (basic math facts and procedures) and supported the following notion: "knowing mathematics is doing mathematics" (NCTM, 1989, p. 4). Not everyone agreed with the kind of instructional changes brought by the 1989 Standards. Greenwald (n.d.) reported the following:

> number lines, charts, counters, and calculators are great tools to introduce addition, subtraction, and multiplication, but the bottom line is the fluency and knowing the correct answers to math facts is essential! If children do not memorize the math facts, they will always struggle with math. (p. 1)

A memorization along with a conceptual understanding of the basic math facts both are needed for mathematics problem solving success (Cavanagh, 2006, 2008; Johnson, 2001; Quirk, 2000b).

NCTM proposed the curriculum standards and mathematics instruction with both 1989 and 2000 Standards for kindergarten through the 12th grade (K-12) to be more discovery-based and hands-on: students needed to acquire an understanding of the computations through the usage of physical materials and modeled procedures rather than just through paper-and-pencil or rote memory methods. Students were expected to develop a deeper understanding of the basic math facts in order to formulate a better mathematical insight, reasoning and problem solving capabilities (NCTM, 1989a). The new redesigned mathematics curriculum brought by the 1989 Standards caused educators to use new instructional techniques that represented a more hands-on approach with physical materials and models rather than just a memorization of the facts through pencil-and-paper procedures and practices.

The early implementations of the 1989 Standards suggested a strong need for professional development for district personnel and teachers. With the adoption of the 1989 Standards within the state's educational policies, the knowledge and resources for the necessary curriculum and instructional changes at the local or district level, were minimal at best. More manipulative materials and in-service training or professional development for the teachers were sorely needed for successful changes for both instructional and assessment practices for mathematics (Edgerton, 1992; Watts, 1993; Cauley, Hoyt, & Van de Walle, 1993). The lack of understanding and resources to implement the 1989 Standards properly, along with a de-emphasis on pencil-and-paper procedures of the basic math facts or computations caused dissension among parents and educators.

The Math Wars

Around the mid 1990s, dissension among the California parents of the current instructional practices, which were based from the 1989 Standards, grew and soon escalated throughout the U.S. and enlisted many college and university professors as a joint effort to petition for mathematics instructional changes (Derbyshire, 2000). The differences in viewpoints among parents, professional mathematicians, and educators about mathematics instruction brought by the 1989 Standards has led to an ongoing intense and philosophical debate which has become commonly known as the “math wars” (Cavanagh, 2008; Schoen, Fey, Hirsch, &. Coxford, 1999).

A classic example of the “math wars” occurred in the state of Texas. In 2007 the Department of Education in Texas decided to reject the usage of *Everyday Mathematics*, the third grade textbook grounded on the principles of the 1989 Standards (Cavanagh, 2007b). Cavanagh reported (2007b) Texas’s state board of education made this decision because “[*Everyday Mathematics*] does not encourage students to memorize multiplication tables and solve problems without calculators” (p. 14). The company of *Everyday Mathematics* rebutted Texas’s statement and adamantly proclaimed students were “required to learn the multiplication facts through 12 times 12, through tables, models, and visual displays” (Cavanagh, 2007b, p. 14). This decision by Texas clearly defined the differences of opinion between traditional and conceptual mathematics instruction.

A number of professional mathematicians argued for a more direct instructional style where students are engaged in the specific rules and procedures of the math basics (addition, subtraction, multiplication, and division). Other math educators or constructivists hold a conceptual approach where students construct their own problem solving strategies and math retention through investigations and an exchange of ideas as the instructor takes more of a facilitator’s role in the classroom (Lewin, 2006). Brewer and Daane (2002) wrote the following: “There are no set guidelines or recipes for teachers to follow to become constructivist teachers" (p. 416). Constructivist teachers are defined with the ability to not only accurately articulate the constructivist's theory or philosophy, but also the capability to effectively implement the core principles of the constructivist theory into practical, instructional strategies within the classroom (Brewer & Daane, 2002). While the new math standards pushed the constructivist learning approach across the United States, many parents, mathematicians and educators had not quite accepted the new standards approach as the premiere decision for effective mathematics instruction and learning. Strong concerns from the New York City Honest Open Logical Decisions (NYC HOLD) on mathematics education reform held a meeting on June 6th, 2001, as an expression of dissatisfaction against the mathematics instruction infused in the classroom. The attendees at the NYC HOLD meeting expressed their concerns about the constructivist teaching philosophy:

> Students use pictures, beads, blocks, and coins to compute, and are discouraged from using the standard operations, such as column addition and subtraction. To measure angles, bent straws serve in place of protractors. Strips of paper, rather than rulers, are used to measure and to learn fractions. Memorization and practice are considered unnecessary; instead, students engage in activities such as skip counting, regrouping into friendly numbers, estimation exercises, games and class discussion. Knowing math facts, such as multiplication tables holds less importance. (Carson & Haffenden, 2001, p. 2)

NYC HOLD supporters addressed their concerns of the current instructional practices and new programs that developed from the 1989 Standards which did not develop accuracy and fluency with a number of mathematical procedures "such as column addition, multiplication of two digit numbers, long division, the division of fractions and procedures for solving algebraic equations" (Carson & Haffenden, 2001, p. 2). A number of basic skill procedures and Algorithms became replaced with a more constructivist or discovery approach as the improved pedagogical instructional method under the implementation of the 1989 Standards.

The Instructional Approaches for Conceptual Learning

Prior to the 1989 Standards, a mathematical instructional approach commonly reflected more of a rote memory of mathematical skills and procedures without much or any conceptual understanding. The 1989 Standards highly de-emphasized instructional practices of rote memory of mathematical skills and procedures and strongly supported a more constructivist instructional approach.

A constructivist instructional approach, defined by Brewer and Daane (2002), emphasized the following characteristics: a) process, b) the exchange of ideas or social interaction, and c) problem solving. Mayer (2004) defined constructivism as "learning as an active process in which learners are active sense makers who seek to build coherent and organized knowledge" (p. 14). Students become "actively engaged in small group and whole class discussions to explain, clarify thinking, agree or disagree, and question various mathematical ideas" (Brewer & Daane, 2002). Although the constructivist instructional method brought a new look at mathematical instruction, not everyone saw the "pure discovery" or constructivist method as the sole appropriate method due to the lack of the student's ability to construct and integrate the new mathematical knowledge with previous mathematical concepts (Mayer, 2004). "In short, when students have too much freedom, they may fail to come into contact with the to-be-learned material"

(Mayer, 2004, p. 17). Mayer (2004) suggested more "guided" discovery learning. Learning required more than just a "doing" or "discussing" as suggested by the 1989 Standards, but rather through teacher-guided lessons (Mayer, 2004). Wu (1999) understood the importance for a deeper or conceptual understanding of mathematics, but skills were also required.

Wu (1999) proclaimed "that skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are requisite vehicles to convey the conceptual understanding" of mathematics (p. 1). Basic math skills through appropriate algorithmic instructional techniques have provided the necessary tools for higher level mathematics learning (Russell, 2000). Ross (1997), mathematics professor at the University of Oregon, has stated algorithms should be the beginning, the focus point of a child's mathematical development. Ross (1997) wrote the following as a non-supporter of the 1989 Standards:

> Standard mathematical definitions and algorithms serve as a vehicle of human communication. In constructivist terms, individuals may well understand and visualize the concepts in their own private ways, but we all still have to learn to communicate our thoughts in a commonly acceptable language. (p. 1)

Wu (1999) exhorted basic math skills as appropriate algorithmic instructional methods are necessary for mathematics understanding: an "algorithm is a shining example of elementary mathematics at its finest and is fully deserving to be learned by every student" (Wu, 1999, p. 6). Algorithms or procedural skills with math computations provided the necessary skills and processes needed to build and understand mathematical knowledge and applications (Perso, 2007).

Ross (1997), Wu (1999), and Russell (2000) regarded algorithms as an important mathematics procedure for higher level mathematics learning. Wu (1999) reported, "If there is any so-called harmful effect in leaning the algorithms, it could only be because they are not taught properly" (p. 6). A conceptual understanding with mathematics could truly exist with algorithms if taught properly (Ross, 1997; Wu, 1999; Russell, 2000). Ross (1997) stated "classroom teachers should watch out for their abuse [of algorithms] as an instrument of mindless drills. They should not be over-emphasized just because they are easy to teach and test" (p. 2). Although the framers of the 1989 Standards de-emphasized algorithms and encouraged a more constructivist or a discovery role of learning, other researchers and studies supported algorithmic instruction as an important mathematical blueprint for conceptual understanding and higher level mathematics learning (Ross, 1997; Wu, 1999; Russell, 2000; Rittle-Johnson, Siegler, & Alibali, 2001).

The 1989 Standards supported a more conceptual understanding of mathematics rather than a rote-memory/procedural instructional approach for a stronger development of mathematics literacy or reasoning skills to solve problems. Chung (n.d.) performed a study that researched both forms of instructional approaches: constructivist or traditional. The study focused on multiplication facts from 0 to 5 through a combination of four third-grade classes grouped into two sections or groups. One group of two classes received a constructivist approach using a 3-tiered instructional strategy: a) usage of concrete materials, b) through visual pictures, and c) through a more abstract nature of words and numbers. The other group of two classes received a traditional approach through procedures and practice worksheets. The results of his research revealed both instructional approaches, constructivist or traditional, improved the student's knowledge and understanding of the multiplication skills. Although both instructional approaches resulted in similar findings, teachers who used the constructivist approach reported issues of classroom management as well as added extra instructional time.

Many districts across the nation who had originally implemented the constructivist's conceptual approach of 'discovery learning' have brought back and incorporated into its math curriculum a more balanced approach that included a combination of conceptual and procedural methods (Ravitch, 2010). Basic math facts memorization integrated with better algorithmic instructional practices and problem solving applications allowed a deeper, conceptual understand of mathematics for better math assessment results (Johnson, 2001); Wong & Evans, 2007; Microsoft and National Broadcasting Company [MSNBC], 2008; Cavanagh, 2006; Wallace & Gurganus, 2005; Wu, 1999). Perso (2007) also believed an incorporation of the two styles of instruction: "It is not a question of either basics or higher-order thinking skills; it is a question of balance" (p. 8). The mathematics curriculums of our schools need to find a balance between the mathematics content (deductive reasoning, math theory, logic, proofs) along with the application and transformation of mathematics knowledge - solving real world problems (Perso, 2007).

History of Mathematics Reform

Most students, who went through the mathematics curriculums from the early 1900's, prior to the launch of Russian's Satellite, Sputnik, graduated high school with a mathematics understanding for grocery clerks, carpenters, and other "practical" or "real-life" applications (Raimi, 2001). Kilpatrick was one of the most influential educational leaders who largely influenced American schools in the early 1900s (Klein, 2003). In

1915, Kilpatrick addressed the National Education Association's Commission on the Reorganization of Secondary Education with his published report, *The Problem of Mathematics*. He noted "nothing in mathematics should be taught unless its probable value could be shown, and recommended the traditional high school mathematics curriculum for only a select few" (Klein, 2003, p. 3). In 1923, the Mathematical Association of America (MAA) and newly founded organization NCTM published, *The Reorganization of Mathematics for Secondary Education*. The document became known as the *1923 Report* to preserve mathematics, especially algebra, for every student at the secondary level (Klein, 2003). The 1923 report made little impact while Kilpatrick's ideas continued throughout the 1920s and into the early 1940s.

By the mid-1940s the educational leaders created the "Life Adjustment Movement" to better prepare high school graduates for everyday living through math programs like consumer math, insurance, taxation and home budgeting rather than algebra, geometry or trigonometry (Klein, 2003).

Although the United States after World War II, realized a need for more technology and a stronger mathematics curriculum to help push the nation further into the technological age, nothing was truly being generated for such a movement. Table 1 provides the percentages of high school students enrolled in high school mathematics courses from 1909 to 1955. The percentages of high school students enrolled in Algebra and Geometry high school mathematics courses declined over a period of 45 years. With less students going into engineering at the University of Illinois, the Dean of the Engineering school created a committee with the help from Beberman who reformed the Illinois high school mathematics programs. The popularity of Beberman's mathematical ideas became known as New Math as he taught and traveled throughout the United States.

Table 1

Percentages of U.S. High School Students Enrolled in Mathematics Courses

School Year	Algebra	Geometry	Trigonometry
1909 to 1910	56.9%	30.9%	1.9%
1914 to 1915	48.8%	26.5%	1.5%
1921 to 1922	40.2%	22.7%	1.5%
1927 to 1928	35.2%	19.8%	1.3%
1933 to 1934	30.4%	17.1%	1.3%
1948 to 1949	26.8%	12.8%	2.0%
1952 to 1953	24.6%	11.6%	1.7%
1954 to 1955	24.8%	11.4%	2.6%

Note. Adapted from "A Brief History of American K-12 Mathematics Education in the 20th Century" by Klein, 2003.

The launch of Sputnik in 1957 sparked an enormous education concern, which ignited a fierce national debate about the inadequacy of the current education curricula, especially in math and science (Hersh, 2009; Cavanaugh, 2007a). "President Eisenhower appointed a Science Advisor and Congress suddenly started to pour money into the National Science Foundation and the National Office of Education, demanding instant science and mathematics" (Raimi, 2001, p. 2). Politicians, military, universities and other technological business of the U.S. wanted students to come out of the K-12 educational system with a stronger mathematics understanding of the concepts (Herrera & Owens, 2001). Beberman's Illinois experiments, American Mathematical Society, and School Mathematics Study Group (SMSG) teamed up and worked over a number of years with mathematics educators to write new math textbooks, enrichment materials, teachers' guides, etc. to improve the U.S. mathematics educational system which later became known as the New Math reformation.

The New Math reform pushed for more of an inquiry-based pedagogy curriculum where students learned how to think and use logic principles rather than "regurgitate" the facts (Hersh, 2009). Unfortunately, SMSG along with Beberman primarily catered the summer institutes to high school math teachers rather than the elementary teacher since high school teachers developed a better mathematical understanding so immediate mathematics achievement could be raised with high school students who were about to enter college (Raimi, 2001). The high school and junior high/middle school teachers experienced the required in-service professional development programs to understand the inquiry-based mathematic instructional approach of logic and theory, but not so with the K-6 instructors. Even though the government did not provide adequate funding and training for the elementary teachers for the New Math curriculum, elementary teachers were still required to implement the New Math concepts (Raimi, 2001). The instructors at the K-6 curriculum not only lacked the appropriate and effective professional training to instruct, but they also lacked the mathematics background to even comprehend the mathematics language. Distrust and shifting values from public schools pushed for the abandonment of the New Math (Pinney, 1977). The publication of Kline's book, *Why Johnny Can't Add*, along with dissension among teachers and parents, pushed for another national mathematics instructional and curriculum change (Pinney, 1977). The desire for change ultimately gave way to a new math reform movement, 'back-to-basics', during the 1970s (Herrera & Owens, 2001).

Not only were mathematics instructors provided new instructional strategies and a curriculum that focused on the math basics, but also well-designed instructional materials for each subject-matter to overcome any lacking of math content information or knowledge by the teacher (Hekimoglu & Sloan, 2005). Herrera and Owens (2001) described the 'back-to-basics' movement instructional technique as the following:

> first answers were given for the previous day's assignment. A combination of the instructor and students worked out the more difficult problems on the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and the problems assigned for the next day. (p. 87)

The National Council of Supervisors of Mathematics (NCSM) published a document titled "Position Paper on Basic Mathematical Skills" along with the writings of Fey and Graeber (Herrera & Owens, 2001) pushed for more problem solving, math applications and technology incorporated into the mathematics instruction. The works by NCTM and Fey and Graeber fueled and initiated NCTM to bring about a new "problem solving approach" mathematics reform, "An Agenda for Action," during the beginning of the 1980s (Herrera & Owens, 2001).

The problem solving approach embedded more of a problem-solving instructional technique within real-world contexts (NCTM, 1980). Influenced by the works of Fey and Graeber, NCTM in 1980 publicized the "Agenda for Action" with eight recommendations (Herrara & Owens, 2001). The "Agenda for Action" included the following eight recommendations: a) more problem solving, b) increase usage of calculators and computers for larger numbers, c) stringent standards for a stronger instructional effectiveness and efficiency, d) a de-emphasis on basic pencil-and-paper computation skills, e) a wide variety of assessments and evaluative methods for instructional improvement, f) flexible instructional methods to meet the diverse needs of all students, g) increase standards of professionalism and accountability with teachers, and h) increased parental and public support (NCTM, 1980). Only a few years after an “Agenda for Action” was published by NCTM for the need to improve the nation’s mathematics curriculum and instruction, the National Commission on Excellence in Education (NCEE) published *A Nation at Risk* in 1983 (Herrara & Owens, 2001).

The publication of *A Nation at Risk* cautioned the educators of our nation for curriculum and instructional change due to the “rising tide of mediocrity that threatens our very future as a Nation and a people” (US DOE, 1983, p. 1). The NCEE strongly pushed for further education reform towards a more constructivist theory of curriculum and instruction. After the publication of *A Nation at Risk*, education researchers rather than mathematicians, because of the New Math era failure that directly blamed the mathematicians (Raimi, 2001), became the primary stakeholders who decided to revise the math standards back to a more constructivist or conceptual approach to math (Garelick, 2005). NCTM published and ratified a new set of math standards in 1989, *The Curriculum and Evaluation Standards for School Mathematics.*

The societal changes of the nation from an industrial age into an informational age caused a change for new goals and standards for better mathematics instruction. The 1989 standards attempted to provide a better opportunity for the next generation of students to meet the job or market place demands for better problem solvers and to improve math scores at a competitive international level.

Frameworks of the 1989 Standards

The framers of the 1989 Standards were discouraged by the “drill and kill” strategy which forces the students to only memorize rather than build understanding and math connections with the basic math facts (Clavel, 2003). Students needed a much better approach to appreciate math so that all students, not just the college bound, could comprehend the mathematics content and instruction (Schoen, Fey, Hirsch, & Coxford, 1999; Herrara & Owens, 2001). Vukmir (2001) wrote the following quote of Sparks, math professor at the University of Wisconsin-Eau Claire and Co-Project Director of the Wisconsin Academy Staff Development Initiative (WASDI), who advocated for a constructivist instructional practice with math in Wisconsin, “Today, we need a curriculum for all, not just the select God-chosen few” (p. 14). Students without having acquired the necessary meaning or usefulness of the concept stimulated and articulated a dislike towards math (Wallace & Gurganus, 2005). The students’ overall attitudes and motivation towards math were counterproductive (French, 2005).

Clopton (n.d.) viewed the 1989 Standards as an effort by NCTM to help the more disadvantaged groups achieve math through more crafts, art, and creative math projects rather than an emphasis on arithmetic and algebraic concept and skill mastery. Educators emphasized or focused on the means or the process (investigational) to possible solutions, more group work, a reliance on calculators, and standardized assessments rather than a direct instructional focus on basic math skill and/or algebraic facts, correct answers, and authentic assessments. The ability to build logical proof arguments, new facts or information derived from known fact(s), as some mathematicians argued as one of the most important fundamental and foundational mathematical skills for higher level mathematics learning (Ross, 1997), essentially, vanquished with the new 1989 Standards (Allen, n.d.). The 1989 Standards discouraged and decreased the attention to some of the following traditional/procedural instructional practices:

> complex paper-and-pencil computations, long division, rote practice, rote memorization of rules, teaching by telling, relying on outside authority (teacher or an answer key), memorizing rules and algorithms, manipulating symbols, memorizing facts and relationships, the use of factoring to solve equations, geometry from a synthetic viewpoint, two column proofs, the verification of complex trigonometric identities, and the graphing of functions by hand using tables of values. (Schoenfeld, 2004, pp. 267-268)

The 1989 Standards valued more individual/group discussions, group work, and investigative learning projects and activities for mathematics understanding of numbers and operations.

The 1989 Standards identified an investigational through cooperative learning atmosphere as a better mathematics instructional approach to problem solving rather than a direct-instructional algorithmic approach. Schoen et al. (1999) characterized four specific classroom interactions and instructional practices characterized by the 1989 Standards:

> the classroom teacher should act as a stimulant, sounding board, and guide in that student problem solving; students should be encouraged to discuss mathematical ideas and discoveries with classmates and with the teacher; classroom activities

> should include frequent challenges for students to develop justifications for their ideas and discoveries; and students should be encouraged to use calculators and computers in their mathematical explorations. (p. 446)

The underlying goal for the mathematics curriculum and instruction of the 1989 Standards reflected the following philosophy that "knowing mathematics is doing mathematics" (NCTM, 1989a, p. 4). The writers of the 1989 Standards desired students to become active participants of their learning through an assortment of different instructional activities: investigations, explorations, group discussions, and problem solving; students needed to become investigators and creators rather than just recipients of knowledge.

The 1989 Standards devoted more attention to operation sense and development for understanding through cooperative learning groups rather than rote-memorization of the number facts. Mathematics learning required a purpose; knowledge creation and learning resulted through some kind of activity or "discovery learning" opportunity. The focus of mathematical instruction and knowledge was through a "doing," rather than a "knowing that" or "procedural" learning process. The Standards called for more attention to problem solving within the mathematics instruction and expected students to analyze problems and build a strong communication of mathematical ideas on a more regular basis. The basic fundamental philosophical idea behind the 1989 Standards was to create students to become mathematically literate or gain a mathematical power: "an individual's ability to explore, to conjecture, and to reason logically, as well as to use a variety of mathematical methods effectively to solve non-routine problems" (NCTM, 1989a, p. 3).

The 1989 Standards implemented a change in three primary areas. The first area focused on more problem solving; learning needed to be more active where students become gatherers, organizers, and interpreters of information from real-world applications, rather than just receivers of knowledge (NCTM, 1989a). The second area focused on a move away from rote-memorization of mathematical procedures to a conceptual understanding of the basic skills that included manipulatives, diagrams or other activities; visuals, activities and periodic small group working arrangements became the new norm of discussion for mathematics learning (NCTM, 1989a). The third area focused on an increased usage of electronic devices and a decrease in paper-and-pencil calculations to improve mathematics literacy, especially with larger numbers (NCTM, 1989a). Technology, which included calculators, computers, and videos, not only provided quicker and more efficient alternative methods to not only learn the material, but also increased the time for other kinds of deeper and richer mathematics classroom investigations and dialogues (NCTM, 1989a).

The impact of computer technology created new pursuits in both business and government employment. As technology grew in the 1970s and 80s, society became more informational rather than industrial. The ability to generate and process information was critical to the advancement of economic change. Equally important to the advancement of economic change was communication. The ability to communicate ideas effectively was vital to society's new pace of economic change. The framers of the 1989 Standards believed and valued mathematics literacy. Mathematics literacy allowed people to communicate and reason mathematical concepts effectively with one another. People with a strong mathematics literacy not only developed mathematical confidence, but also incurred a better opportunity to succeed within this new technological, social, and economic shift of the 21st century around the world. NCTM believed in the availability of calculators and computer labs for problem solving rather than for basic algorithm procedures and other noncomplex calculations. Although NCTM understood the importance of computational algorithms, “such knowledge should grow out of the problem situations that have given rise to the need for such algorithms” (NCTM, 1989a, p. 5). The usage calculators and computers were an important inclusion of technology within the classroom throughout the K-12 mathematics curriculum.

Another important belief developed from the 1989 Standards was mathematical connections. NCTM (1989a) believed an integration of mathematics through the following mathematics topics: number concepts, computation, estimation, functions, algebra, statistics, probability, geometry, and measurement played an important part to the process of mathematical literacy.

Educational Concerns with the 1989 Standards

Cauley et al. (1993) performed a research study with a number of schools within Virginia’s Metropolitan Educational Research Consortium (MERC) on the implementation of the 1989 Standards. Although a majority of teachers and administrators were aware and agreed with the 1989 Standards, a large percentage of teachers incorrectly implemented the Standards. “Teachers are in serious need of support as well as education concerning what the Standards are actually saying” (Cauley et al., 1993, p. 29). The study summarized a number of reasons for the lack of the Standards implementation. The first issue involved a lack of time. The employment demands did not allow sufficient time for instructors to fully comprehend and implement the 1989

Standards. Instructors expressed a lack of professional development because they attended minimal instructional service training. The standards of assessment did not align with the 1989 Standards of instruction. The availability of resources was limited. Instructors expressed that students with low mathematical skill abilities had difficulty with the new “discovery” instructional approach. Finally, the district policies for instruction did not truly reflect a change towards the 1989 Standards. Although the 1989 Standards revolutionized curricular and instructional change with mathematics, the need to effectively implement the Standards became problematic.

The National Science Foundation (NSF) quickly adopted and encouraged the use of the 1989 Standards through professional learning workshops. The direction for curriculum and instructional change brought about by the 1989 Standards required a considerable amount of resources, especially with a low educator buy-in at first, to train and educate administrators and teachers of the instructional philosophies and goals of the 1989 Standards. However, a vast number of administrators and teachers, across the nation, did not truly or effectively comprehend how to implement the new standards. Many or most staff members initially did not understand the needed curriculum and instructional change of the 1989 Standards. Although the 1989 Standards brought such a radical change to the overall process of mathematics teaching and learning, data still revealed minimal gains in student achievement (US DOE, 1999).

A study conducted by Edgerton (1992) revealed only one out of four teachers consistently and appropriately implemented the constructivist approach of teaching and learning as outlined and defined by the 1989 Standards. The other three teachers who used aspects of a constructivist instructional approach primarily utilized a more lecture or direct-instruction with teaching and learning. A different study conducted by Watts (1993) with K-3 mathematics specialist teachers revealed a large percentage (60%) of assessment practices using a more traditionalist instructional approach: knowledge level assessment questions along with a right or wrong assessment scoring. Edgerton (1992) explained assessments should be a reflection in not just what, but also how the students have learned in the classroom: "Teachers cannot develop into interactive, conceptually-oriented, constructivists without having assessment tools which support them. Teachers may progress instructionally only as far as their assessment techniques allow" (p. 25). Edgerton's (1992) and Watt's (1993) studies are only two of many others who have revealed a lack of an early integration of the 1989 Standards.

All four teachers with Edgerton's (1992) study expressed a limitation of time towards professional development to learn and/or an implementation to teach a constructivist instructional approach of the 1989 Standards. Edgerton (1992) wrote about the implementation of the 1989 Standards: "Teachers need time to process what they are learning and to adapt it to their situations. Teachers have busy lives: the curriculum is crowded, classrooms are being populated by increasingly diverse students, and regular teaching duties take a lot of time" (p. 27). Edgerton (1992) added the following statements about teacher professional development for the 1989 Standards:

> Limiting ourselves to evenings, weekends, and summers for the kind of work we want teachers to do can only make the process more difficult for them. They need extended periods of time to work on mathematics in problem situations, talk with colleagues, observe other teachers at work, and try out their ideas with ample opportunities for reflection, feedback, and revision. (p. 27)

A lack of professional development for teachers and administrators, lack of funding, lack of deep conceptual understanding of mathematics, and/or a lack of administrative support/change of district policy are all deemed plausible causes to the lack of success with student achievement under the 1989 Standards. This lack of success in the classroom with the 1989 Standards had repercussions outside the classroom.

Negative Effects with Mathematics Achievement

The 1989 Standards challenged a vast numbers of school districts who implemented changes in support of the 1989 Standards with their own curriculum and policies. On the down side, many school districts did not effectively implement the 1989 Standards. As such, a number of negative effects such as students unprepared for college level mathematics courses resulted (Carson & Haffenden, 2001). Data also showed a number of students who took the standardized tests assessed poorly and decreased in the area of basic computational skills (Herrara & Owens, 2001).

This decade of reform math during the 1990s challenged students to construct their own mathematical understanding through discovery or investigational activities. This kind of instructional practice(s) seemed to have lowered students' basic understanding of whole number operations (Mathematically Correct, n.d.). Not only the students' basic fact fluency of whole numbers lowered, but also achievement gap between Black students and White students expanded in every math computational area in the 1990s (Byrd, 1997). This discovery approach caused students to lose ground in the computation skills in all four areas of our basic math skills: addition, subtraction, multiplication, and division of whole numbers (Loveless, 2003).

Mathematically Correct (n.d.), a nationwide organization founded through the Internet by a number of California parents due to their frustration of the mathematics instructional practices of modeled math standards of the 1989 Standards. Mathematically Correct (n.d.) endorsed a counterrevolutionary campaign on their website towards the constructivist's style instructional practices as the new mathematics standard of instruction:

> The advocates of the new, fuzzy math have practiced their rhetoric well. They speak of high-order thinking, conceptual understanding and solving problems, but they neglect the systematic mastery of the fundamental building blocks necessary for success in any of these areas. Their focus is on things like calculators, blocks, guesswork, and group activities and they shun things like algorithms and repeated practice. The new programs are shy on fundamentals and they also lack the mathematical dept and rigor that promotes greater achievement. (p.1)

"Fuzzy math" reflected the mathematics instructional practices where the process was more important than simply the correct answer to a problem. Student problems were often deliberately ambiguous to solve (Allen, n.d.). The new goals of the 1989 Standards emphasized less paper-and-pencil algorithms and more opportunities for students to undergo investigative instructional strategies that allowed the students to foster an increase of mathematical reasoning, communication and problem solving. Rote activities that emphasized pencil-and-paper computations were discouraged.

Schoen et al. (1999) advocated for the 1989 Standards. They wrote the following in summary of their research of the Core-Plus Mathematics Project (CPMP), a Standards-based high school mathematics program:

> But seldom are the critics' attacks supported by a careful analysis of the complete curriculum and evaluation evidence. The critics also usually fail to acknowledge that the motivation for the whole reform movement is deep concern about the inadequate effects of long-standing American traditions in curriculum, teaching, and testing. (Schoen et al., 1999, p. 449)

After the implementation of the 1989 Standards, U.S. assessment scores showed a minimal increase during the 1990s. Although the1989 Standards brought strong guidelines for mathematics reform for the U.S., the mathematics curriculum across the nation still lacked focus and quality (Lindquist, 2001).

The 1989 Standards brought about such a drastic change in education resources, state and district policies, curriculum, instruction, and assessment that the magnitude of such change produced a paradigm shift in how our educators teach, and students learn mathematics. With a rise with public dissension from numerous stakeholders (concerned parents, teachers, and other professional educators) along with the publications of the TIMSS and NAEP reports of the 1990s, NCTM (2000a) revised the 1989 Standards. In 2000, NCTM publicized the 2000 Standards: "Principles and Standards for School Mathematics" (PSSM).

Frameworks of the 2000 Standards

NCTM's new document, PSSM, identified and strongly stressed minority group inclusion as part of each state's AYP goal for "equity efforts" (p. 47) in mathematics; mathematics pedagogy (curriculum and instructional frameworks and practices) should take into account the student's diverse cultural and socio-ethnic background (Matthews, 2005). As it was in the 1989 Standards, the vision for the 2000 Standards incorporated a common mathematical foundation for all students. “All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding” (NCTM, 2000d, p. 2). The 2000 Standards by NCTM still recognized mathematics to be a necessary foundation for students to be better citizens, equipped and productive for success within this changing and advancing technological era: “there is no conflict between equity and excellence” (NCTM, 2000d, p. 2).

The 2000 Standards incorporated five content and five process standards as new goals to reach mathematics proficiency. The first five content standards included the following: numbers and operations, data and probability, measurement, geometry and algebra, while the second five process standards included the following: problem solving, reasoning and proof, connections, communication, and representation (NCTM, 2000d). Through the numbers and operations content standard, students by the end of fourth grade should have developed a ‘quick recall’ of the multiplication facts of whole numbers (Cavanagh, 2006).

Like the 1989 Standards, the 2000 Standards discouraged rote memorization of the basic math facts. The framers of the 2000 standards understood the importance and defined computational fluency as the student’s ability to have “efficient, accurate, and generalizable methods (algorithms) for computing” (NCTM, 2000e, p. 1). Although the framers of the 2000 Standards recognized the importance of the basic math facts, a conceptual understanding rather than a memorization of math facts and procedures proved to be better for problem solving. The 2000 standards incorporated algorithms only as a mathematical tool for problem solving, rather than a focus or an emphasis of the mathematics learning process (NCTM, 2000e). Quirk (2000b) stated the following:

> PSSM fails to clearly acknowledge that the ability to instantly recall basic number facts is an essential skill, necessary to free up the mind, first for mastery of the standard algorithms of multi-digit computation, and next for mastery of fractions. Then once this knowledge is also instantly available for memory, the mind is again free to focus on the next level, algebra. (p. 2)

Quirk (2000b) emphasized the development of computation fluency with addition, subtraction, multiplication, and division as important fundamental mathematics foundations for higher level mathematics learning. The 2000 Standards also emphasized the importance and development of computational fluency through a conceptual instructional approach that characterized an understanding rather than a memorization of numbers and operations. The 2000 Standards idealized mathematics learning to be about the development of mathematical ideas and the acquirement of necessary skills and insights for problem solving.

Mathematics Achievement at the International Level

The 2007 study by TIMSS revealed the math skills of eighth graders in the U.S. are behind numerous countries worldwide, especially the Asian countries. Asian students revealed superiority in mathematic understanding. Cai and Wang (2006) examined the instructional practices with the concept of ratio between U.S. and Chinese teachers. The study revealed a number of differences. First, Chinese teachers created similar lesson plans that focused exclusively on the one concept, ratio, and highlighted possible areas of student learning difficulties, while the U.S. teachers created varied lesson plans that introduced not only the original concept, ratio, but also an additional concept, proportion. The U.S. teachers expected the students to apply ratios immediately for problem solving. In addition, the U.S. teachers did not predict the possibilities of learning difficulties. Second, although both the Chinese and U.S. relied on concrete representations for their introduction with ratios, the Chinese teachers introduced mathematical terms and symbolic representations with their one concrete example of ratios, while the U.S. teachers focused more on the physical representations of many different problems that related to possible representations with a typical students' life. Third, the Chinese teachers summarized the contents of the lesson, while the U.S. teachers did not. In summary, while the Chinese lesson plans included a more in-depth mathematical analysis of ratio with fractions and division, the U.S. lesson plans focused more on practical applications and problem solving rather than a deeper conceptual understanding of the concept.

NCES conducted a 2009 report that highlighted the 2007 TIMSS technical report of Grade-4 and Grade-8 scale scores for a number of participating countries around the world for math and science. The Grade 4 students from the U. S. ranked 11th while the Grade 8 students from the U.S. ranked ninth worldwide out of 30 countries (US DOE, 2009a). The scale scores of the following countries: Chinese Taipei - 598, Republic of Korea - 597, Singapore - 593, Hong Kong - 572, Japan - 570, Hungary - 517, England - 513, and the Russian Federation - 512 ranked higher than the U.S. - 508 (Bailey, 2010).

Under the advanced range (625 and above), "students can organize and draw conclusions from information, make generalizations, and solve non-routine problems" (US DOE, 2009a, p. 13). Under the high range (550 – 624), "students can apply their understanding and knowledge in a variety of relatively complex situations" (US DOE, 2009a, p. 13). Under the intermediate range (475 – 549), "students were able to apply basic mathematical knowledge in straightforward situations" (US DOE, 2009a, p. 13). A 508 scale score ranked within the intermediate scale score of 475 while Chinese Taipei, Republic of Korea, Singapore, Hong Kong, and Japan all ranked within the high scale score of 550. The finding from Cai and Wang (2006) realized the difference of cultural beliefs with teaching and problem representations between the U.S. and Chinese teachers: the U.S. teachers valued responses that included using concrete strategies or representations, while the Chinese teachers valued more abstract or symbolic representations, for problem solving. Instructional strategies that emphasized a conceptual understanding of the facts provided students better problem solving skills (Cai & Wang, 2006) and performance on standardized tests (Wallace & Gurganus, 2005) than students without a strong conceptual understanding of the facts.

National Assessment of Educational Progress Mathematics Assessment Results

The National Assessment of Educational Progress (NAEP, often called the nation's report card) created reading and mathematics standardized assessments. Both reading and mathematics assessments are administered every two years to randomly selected representative populations of fourth, eighth, and 12th grade students in each state. NAEP created and designed the assessment as an academic snapshot of each state and as a nation. The percentage of students in Missouri who performed at or above the NAEP Proficient level was 32% in 2011. The 2011 proficient level of 32% was down 3% from 2009 and 20% higher than the proficient percentage of 1992. Table 2 summarized the achievement level percentages and average score results for Missouri Grade 8 public schools in mathematics from 1992 to the most recent year, 2011.

Table 2

Missouri's NAEP Grade 8 Mathematics Achievement Level Percentages and Average Scores

Year	Below Basic Percent	Basic Percent	Proficient Percent	Advanced Percent	Average Score
1992[a]	38	43	17	2	271
1996[a]	36	42	19	2	273
2000[a]	33	45	19	2	274
2000	36	42	19	2	271
2003	29	43	24	4	279
2005	32	42	22	4	276
2007	28	42	25	5	281
2009	23	41	29	7	286
2011	27	41	25	7	282

Note. Accommodations not permitted with the following years denoted by a superscript **a**. Adapted from Mathematics 2011 State Snapshot Report Missouri Grade 8 Public Schools (MO DESE, 2011c).

The 2011 NAEP results revealed 32% of Missouri students scored a proficient or advanced descriptor – a 4% drop from 2009 NAEP results. Table 3 summarized the 2011 percentage breakdowns for White, Black, and Hispanic student groups in Missouri. In 2011, Black students achieved an average score of 33 points, while the Hispanic students achieved an average score of 21 points, both lower than the White students. The results from Table 3 revealed a smaller percentage of Missouri students in all student groups placed at or above mathematics proficiency as compared with the total national percentage of students placed at or above mathematics proficiency in Table 3.

Table 3

Missouri's 2011 NAEP Results by Race/Ethnicity

	Percentage of Students					
Ethnic Groups	Percent of Students	Ave. Score	Below Basic	At or above Basic	At or above Proficient	At Advanced
White	78	288	21	79	36	8
Black	16	254	60	40	8	#
Hispanic	3	267	42	58	16	#
Asian	2	‡	‡	‡	‡	‡
Indian	#	‡	‡	‡	‡	‡

Note. ‡ Reporting standards not met; sample size insufficient to permit a reliable estimate. # symbol rounds the percentage to zero. Adopted from "The Nations Report Card Mathematics 2011 National Assessment of Educational Progress at Grades 4 and 8" by the National Center for Education Statistics (US DOE, 2011a).

The NAEP mathematics assessment revealed more than 50% of the nations' eighth grade students are basic and below, especially for the Black, Hispanic, and Indian subgroups. NAEP defined the basic mathematics achievement level as a partial mastery of prerequisite knowledge and skills. NAEP described a proficient student who demonstrated competency over challenging subject matter, while an advanced student demonstrated superior performance. Asian students in 2011 scored higher than the scores for all the other reported racial/ethnic groups. The average mathematics score in 2011 was one point higher than in 2009, and 21 points higher than in 1990. Although the NAEP assessment results revealed a percentage increase for students who achieved a proficient or advanced score since 1990, the overall percentages revealed a needed growth with mathematics understanding. Educational writer, Klein, has questioned the validity of the NAEP assessment as an indicator for mathematics achievement.

Klein (2011) stated from his research that the math assessment scores from the NAEP test might not be a true measure of a student's mathematics achievement, but rather the estimation of the student's IQ. Klein (2011) reported "many of the questions appear to be IQ items, rather than math problems, in the sense that their solutions rely on almost no education or knowledge of mathematical techniques" (p. 3). Klein (2011) argued the NAEP assessment results focused more on math logic or ability rather than math knowledge or achievement; hence, a different comparison of results between the NAEP and individual state's assessment scores (Klein, 2011). Table 4 summarized the nation's percent of eighth-grade students who either received a proficient or advanced descriptor NAEP mathematics achievement-level by race/ethnicity.

From the *State of the States Report*, October 5, 2011, by the U.S. Department of Education, Missouri nationally placed in the top 10 for eight categories out of 27 total categories. Four of the percentage categories included Hispanic students at or above proficient in fourth and eighth grade reading and mathematics. The fifth percentage category included students with disabilities at or above proficient in fourth grade mathematics. The last three percentage categories included four-year high school graduation rates for all students and, specifically, for White and Hispanic students (US DOE, 2011b). Missouri nationally ranked in the middle for the remaining 19 categories.

Table 4

Eighth-grade NAEP Math Achievement-level Percentages by Race/Ethnicity

Year	White Proficient (Advanced)	Black Proficient (Advanced)	Hispanic Proficient (Advanced)	Asian Proficient (Advanced)	Indian Proficient (Advanced)
1990	16 (2)	5 (0)	7 (1)	23 (6)	n/a n/a
1992	22 (4)	2 (0)	6 (1)	30 (14)	n/a n/a
1996	25 (5)	4 (0)	7 (1)	n/a n/a	n/a n/a
2000	28 (6)	5 (0)	8 (0)	29 (12)	8 (2)
2003	30 (7)	7 (1)	10 (1)	31 (13)	13 (2)
2005	31 (8)	8 (1)	12 (1)	31 (16)	12 (2)
2007	32 (9)	10 (1)	14 (2)	32 (17)	14 (2)
2009	33 (11)	11 (1)	15 (2)	34 (20)	15 (3)
2011	33 (11)	12 (2)	18 (3)	33 (22)	14 (3)

Note. Adopted from "The Nations Report Card Mathematics 2011 National Assessment of Educational Progress at Grades 4 and 8" by the National Center for Education Statistics (US DOE, 2011a).

Missouri's Assessment Program Timeline

Table 5 summarized the timeline for both the MAP grade-span and grade-level assessments for the state of Missouri (MO DESE, 2010).

Table 5

Missouri Assessment Program Grade-Span and Grade-Level Timeline

Year	Event
1996	Show-Me Standards Approved
1996	Frameworks for Curriculum Development published
1997	Annotations to the Curriculum Frameworks published
1998	First operational administration of Mathematics MAP (Grades 4, 8, and 10)
1999	First operational administration of Communication Arts MAP (Grades 3, 7, and 11) and Science MAP (Grades 4, 8, and 11)
2000	First operational administration of Social Studies MAP (Grades 4, 8, and 10)
2001	Mathematics Curriculum Supplement published
2004	Grade-Level Expectations published
2005	Communication Arts and Mathematics Field Test
2005	Last year of grade-span MAP
2005	Standard Setting for Communication Arts and Mathematics
2006	First Operational Communication Arts and Mathematics
2007	Science Field Test
2008	First Operational Science MAP
2008	Last Operational Administration of High School MAP
2008	Version 2.0 Grade-Level Expectations (GLEs) published
2009	Last Operational Administration of MAP based on Version 1.0 GLEs
2010	First Operational Administration of MAP based on Version 2.0 GLEs

Note. Adapted from Missouri Department of Elementary and Secondary Education Missouri Assessment Program Technical Report, 2010. (MO DESE, 2010).

As an outcome of the Outstanding Schools Act of 1993, the MO DESE worked with numerous stakeholders throughout the state to create both the Missouri Show Me Standards and the MAP tests for academic accountability of student learning (MO DESE, 2010). As a direct response to the enactment of the No Child Left Behind legislation of 2001, the state's MAP tests changed from grade-span tests to grade-level tests. MO DESE described within the Missouri Assessment 2010 Technical Report the purpose of the Missouri Assessment Program as follows:

> The MAP is designed to measure how well students acquire the skills and knowledge described in Missouri's Grade-Level Expectations (GLEs). The assessments yield information on academic achievement at the student, class, school, district, and state levels. This information is used to diagnose individual student strengths and weaknesses in relation to the instruction of the GLEs and to gauge the overall quality of education throughout Missouri. (MO DESE, 2010, p. 14)

For the 2005-06 school year, Missouri created assessments that matched the grade-level expectations for Grades 3-8 and course-level expectations for high school students.

Missouri's Grade Level Expectations for Grade 8 Mathematics

Missouri adopted its mathematical curriculum strands from NCTM's 2000 standards. The content standards for mathematics are divided into five mathematical strands. The sources for each mathematical strand description came from the MO DESE. The first mathematical strand, numbers and operations, included the following mathematical concepts: basic math facts of addition, subtraction, multiplication, and division; estimation and computing techniques; number representations, systems, and relationships, along with the use of these operations and concepts in the workplace and real-world applications (MO DESE, 2011b). The second mathematical strand, algebraic relationships, included the following mathematical concepts: algebraic concepts including patterns, relationships, and functions; represent and analyze mathematical structures using algebraic symbols; understand quantitative relationships; analyze change in various contexts (MO DESE, 2011b). The third mathematical strand, geometric and spatial relationships, included the following mathematical concepts: geometric and spatial sense including analysis of characteristics as well as properties of geometric shapes; arguments about geometric relationships; coordinate geometry, symmetry and transformations; visualization, spatial reasoning, and geometric modeling (MO DESE, 2011b). The fourth mathematical strand, measurement, included the following mathematical concepts: measurable attributes of objects and the units, systems, and processes of measurement; use of appropriate techniques, tools, and formulas to determine measurements (MO DESE, 2011b). The fifth mathematical strand, data and probability, included the following mathematical concepts: data collection and statistical reasoning; formulating questions that addressed data analysis and statistics; develop and evaluate inferences based on data; understand and apply probability concepts (MO DESE, 2011b).

Missouri's Course Level Expectations for Algebra I

Missouri's Algebra I course level expectations adopted the same five mathematics content standards from the Principles and Standards for School Mathematics. The first mathematical strand, numbers and operations, included the following mathematical concepts: understand numbers, ways of representing numbers, relationships among numbers and number systems; understand meanings of operations and how they relate to one another through mental math, or pencil/paper for simpler calculations, or technology (calculators) for more complex complications; compute fluently and make reasonable estimates which includes proportions (MO DESE, 2011a). The second mathematical strand, algebraic relationships, included the following mathematical concepts: understand patterns, relations and functions (linear, quadratic, and exponential); represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; analyze change with linear and quadratic functions by investigating rates of change, intercepts and zeros (MO DESE, 2011a). The third mathematical strand, geometric and spatial relationships, included the following mathematical concepts: specify locations and describe spatial relationships using coordinate geometry and other representational systems; use visualization, spatial reasoning and geometric modeling to solve problems (MO DESE, 2011a). The fourth mathematical strand, measurement, included the following mathematical concepts: apply appropriate techniques, tools, and formulas to determine measurements (MO DESE, 2011a). The fifth mathematical strand, data and probability, included the following mathematical concepts: a) formulate questions that can be addressed with data as well as collect, organize and display relevant data to answer them, b) select and use appropriate statistical methods of central measure to analyze data and determine equations of scatter plots from the line-of-best-fit, and c) make general conclusions about possible relationships between two characteristics of samples on the basis of scatter plots of the data (MO DESE, 2011a).

Computational Fluency with Missouri Public School Students

Under Missouri's grade- and course-level expectations, students are required to develop and demonstrate fluency with basic number relationships with addition, subtraction, multiplication and division. By the end of second grade, students should have developed not only a fluency, but also a quick recall of addition and subtraction for sums up to 20 (MODESE, 2008). At the end of third grade, students should have developed basic number relationships of single digits (9 × 9) with division and multiplication. After third grade students are expected to utilize and demonstrate their knowledge and understanding of computation fluency with double digit numbers in the fourth grade, and decimal numbers and fractions that include unlike denominators in the fifth grade (MODESE, 2008). By the time students completed elementary school, kindergarten through the fifth grade, students are expected to not only have developed computation fluency, but also demonstrated computation fluency with higher mathematics learning.

Importance of Mathematical Computation Automaticity

Bratina and Krudwig (2003) recognized the automaticity or quick recall of basic math fact computations with whole numbers to be essential tools and important mathematical achievements for higher level mathematics learning. "The development of [mathematic] automaticity enables standard mathematical processes, such as facts about families of functions and formulas, to become useful tools for facilitating higher-order thinking" (Bratina & Krudwig, 2003, p. 47). The function of automaticity seemed to help students make the necessary connections more quickly with other mathematical processes for higher level mathematics learning; whereas, concrete strategies like counting with fingers hindered higher level mathematics learning (Kim, n.d.).

Students who used counting as their primary strategy for problem solving resulted in slower responses and less accuracy with their mathematics problem solving (Steel & Funnel, 2001; Hecht, 2002; Henry & Brown, 2008). The 2008 study by Henry and Brown revealed students who relied more on counting tended to score lower on the Number Sense Proficiency test; furthermore, the researchers also revealed more than two thirds of the assessed participants still used counting as the primary method for basic-fact problem solving with only a few weeks left in the school year. Although the 2000 Standards emphasized conceptual knowledge through a variety of instructional strategies like counting as a valid introductory instructional basis for number sense understanding, the 2000 Standards defined computation fluency as the ability to compute whole numbers accurately and efficiently without a sense of timeliness. Bratina and Krudwig (2003) stated the following: “Without automaticity, students will expend too much time and energy focusing on basic skills rather than on processes such as, but not limited to, understanding, representing, interpreting, and selecting appropriate operations for problems solving” (p. 60). The participants of Hecht’s study (2002) who used counting as the primary strategy for computing and problem solving substantially overloaded their working memory. The performance on both tasks tended to be impaired for participants with an overloaded working memory. In contrast to counting as a primary problem solving strategy, Steel and Funnel’s study (2001) revealed 10-12 year-old students who were able to accurately and quickly retrieve their multiplication facts performed better on mathematics assessments than other students who could not quickly retrieve their multiplication facts. Crawford (2003) stated the following:

> students who are automatic with their math facts can't help but think of the answer to a math fact when they say the problem to themselves. This automaticity allows them to focus their mental energies on the problem solving step rather than the facts. (p. 43)

Students who showed a lack of automaticity also used considerable amounts of additional time and energy that normally resulted in an overload of working memory (Bratina & Krudwig, 2003).

Working Memory

Hecht (2002) showed working memory affected the student's learning ability with mathematics concepts; his study statistically revealed a positive correlation with working memory and general math computation. This correlation suggested that working memory ability influenced a higher order mathematics learning.

Quirk (2000a) recognized certain math content needed to be stored in the brain as a precondition for the understanding of other math concepts. "Working memory refers to a brain system that provides temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning" (Baddeley, 1992. p. 556). Rasmussen and Bisanz (2005) noted within the research of their study "that working memory is related in some way to mathematical development" (p. 140). In regards to working memory and mathematics learning, Tronsky and Royer (2002) concluded a positive significant relationship existed between basic math fact automatization and higher level problem solving ability. Hasselbring, Goin, and Bransford (1988) connected the ability of math execution and cognitive processing capacity:

> If they do not have to use part of this limited capacity for performing basic skills, they have more capacity remaining for understanding higher-order concepts. Thus, the ability to succeed in higher-order skills appears to be directly related to the efficiency at which lower-order processes are executed. (p. 1)

Both Hecht's study (2002) and Tronsky's study (2005) confirmed "that working memory does not affect computation processes" (p. 454); hence, computation processes or automaticity of the math facts allowed the necessary working memory for other mathematical processes to occur for higher level mathematics learning.

A study by Rasmussen and Bisanz (2005) observed the relationship between working memory and arithmetic performance between preschoolers (34) and Grade 1 students (29). The results for preschoolers showed a poorer performance on verbal math problems due to the higher demands of working memory. Since the preschoolers relied more on mental models to perform arithmetic, the students most likely used additional working memory (the central-executive) to convert the verbal problems back into a visual-spatial code. The preschoolers' additional usage of the central-executive demanded a considerable part of the students' overall working memory to answer the math problems correctly. On the other hand, the first graders created additional strategies that helped solve verbal problems. The first graders used their hands as their external strategy. Rather than process the math mentally like the preschoolers, which would have required additional working memory, the first grade students used their fingers to transform verbal information into a quantitative means to solve the math problems.

Rasmussen and Bisanz (2005) revealed the preschoolers did not have the necessary learned tools to translate verbal statements very easily; thus, the preschoolers not only struggled to answer the verbal math problems accurately, but also overloaded their working memory to work through additional mathematical processes. The more mathematical processes that occupied the working memory portion, the more difficulty the students exercised in accurate problem solving skills.

Another study by Tronsky (2005) investigated the development of strategy used in complex multiplication and related working memory effects with 23 undergraduates from Massachusetts University. Each participant used the Computer-Based Academic Assessment System that collected response times and accuracies with non-working and working memory tasks. The participants completed three 1-hour practice sessions. During the practice sessions, participants primarily chose the retrieval method as the primary method for problem solving.

For non-working memory tasks, participants performed simple multiplication, complex multiplication, and strategy assessment tasks. The participant's response stopped the computer's timing mechanism, while the scorer pressed a button for problem accuracy. For the strategy assessment task, participants provided verbal reports of the strategies that he or she used. The verbal description responses included the following: retrieval, decomposition (e.g., 3×17 into $3 \times 10 + 3 \times 7 = 51$), standard algorithm (mentally carried out the standard multi-digit multiplication algorithm), repeated addition (2×4 into $4 + 4 = 8$), 10s (multiplied the 10s column, then the ones column and added both products), or other. For working memory tasks, participants read the six-letter consonant string aloud at a rate of about two letters per second. After the participants finished reading the letters, the participants voiced the product of a multiplication problem, and then recalled the letters from the first screen in the same exact order.

The study revealed the participants at first loaded their working memory during the pre-practice session since a majority of the participants used a standard algorithm method or nonretrieval strategy. At post-practice, the participants primarily used the retrieval strategy which revealed no effect of work memory load. The participants also loaded their working memory with letter recall in the dual-task (verbally ordering the letters after a multiplication problem) rather than the single-task (only verbalizing the order of the letters) condition. Tronsky (2005) concluded from the results of the study that automaticity did not affect working memory; automaticity freed necessary working memory for other cognitive processes to occur.

Instructional Improvement Methods for Multiplication Fact Fluency

One of the basic strategies for increased multiplication fact recall from a study by Knowles (2010) emphasized systematic timed practice drills. Knowles implemented an 8-week long study that involved 227 regular education sixth grade math students from three teachers who instructed one of the three groups. One group received, on a daily basis, a 3-minute drill or intervention for 8 weeks. A second group received a 3-minute drill or intervention once a week for 8 weeks, while the third group received no interventions. All three groups took a pretest and posttest. The results revealed time practice drills statistically improved the rate with basic multiplication fact recall with sixth grade students.

A study by Pegg, Graham, and Bellert (2005) examined the effect of increased accuracy and automaticity in basic mathematics on more difficult mathematics questions with middle-school students, year five (11 years old) and year seven (13 years old), who had exhibited long-term poor performance in mathematics. The study involved an intervention program called *QuickSmart*. The theory of *QuickSmart* came from the Assessment and Training of Academic Skills at the University of Massachusetts along with some related work from the National Centre of Science and Mathematics Education for Rural and Regional Australia at the University of New England in Armidale, Australia. The *QuickSmar*t interventions emphasized strategies for students to move away from slow, counting strategies to more efficient automatic recall strategies.

A total of 12 students, six boys and six girls, participated in the study. The *QuickSmart* program ran 26 weeks for year-five students and 24 weeks for year-seven students. All the students worked with five 30-minute lessons over a two-week period with the same instructor, in pairs who had similar instructional needs. Each lesson from *QuickSmart* usually involved four components. With the first component, students began with a review of the previous lesson. In the second component, the teacher guided a discussion about the relationships between the number facts that often involved highly focused games for the students to practice as a motivational way to learn the facts. With the third component, students took timed performance activities to strengthen their memory and retrieval strategies. With the fourth component, students not only practiced on selected worksheets that closely related the recent content of study but used a small computer-based academic assessment (Pegg et al., 2005).

Year-five students focused on addition, while year-seven students focused on multiplication. The results of the study revealed year-five students improved their ability to answer addition sums from an average of 5.2 seconds to an average time of 1.7

seconds and year-seven students improved their ability to answer multiplication problems from an average time of 2.6 seconds to an average time of 1.15 seconds. After the *QuickSmart* interventions, both year-five and year-seven students also improved their standardized mathematics test scores. The students within the study who utilized an increased-speed to recall the basic math facts also revealed an improvement with more difficult mathematics tasks (Pegg et al., 2005).

Dr. Crawford of Otter Creek Institute acknowledged the importance of fluency or automaticity of the basic math facts. “An essential component of automaticity with math facts is that the answer must come by means of direct retrieval, rather than following a procedure” (Crawford, 2003, p. 10). To acquire math fact automaticity, Dr. Crawford authored, *Mastering Math Facts*; the program entailed a three-stage instructional practice for the fluency, or automaticity, of the basic math facts.

The first stage emphasized the importance of a “conceptual” or “procedural” understanding of the math facts. The second stage characterized the development of specific instructional strategies, rules, procedures, and relationship activities of the numbers for accuracy, rather than speed. The third stage emphasized the development of automaticity or mastery of the math facts rather than the strategies learned from stage one or two; students developed the capacity of fast recall or direct retrieval for the answer. Students achieved automaticity if he or she verbally responded within one second, or completed within the range of 30 to 40 problems per minute in writing (Crawford, 2003).

Two separate studies, Lehner (2008) and Hastings (2010), researched the relationship between Fluency and Automaticity through Systematic Teaching with Technology (FASTTT Math), a software program by Tom Synder Productions as a possible means with the recall improvement for basic mathematics computational skills. Although Lehner (2008) only used FASTT Math, Hastings (2010) also incorporated Dr. Crawford's intervention program, *Mastering Math Facts*.

With Lehner's (2008) study, 25 fifth-grade students participated daily with a 10-minute *FASTT Math* lesson over a period of 44 days with multiplication facts 0-12. *FASTT Math* performance ratings graded the students' scores as fluent, near fluent, developing, and underperforming. Prior to any instructional practices, 100% of the total participants achieved an underperforming score with *FASTT Math*. The results of the study revealed 8% achieved near fluent, 72% achieved developing, and 20% remained underperforming. Although no one achieved fluent status, 80% of all the students showed progress or improvement from an underperforming score.

Hastings (2010) conducted a similar Lenher's (2008) study, but also incorporated Dr. Crawford's *Mastering Math Facts* intervention instructional strategies. Hastings incorporated within the study the third component of *Mastering Math Facts* - the development of automaticity or mastery of the math facts along with *FASTT Math*. The instructional process of the third component with *Mastering Math Facts* required the students to be in pairs and ended with a quick individual 1-minute timed test. Eight "at-risk" fourth-grade students participated with this study. Of the eight, half of them qualified for special education in the area of reading while two additional students qualified for special education testing. The students participated in a two-part instructional strategy: a) *FASTT Math* and b) *Mastering Math Facts*.

With *FASTT Math*, students worked for about 15 minutes on computers. Afterward, students paired up and obtained their individual folder for oral practice. New student pairings changed every week. Each folder had a rocket picture on one side and bar graph where the students charted their weekly results and goals. Each folder contained 23 sets of facts, labeled A-W. Each page of facts was divided into a top and bottom section. The students orally practiced with the top section and the students took a timed-test with the bottom section. Within each folder, students had access to an answer booklet for every set of facts.

The time was set for 1 minute and 30 seconds; one student recited while the other student listened to and checked the answers on the answer page. If the practicing student ever missed or even hesitated, the checker interrupted and immediately provided the answer. The checker asked the practicing students to repeat the problem and the correct answer at least once and as high as two or three times. The checker had the practicing student back up two problems, and then, the practicing student started over from there. After 90 seconds, the students switched roles. At the end of oral practice, each student took a one-minute assessment with the bottom half of the worksheet as his/her quiz. After the students corrected their quiz with their answer booklets, students graphed the results of their assessment. Whenever an individual did not achieve their goal, the teacher instructed the students to take the page home for study, while the students who achieved his or her goal returned their sheet back into their folder. Hastings (2010) concluded multiplication fact fluency increased due to both instructional methods: *FASTT Math* and *Mastering Math Facts*.

While the previous studies outlined specific instructional interventions or procedures for multiplication fact automaticity, the study by Wong and Evans (2007) compared the effectiveness of multiplication fact recall improve between pencil and paper (PPI) and computer-based instruction (CBI) practice. Primary students in year-five classes at four inner-city primary schools in Sydney, Australia, participated in the study. Thirty-seven primary students composed the CBI group, while 27 primary students composed the PPI group. The number of multiplication facts answered correctly in one minute determined the participant's score for both groups. Both groups went through 11 practice sessions. Each session lasted 15 minutes for both groups. The researchers administered a pretest two days prior to the start of the practice sessions, post test two days after the completion of the practice sessions, and maintenance test approximately four-weeks after students finished the practice sessions.

The results of the study by Wong and Evans (2007) revealed the PPI group significantly performed better on the post-test than the CBI group. Although the PPI intervention revealed better results, the researchers wondered whether the writing practice provided a possible unfair advantage with the PPI group; the researchers took no action with this assumption. With the maintenance test, the CBI's mean multiplication score differed significantly – increased by 1.92 facts per minute, while the PPI's mean multiplication score did not differ significantly – decreased by 1.63 facts per minute. The study concluded the PPI intervention proved to be the more effective method for improving multiplication fact recall. The study overall concluded a systematic practice with both interventions of the basic multiplication facts proved to be an effective method.

Developing Mathematics Automaticity with Learning Disabled Students

A study by Hasselbring et al. (1988) researched whether learning disabled (LD) children in mathematics could achieve multiplication fact fluency. The results of Hasselbring et al. (1988) revealed learning disabled children in mathematics developed an automaticity with the basic math facts if the mathematics instruction deemed appropriate with six instructional principles. The first step determined the students' area or level of automaticity. The second step built on existing declarative knowledge: "an interrelated network of relationships containing basic problems and their answers" (p. 2). The third step incorporated only a small set of target facts for concept mastery. The fourth step used controlled response times – "amount of time allotted to retrieve and provide the answer to the fact" (p. 4) – that forced students to abandon strategies of procedural knowledge – methods (finger counting strategies) utilized by students to derive answers for math problems. The students' range of controlled response times occurred between 3 to 1.25 seconds. If the students answered incorrectly under the use of controlled response times, the instructor provided the answer along with the same math problem. The instructor repeated the answer and problem until the students correctly answered the problem. The fifth and final step interspersed automatized math facts with target non-automatized math facts. When the students accurately retrieved the answers to the selected learned math problems or facts, the students practiced these facts with a computer-based drill and practiced until the students retrieved the facts from memory with ease. The research from Hasselbring et al. (1988) concluded a combination of recall training and drill (computerized) provided the necessary instructional mechanisms for developing automaticity with learning disabled students in mathematics.

Although the study by Poncy, Skinner, and Jaspers (2007) used an entirely different instructional strategy, like in Hasselbring et al. (1988), the researchers also used a small set of target facts for each instructional period. The study by Poncy et al. (2007) worked with one 10-year old female learning disabled student who had a full scale IQ of

44. The study researched instructional interventions: a) cover, copy, and compare (CCC) and b) taped-problems (TP) from an audiotape for math fact accuracy and fluency. One of the instructional interventions, CCC and TP, occurred in the morning, while the other occurred in the afternoon. Within each instructional intervention, the participant worked on four problems.

Under the CCC intervention, the participant worked off set of targeted math problems or facts, one at a time. The participant studied the problem and answered on the left side of the page. The participant, then, covered the problem and answered on the left, and from memory, the participant wrote the problem and answered on the right side of the page. The participant uncovered and evaluated her response. If the problem was correct, the participant verbalized the answer three times. If the response was incorrect, the instructor pointed to the model and stated the problem and correct answer; the instructor also instructed the participant to record the problem with the corrected answer (Poncy et al., 2007).

Under the TP intervention, the participant listened to a tape that corresponded with the problems on the worksheet. A 4-second delay occurred between the stated problem and answer. The participant attempted to write down the correct response before the tape verbalized the answer. If the participant incorrectly responded, the instructor paused the tape and allowed the participant to correct the response. The participant practiced six times with each math problem. Even though the participant increased in accuracy and fluency under both interventions, the participant spent 30% less time on the TP, rather than the CCC problems; thus, the TP intervention proved the superior intervention (Poncy et al., 2007).

Although the study was similar to the CCC/TP study by Poncy et al. (2007) and McCallum, Skinner, Turner, and Saecker (2006) of the University of Tennessee, Knoxville researched the TP intervention with multiplication facts 2-9 instead of single-digit addition facts. The participants involved 18 students (11 Caucasian, five African American, and two Hispanic) who were either eight or nine years old. Ten students were male and eight students were female. Each session consisted of three different sets. Each set underwent four different series, as each session lasted approximately 12 to 15 minutes. The results of the study revealed an increase in the digits correct per minute for each assessment with the TP procedure. The students also reported the TP intervention as an acceptable instructional practice to learn and improve their speed and accuracy with the multiplication facts.

A study by Woodward (2006) examined and compared the impact of an integrated instructional approach (conceptual strategies along with timed-practice drills) with just a timed-practice approach with teaching multiplication facts. The participants involved fourth-grade students with and without a learning disability. Thirty students created the intervention group (integrated instructional approach), which included eight students with a LD in mathematics. Twenty-eight students created the comparison group (timed-practice drills only), which included seven students with LD in mathematics. Although the students from both groups received 25 minutes of instruction daily, every day of the week, for four consecutive weeks, students from the integrated group included relationships between facts and extended facts with arrays and number lines, and approximation skills with number lines. The activities with the integrated group reinforced the role of facts and extended facts in number-sense tasks; whereas, the timed-practice group worked on traditional multiplication algorithm worksheets. The timed-practice group never worked outside the instructional components of computational problems instructed by the teacher.

The results of the study revealed statistical gains in scores from pretest to posttest. The non-learning students yielded better assessment on performances of the facts than the LD students within each group. A significant difference between each group occurred with the "Extended Facts and Approximation" assessment. Since the integrated students had the opportunity to see and discuss connections between the basic facts, extended facts, partial product algorithms and methods for approximations, the integrated group outperformed the timed-practice group. The same idea applied with the computation test. Although the results of the computation test yielded no significant difference between both groups, a noticeable mean percent difference occurred and favored the timed-practice group over the integrated group because the timed-practice group had additional or more focused practice in this specific area. Both methods of instruction were comparably successful and effective for multiplication automaticity (Woodward, 2006).

Mastering Multiplication Facts with a Conceptual Instructional Purpose

The 2000 Standards with NCTM defined multiplication mastery, or fluency that included a flexible approach and understanding of the concepts, rather than rote memorization of the facts. Wallace and Gurganus (2005) agreed with the principles of the 2000 Standards and outlined the following instructional practices as the most effective sequence towards multiplication fact fluency. The first step introduced multiplication through realistic problems that involved manipulatives with repeated addition. The second step not only promoted students' drawings, including boxes or circles with tallies that represented the number of groups and how many in each group, but also an understanding of certain properties - commutative, zero, identity, along with the distributive (French, 2005). The third and final step involved a more deepened understanding of the facts through additional concrete and representational experiences. The third and final step only occurred after the students achieved or developed a quick recall of the multiplication facts through a continued drill and practice instructional techniques. Students with an increased speed and accuracy of the multiplication facts not only improve their attitude, but also their overall mathematics experience (Wallace & Gurganus, 2005).

A multiplication fact instructional strategy implemented by Caron (2007) changed the attitudes and confidence of eighth-grade students enrolled in his math class. Caron (2007) utilized an innovative instructional technique of rote-memorization for multiplication fact fluency. The students of Caron had none to little knowledge of the multiplication table. Caron's innovative instructional method used practice problem worksheets that included examples of each problem and its product at the very top of the paper. Each practice problem worksheet focused on one factor at a time. Each student worked through the problems and looked at the top if they needed any assistance to complete the worksheet accurately and successfully. Although the students resisted at first, they quickly realized multiplication retention was possible. This motivated the students to practice with other practice problem sheets two to three times a day. The results of Caron's (2007) innovative instructional intervention quickly provided an efficient and fun learning opportunity that caused a change in the student's self-perception and attitude for future mathematics progress and achievement.

A deeper understanding of the number sense of the multiplication facts not only provided stronger mathematics confidence, but also produced the ability for students to generate ideas to solve new problems. A study by Williamson (2007) examined the effects between conceptual instruction and rote memorization of the multiplication table. The participants included 32 pupils split into two groups: group A and group B. Group A, through adapted mental and discussion-oriented math lessons and group B, through rote memory, learned the following multiplication factors: 2, 3, 4, 5, and 10. Although both groups received the same amount of instructional time, group A covered the six-factor as well.

The results of the study revealed from a 50 rapid, recall question assessment, the students from group A answered 48 correctly, while the students in group B answered only 31 correctly with more frequent and much longer hesitations. While both groups never learned the 7, 8, and 9 factors of the multiplication table, group A worked out the factors successfully and confidently through the mathematics connections and understanding with the other learned factors; whereas, group B quite quickly complained and stopped without any effort applied with the 7, 8 and 9 factors. The study by Williamson (2007) emphasized the importance of the process through a conceptual understanding, rather than just straight memorization of the facts, allowed group “A” a more willing effort to solve new problems.

Conceptual/Procedural Instructional Blend for Computational Fluency

Students instructed purely conceptually through discovery methods do not efficiently learn the rule or principle. Students responded better if enough guidance supported the learning process as well (Mayer, 2004). “Students need enough freedom to become cognitively active in the process of sense making, and students need enough guidance so that their cognitive activity results in the construction of useful knowledge" (Mayer, 2004, p. 16). A conceptual instructional approach through investigations and questions provided needed necessary guided structure and support for mathematic understanding.

Kotsopoulos (2007), a mathematics instructor at the University of Western Ontario, Canada, realized student difficulty with basic multiplication table fact retrieval when students factored simple quadratics. Through her own literature research and experience, Kotsopoulos (2007) suggested a need for a blending of procedural and conceptual instructional practices for a deeper mathematical understanding of the concepts. Both hands-on activities and guided instruction illustrated aspects of Piaget's theory of cognitive development for student understanding (Ojose, 2008) which some researchers addressed to be a strong rationale for building automaticity with mathematics learning (Bratina & Krudwig, 2004).

Piaget's theory identified four primary stages of cognitive development: sensorimotor, preoperational, concrete operational, and formal operational (Ojose, 2008). The sensorimotor stage characterized the child's development of counting and a conceptual understanding of numbers as it relates to the total number of objects. The preoperational stage characterized student engagement with beginning problem-solving tasks that might include dimensions of objects. The concrete operations stage characterized basic skills acceleration; usage of mathematics manipulatives produced a visual, concrete foundation for students to develop meaningful conceptual understanding of the mathematics concepts. This stage of cognitive development also characterized the opportunity for students to develop accuracy of the mathematic concepts. Research identified accuracy of the mathematics facts or concepts as a necessary but critical step before the development and occurrence of basic-fact automaticity (Crawford, 2003). The development of basic-fact fluency perpetrated from a regular practice of the mathematics concepts. The last stage of cognitive development called the formal operations stage characterized the students' ability to construct and formulate mathematical ideas through clarification, inference, evaluation and application. Each step of the Piaget's cognitive development model constructed an explanation of the students' frames or stages of learning for the effectiveness to not only learn mathematics with accuracy, but also fluently and efficiently.

A study by Rittle-Johnson et al. (2001) researched how conceptual knowledge, procedural knowledge, and correct problem representation related to one another along with the overall effectiveness on students' mathematics learning. Rittle-Johnson et al. (2001) defined conceptual knowledge as generalized knowledge; knowledge not tied down or hindered to a specific problem type, whereas procedural knowledge addressed specific problem types without much generalization. The study involved two experiments.

The first experiment of the Rittle-Johnson et al. (2001) study hypothesized conceptual knowledge of decimal fractions at the pretest predicted an increased gain in procedural knowledge from pretest to posttest, while the increased gain in procedural knowledge predicted an increased gain in conceptual knowledge from pretest to post test. Of the 74 fifth-grade students (33 girls and 41 boys) who participated with the first experiment, the researchers excluded 25 students due to high pretest scores.

In the first experiment, the students worked with decimal fractions on the pretest, classroom interventions, posttest and transfer assessment. Each assessment had specific questions that signified conceptual and procedural knowledge. While the pretest, posttest and transfer assessment involved paper-and-pencil responses, the problem-solving interventions involved a computerized game, “Catch the Monster”, that was developed by the researchers of the study. Participants worked on three different tasks for the pretest/posttest assessments and intervention phase. One task required students to mark the position of a decimal fraction on a number line from zero to one that included tenths-mark. Another task required the students to mark the position of a decimal fraction for a given position on a number line from zero to one that did not have the tenths-mark. The third task required the students to choose the decimal fraction for a given position on a number line from 0 to 1 that did not have the tenths-mark. For the transfer assessment, students marked positions of a pair of decimal fractions on a line from zero to one that did not have the tenths-mark and a pair of decimal fractions that was greater than one on a number line from zero to 10 with only the end-points marked. The results of the first experiment revealed conceptual knowledge predicted gains in procedural knowledge, while gains in procedural knowledge predicted further improvements with conceptual knowledge (Rittle-Johnson et al., 2001).

The second experiment researched the causal evidence for the relation between correct problem representation and the development of procedural knowledge. One manipulation from experiment one involved prompts for the students to notice the tenths digit of the target numbers and the second manipulation from experiment one involved presenting number lines with 10 equal sections. The researchers predicted that both manipulations provided improved problem representation which, then, improved the students' procedural knowledge. Of the 59 fifth graders (33 girls and 26 boys) and 58 sixth graders (28 girls and 30 boys) who participated in the second experiment, the researchers excluded two fifth graders and seven sixth graders due to high pretest scores (Rittle-Johnson et al., 2001).

In the second experiment, students first completed the conceptual and procedural knowledge pretest, two new measures of problem representation, and a mathematics motivation assessment. Researchers randomly assigned the participants to one of four intervention conditions: a) prompts to notice the first digit along with number lines marked with 10 sections, b) prompts only, c) marked number lines only, or d) no prompts or marked number lines. After the participants completed the 40-minute intervention phase, the participants took the procedural knowledge post test, conceptual knowledge posttest and a transfer test. The results of experiment two revealed both forms of problem representational support (prompts and number lines marked with 10 sections) increased the students' procedural knowledge. Both forms of problem representational support also increased the conceptual knowledge of students who started off low. Correct problem representation (prompts and number lines marked with 10 sections) created a strong link between increased gains in conceptual and procedural knowledge (Rittle-Johnson et al., 2001).

The study by Rittle-Johnson et al. (2001) not only concluded both forms of conceptual and procedural knowledge developed a "hand-over-hand" process, but also specific problem representations increased both forms of conceptual and procedural knowledge. Both processes of conceptual and procedural knowledge formed bidirectional support: improved procedural knowledge led to improved conceptual knowledge and vice-versa. The study by Rittle-Johnson et al. (2001) concluded the inculcation of both types of knowledge along with improved problem representations improved student learning.

Self-Assessment Strategies for Improved Computational Fluency

Bratina and Krudwig (2003) researched and rationalized the importance to move students from mathematical accuracy of the basic math facts and other mathematics formulas to automaticity. One of the rationales for building math fact fluency involved increased motivation and self-esteem through self-assessment strategies. Caron's (2007) innovative, multiplication fact learning intervention provided the necessary increased motivation and self-esteem for higher level mathematics learning. Bratina and Krudwig (2003) emphasized self-monitoring; self-correction allowed students to avoid faulty-thinking and self-graphing provided the necessary and "relevant real-world link between academic material and students' own, measurable self-improvement" (p. 60).

A study of student self-assessment by Brookhart, Andolina, Zuza, and Furman (2004) determined whether self-assessment strategies provided a positive experience for an increased motivation in mathematics learning. Brookhart et al. (2004) determined third-grade students developed a strong retention of the multiplication facts, even through rote memory lessons, if teachers involved the students with their own assessment. The participants of the study involved two classes of third-grade students (20 and 21). Participants took 5-minute multiplication fact tests (0-9 tables) once a week for 10 weeks. After each week's multiplication fact test, the participants performed the following three tasks: a) graphed their actual score, b) graphed their predicted score for next week and filled out their Goals, Plan, Action, Reflection (GPAR) sheet.

The participants enjoyed participating in the self-assessments and seeing their progress. One teacher reported this year's third-grade students learned the multiplication tables better this year than with any previous year. The results of the study revealed increased growth with the multiplication facts along with student determination and enjoyment. "This study suggests that student involvement in their own assessment can, indeed, add reflection and metacognition ('thinking about thinking') to rote memory lessons like learning the multiplication tables" (Brookhart et al., 2004, p. 225). Whenever students viewed their assessment results and consistently charted their results, student retention and achievement of multiplication facts excelled (Bratina & Krudwig, 2003).

Relationship between Fluency and Higher level Mathematics Learning

A 2005 study by Lin and Kubina, Jr. analyzed four variables: component skill fluency, component skill accuracy, composite skill fluency, and composite skill accuracy. The participants of this study involved 157 fifth graders. The study defined components as basic or foundational skills and composites as more complex problems. The number of correct digits per minute, regardless of the total number of completed digits and errors, determined the fluency variable. Students achieved component skill fluency for 80 to 120 correct digits per minute. Students achieved composite skill fluency for 40 to 60 correct digits per minute. The percentage of correct digits and total completed digits determined the accuracy variable. Both component skill and composite skill accuracy required students to be 100% correct.

The study by Lin and Kubina (2005) revealed a strong correlation between component and composite fluency; "students trying to become fluent with the composite skill, such as multi-digit multiplication, will encounter a more demanding task if they are not fluent with the correct algorithm for solving the problem in the first place" (Lin & Kubina, Jr., 2005, p. 82). Students who struggled with component fluency (quickly and accurately multiplying single-digits) also struggled with composite fluency (quickly and accurately multiplying multi-digits). Lin and Kubina, Jr. (2005) concluded basic math fact fluency, rather than just accuracy, to be a possible "alternative solution for cumulative mathematical deficits" (p. 85). The study revealed an automaticity of the facts rather than just an accuracy of the facts yielded better achievement results for higher level mathematics learning.

Reading and Basic Mathematics Computation with Problem Solving

The 1989 Standards from NCTM defined and established a goal for students to become mathematical problem solvers. With the 1989 Standards, NCTM emphasized problem solving as an essential instructional focus with mathematics instruction (NCTM, 1989a). Problem solving not only reflected the students' reading ability, but also their mathematics ability. Fite's (2002) literature review between reading and math allowed Fite to conclude that a presumable difference existed between reading running text that involved narratives and reading math problems. "The math teacher is a reading teacher…a reading teacher that teaches the student to read math" (Fite, 2002, p. 9). Solving math word problems depended on the student's ability to think mathematically: a focus on seeking solutions, not just memorizing; exploring patterns, not just memorizing; and predicting and evaluating answers, not just doing math exercises without checking or understanding the solution (Fite, 2002). A translation of math word problems-pictorials, reworded statements or phrases, outline or table summaries– provided a better success for math word problems. "Success with math problems requires both reading for comprehension and computational skills" (Fite, 2002, p. 10). Both reading fluency skills and math skills played an important role for students to not only process, but also translate verbal language within math word problems into symbolic mathematical expressions. The ability to translate effectively the reading and math symbolism with math word problems required students to not only achieve reading, but also a math fluency: algorithmic, procedural, and conceptual.

Fuchs, Fuchs, and Prentice (2004) performed a study about the responsiveness of learning disabled and non-disabled students to mathematical problem-solving instruction. The researchers of this study used the *TerraNova* state assessment (an assessment developed in 1997 by CTB/McGraw-Hill of different subject areas for all different grade levels). Based off of the *TerraNova* state assessment scores, the researchers formed four different third grade student groups: NDR (no disability risk), MDR/RDR (with a combination of mathematics and reading disabilities), MDR only (mathematics disability risk), and RDR only (reading disability risk) created the participants of the study. The four groups were created from the students percentile score in computation and reading comprehension. The study revealed MDR/RDR, MDR-only, and RDR-only improved less than the NDR students on computation and labeling (answers that included words, mathematical symbols, money signs or brief explanations), and the MDR/RDR improved less than all the other groups in mathematics understanding. The study concluded that students with MDR only, RDR only, and MDR/RDR required additional supplementary instruction

An exploratory analysis investigated whether the scores of the MDR/RDR group was due to mathematics computation or to reading comprehension difficulties. The result of a two hierarchical regression analyses between computation and reading comprehension revealed when the computation scores were entered after the reading comprehension scores, the unique variance substantially increased from 1.5% to 21.0% on the "immediate transfer" pre and posttest and 0% to a 13.4% on the "near transfer" pre and posttest. The results of the exploratory analysis allowed the researchers of this study to hypothesize the following: "that mathematics difficulties (or the underlying deficits associated with mathematics difficulties) may contribute more to mathematical problem-solving learning problems than do reading comprehension difficulties (or the underlying deficits associated with reading comprehension difficulties)" (Fuchs et .al., 2004, p. 305).

A study by Capraro and Joffrion (2006) investigated the ability to translate math word problems into appropriate mathematics symbols using conceptual or procedural indicators. Capraro and Joffrion proposed "mathematics students must possess conceptual understanding so that once the words have mathematical meanings they can accurately translate those words into mathematical symbols = linear equations" (p. 150). Both a procedural and vocabulary understanding created a mathematical conceptual understanding. The results of the study revealed vocabulary success attributed with students who had a stronger conceptual understanding that involved both procedural and vocabulary knowledge. "Reading in mathematics necessitates that one understand the meaning of the words" (Capraro & Joffrion, 2006, p. 162). Both reading fluency and a mathematics conceptual understanding provided the necessary ability to translate words from math word problems into the proper symbolic mathematics representation. The new and upcoming Common Core State Standards, CSSS, also reiterated mathematics proficiency resulted from a mathematics understanding of concepts rather than just a mathematics memorization of facts and formulas.

2010 Common Core State Standards

Both the National Governor's Association (NGA) and Council of Chief State School Officers (CCSSO) initiated and developed the new and revised the CCSS for mathematics and the English language. As of June 15, 2010, 48 states accepted the CCSS as the new national standards for English language and mathematics. The CCSS standards addressed the necessary knowledge and skill requirements students have within the K-12 curriculum for not only high school graduation, but also the readiness to enroll in college credit classes or immediate employment that prepared the students for the workforce training programs. The CCSS focused on "mathematical understanding" defined as "the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from" (CCSSI, n.d., p. 4). While the 1989 Standards and 2000 Standards focused primarily on "mathematical understanding," the CCSS identified procedural skills to be equally important for mathematics problem solving.

Some of the five content standards (Numbers and Operations, Data and Probability, Algebra, Geometry, Measurement) from the 2000 Standards changed with the CCSS. The 2000 standards used the same five content standards from kindergarten to the eighth grade. The CCSS incorporated some content similarities and differences from kindergarten through the eighth grade. The CCSS identified these following five areas of mathematics importance in kindergarten: counting and cardinality, operations and algebraic thinking, number and operations in base 10, measurement and data, and geometry. With Grade 1 through Grade 5, the CCSS identified these following five areas of mathematics importance: operations and algebraic thinking, number and operations in base 10, number and operations with fractions, measurement and data, and geometry. With Grade 6 through Grade 8, the CCSS identified these following five areas of mathematics importance: rations and proportional relationships, the number system, expressions and equations, geometry, and statistics and probability.

The CCSS for Mathematics Practices not only adopted the process standards (problem solving, reasoning and proof, connections, communication, and representation) from the 2000 Standards but also the five strands characterized for mathematics proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition outlined in the book published by the National Academy Press, *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001). Kilpatrick et al. (2001) defined conceptual understanding as the student's level of "math comprehension of concepts, operations, and relations" (p. 5). Kilpatrick et al. (2001) defined procedural fluency as the student's "skill to carry out mathematics procedures flexibly, accurately, efficiently, and appropriately" (p. 5). Kilpatrick et al. (2001) defined strategic competence as the student's "ability to formulate, represent, and solve mathematical problems" (p. 5). Kilpatrick et al. (2001) defined adaptive reasoning as the student's "capacity for logical thought, reflection, explanation, and justification" (p. 5). Kilpatrick et al. (2001) defined productive disposition as the student's "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 5). Kilpatrick et al. (2001) illustrated the five strands for mathematics proficiency as "interwoven and interdependent" (p. 5). Through the five strands of learning strategies, students developed the necessary skills for mathematical proficiency with the concepts.

The CCSS for Mathematics Practices interlinked mathematical processes with mathematics proficiency. The CCSS created and described the following instruction and learning practices as essential within the mathematics curriculum. The first practice required students learning to "make sense of problems and persevere in solving them" (CCSSI, n.d., p. 6); students developed the ability to explain equations, verbal descriptions, tables, graphs mathematically as well as question their understanding through the mathematical process. The second practice required students learning to "reason abstractly and quantitatively" (CCSSI, n.d., p. 6); students developed a "quantitative reasoning" skill - an understanding of the quantities and their relationships within the problem. The third practice required students to "construct viable arguments and critique the reasoning of others" (CCSSI, n.d., p. 6); students developed the ability to construct arguments and critique other students' reasoning from previously established and stated information. The fourth practice required students to "model with mathematics" (CCSSI, n.d., p. 7); students developed the ability to apply mathematics to everyday life, society and workplace, along with the ability to simplify complicated math problems with appropriate generalizations for better understanding and grasp of the problem. The fifth practice required students to "use appropriate tools strategically" (CCSSI, n.d., p. 7); students developed a recognition of other resources (websites or other technological tools) to help solve problems. The sixth practice required students to "attend to precision" (CCSSI, n.d., p. 7), students developed the ability to communicate mathematical concepts clearly and accurately to others. The seventh practice required students to "look for and make use of structure" (CCSSI, n.d., p. 8); students developed the ability to identify patterns or breakdown existing mathematical complex structures or problems into simpler steps or generalizations. The final and last practice required students to "look for and express regularity in repeated reasoning" (CCSSI, n.d., p. 8); students developed the ability to identify other methods or "short-cuts," to solve the problem as well as understand and evaluate the process of their work with the problem. Each of the eight mathematical practices represented important developmental goals for students within their grade-level mathematical content standards for mathematics proficiency.

Another subtle difference between the 2000 Standards and the CCSS reflected the development of whole number operations. Although both standards argued for mathematics understanding with whole number operations, the standards somewhat disagreed on the purpose with algorithms. Kilpatrick et al. (2001) defined algorithms as a set of procedures to solve a variety of problems. The 2000 standards observed "algorithms as tools for solving problems rather than as the goal of mathematics study" (NCTM, 2000e, p. 1); whereas, Kilpatrick et al. (2001) iterated the following for mathematics proficiency with the CSSS: "learning to use algorithms for computation with multi-digit numbers is an important part of developing mathematical proficiency" (p. 7). The 1989 Standards, 2000 Standards and now, the 2010 CSS standards all emphasized the importance of mathematics understanding through a variety of instructional strategies that involved active learners with problem solving.

Conclusion

The mathematics standards have undergone a number of paradigm shifts over the past 50 years. Although individuals discussed new mathematics standards after World War II, nothing truly resulted until the Russians launched *Sputnik* in 1957. Beberman's Illinois experiments, American Mathematical Society, and School Mathematics Study Group (SMSG) wrote new math textbooks, enrichment materials, teachers' guides, etc. to improve the U.S. mathematics educational system, which later became known as the New Math reformation.

The New Math standards focused on logic, proofs, properties, and symbolic notations. After implementing with the New Math standards from 10 to 15 years, educators went "back to the basics" where the curriculum focused on more computation skills and procedures. The publication of *A Nation at Risk* in 1983 by the National Commission on Excellence in Education strongly pushed for another education reform that reflected a more constructivist theory of curriculum and instruction and less instructional drills on computation skills and procedures (US DOE, 1983). The publication of A *Nation at Risk* led to new mathematics standards commonly known as the 1989 Standards.

The 1989 Standards encouraged a conceptual model of thinking and understanding through investigations and discussions rather than a traditional instructional model of teaching. The 1989 Standards desired less "skill and drill" procedural practices and more problem solving through a cooperative learning effort.

After a few years of the new implemented 1989 Standards, a number of university mathematicians, parents and other educators disliked the “investigational” practices of instruction that especially involved the basic four operations: addition, subtraction, multiplication, and division. This ultimately led to the “math wars.”

The math wars resulted from the changes brought by the 1989 Standards and continued with the implementation of the 2000 Standards. The mathematics’ curriculum needed to include some of the traditional instructional procedures of basic fact computations and rules rather than overlooked with only conceptual instructional practices. During the last five to 10 years, researchers like Bratina and Krudwig (2003); Ross, 1997; Wu, 1999; Russell, 2000; and Rittle-Johnson et al. (2001) supported algorithmic instruction like the multiplication fact fluency as an important mathematical blueprint for conceptual understanding and higher level mathematics learning; the mathematics curriculum needed a balance with both instructional classroom practices, conceptual and traditional.

The design and implementation of the No Child Left Behind became a law in 2001 to not only improve student achievement, especially with reading and mathematics, and close achievement gaps but also to hold school districts across the nation accountable to meet annual performance targets: commonly understood as Adequate Yearly Progress. The No Child Left Behind law required 100% proficiency descriptor for all students in reading and mathematics by the year 2014. In order to meet the demands of the No Child Left Behind, states and school districts have adopted and implemented the latest standards and instructional practices into policy. The states adopted the CCSS as the new standards for mathematics curriculum, instruction and assessment.

Although the CCSS adopted the instructional principles of the 1989 and 2000 Standards, the standards also changed some grade level expectations for mathematics proficiency. Students are required to have multiplication fact fluency by the end of third grade.

Chapter 2 illustrated not only the importance of mathematics computations with the basic operations (addition, subtraction, multiplication, and division) for number sense and understanding but also as a balanced conceptual and traditional instructional approach for the development of mathematics understanding and proficiency. With the growing demands from the No Child Left Behind, for all students to have mathematics proficiency, does multiplication fact fluency become a necessary tool to learn and understand higher level mathematics concepts?

Chapter 3 outlines the methodology, instrumentation, populations and samples used in this quantitative study for not only the investigation of a possible relationship between the multiplication fact speed-score and higher level mathematics learning, but also investigation of the GMRT grade equivalency score of eighth grade students in Algebra I and Pre-Algebra. This quantitative study investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and a GMRT equivalency grade at eighth grade or higher.

Chapter 3 – Research Methodology

Overview

A number of studies have revealed the importance of multiplication fact fluency and higher level math achievement (Caron, 2007; Cavanagh, 2008; Wong & Evans, 2007; Loveless & Coughlan, 2004; Robinson, 2009; Wu, 1999). This quantitative study investigated a possible relationship between the multiplication fact speed-score and higher level mathematics learning and a) eighth grade combined average math score of first and second semester assessment scores, b) eighth grade 2011 mathematic MAP test scale scores, and c) with the student's Gates-MacGinitie Reading Test (GMRT) grade equivalency for a population of eighth grade students in Algebra I and Pre-Algebra. This quantitative study also investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and a GMRT equivalency grade at eighth grade or higher.

Both reading fluency skills and math skills played an important role for students to not only process but also to translate verbal language within math word problems into symbolic mathematical expressions. "Success with math problems requires both reading for comprehension and computational skills" (Fite, 2002). The ability to translate effectively the reading and math symbolism in word problems required students to not only achieve appropriate reading levels, but also to achieve algorithmic, procedural and conceptual math fluency. This researcher investigated the potential relationship between multiplication fact fluency and math achievement, and also investigated a possible relationship between multiplication fact fluency and reading achievement, as measured by the GMRT grade equivalency. Chapter 3 outlines the demographics of the middle school, methodology, and instrumentation used in this study.

Demographics of Middle School of Study

Table 6 summarizes the percentage breakdown of the students' racial backgrounds. The White/non-Hispanic population group averaged the last five years as the largest ethnic category of attendance in the school. The percentage of students on free/reduced-price lunch rose slightly more than 6% between 2007 and 2011.

Table 6

Missouri Middle School Student Racial Profile Breakdown

Missouri Middle School					
Year	2007	2008	2009	2010	2011
Total Enrollment	776	689	707	705	670
	Percent	Percent	Percent	Percent	Percent
Asian	2.3	2.0	1.7	1.8	0.7
Black	6.3	5.4	6.9	7.8	6.3
Hispanic	1.3	1.3	1.1	1.3	1.5
Indian	0	.1	0	0	0
White	90.1	91.1	90.2	89.1	90.9
Free/Reduced-Price Lunch	12.1	10.0	14.6	18.3	18.2

Note. Adapted from Missouri Department of Elementary and Secondary Education (MO DESE, 2011e).

Table 7 summarizes the percentage of students who achieved a proficient or advanced score for the past five years of the mathematics MAP test. This school of study generally increased the percentage of proficient students from year-to-year.

Table 7

2011 Missouri Middle School Student Racial Mathematics MAP Proficient Percentages

Missouri Middle School					
Year	2007 Percent	2008 Percent	2009 Percent	2010 Percent	2011 Percent
Annual Performance Target	35.8	45.0	54.1	63.3	72.5
All School	49.6	57.7	62.6	68.4	69.4
Asian	46.2	50.0	81.8	68.8	72.7
Black	23.3	40.6	33.3	46.2	51.4
Hispanic	44.4	37.5	50.0	77.8	66.7
White	51.6	59.2	64.7	70.1	70.5
Multi-Racial	n/a	n/a	n/a	n/a	100
Free/Reduced-Price Lunch	25.6	37.3	38.8	47.2	49.5
IEP	23.8	21.0	27.4	21.7	20.4
LEP	14.3	0.0	50.0	50.0	25.0

Note. Adapted from Missouri Department of Elementary and Secondary Education (MO DESE, 2011f).

Although the district mathematics MAP performance missed the annual performance target in 2011, this school of study received an Annual Proficiency Target

Met. Students with an individualized education plan (IEP), and limited English proficiency (LEP), revealed no improvement from the previous 2010 school year which caused this school of study to not meet Adequate Yearly Progress (AYP) for all five sub-groups.

The obtainment of Adequate Yearly Progress required this school of study to have successfully achieved the target in all five subgroups – school total, race/ethnicity, LEP, IEP, and Free/Reduced. The mathematics department in this school of study in 2011 met four out of the five sub-groups defined under the state's AYP requirements. This school of study did not meet the IEP sub-group.

This school of study exceeded its attendance goal of 95.1% for the 2010-11 school-year by 0.2 percentage points. In order for the school to meet this additional indicator target, the school of study had to demonstrate an attendance rate of at least 93% or an improvement from the previous year. Table 8 summarizes the attendance percentage between the state and this school of study. The state's overall attendance percent remained somewhat flat for the last three years, while this school of study demonstrated an increase over the course of two years, since 2009.

Table 8

Attendance Rate Comparison between the State and School of Study

Year	2007	2008	2009	2010	2011
Missouri	94.0	94.0	94.4	94.2	94.4
School of Study	94.8	94.5	94.3	94.7	95.3

Note. Adapted from Missouri Department of Elementary and Secondary Education (MO DESE, 2011g).

The student-to-classroom teacher ratio for the school of study revealed fewer than 20 students to one teacher. Table 9 summarizes the student/teacher ratio, student/classroom teacher ratio and student/administrator ratio. Over the past five years, the students to classroom teacher ratio maintained, on average, the same the past four years at 18 students per classroom teacher.

Table 9

Missouri Middle School Student/Staff Ratios

Missouri Middle School			
Year	Students per teacher	Students to Classroom Teacher	Students to Administrator
2007	15	20	259
2008	13	17	230
2009	14	18	236
2010	15	19	235
2011	14	18	223

Note. Adapted from Missouri Department of Elementary and Secondary Education (MO DESE, 2011h*)*.

State, School District, and Middle School of Study Mathematics MAP Achievement

Although the school of study did not make AYP in 2011, the school received an assessment of Non -Title I School Improvement Year 4, Delayed. Table 10 summarizes and compares the 2011 mathematics MAP descriptors and achievement level percentages for the state, district, and middle school. The Missouri school of study produced similar results within the school district except for Algebra I (A1). For all eighth grade students, who took the Algebra I EOC test at the Missouri school of study, achieved proficient or advanced on the Algebra I EOC test. Both the school district and school of study performed better on each mathematics MAP descriptor than the state average.

Table 10

2011 State/District/Middle School Mathematics MAP Achievement Percentages

Grade	Achievement Level	State %	District %	School Count	School Total	School %
6	Advanced	17.00	24.20	54	210	25.71
6	Proficient	40.52	43.47	99	210	47.14
6	Basic	35.04	28.83	51	210	24.29
6	Below Basic	7.44	<5%	*	210	*
7	Advanced	17.16	26.36	49	211	23.22
7	Proficient	39.22	41.15	87	211	41.23
7	Basic	33.22	25.62	56	211	26.54
7	Below Basic	10.41	6.87	19	211	9.00
8	Advanced	20.35	28.62	78	244	31.97
8	Proficient	31.15	34.91	83	244	34.02
8	Basic	33.66	29.59	67	244	27.46
8	Below Basic	14.84	6.88	16	244	6.56
A1	Advanced	19.91	30.81	58	70	82.86
A1	Proficient	39.91	44.99	12	70	17.14
A1	Basic	30.49	19.82	-	70	-
A1	Below Basic	9.69	<5%	-	70	-

Note. * Indicates cell contents suppressed to protect student confidentiality. Adapted from Missouri Department of Elementary and Secondary Education (MO DESE, 2011d).

Research Design

This researcher used a quantitative research design method for this study. This study investigated whether or not a relationship existed between multiplication fact recall and higher level mathematics learning. A correlational analysis was used for the first hypothesis to determine if the independent variable, multiplication fact speed-recall, shared a relationship with each individual dependent variable. This researcher used a Pearson Product Moment Correlation Coefficient (PPMCC) analysis for the first hypothesis. From this calculation this researcher examined whether a strong relationship, or pattern, existed between the multiplication fact speed-recall score and each of the following assessments: a) 2010-11 combined average score of first and second semester mathematics assessment score, b) 2011 mathematics MAP test scale score, c) 2011 Algebra I EOC raw score, and d) GMRT grade equivalency. Null hypothesis # 1 stated: There will be no relationship between the speed-recall score and 2010–11 combined first and second semester average mathematics assessment score, 2011 mathematics MAP test scale score, 2011 Algebra I EOC raw score, and GMRT grade equivalency. Creswell (2008) defined a correlation analysis design as "procedures in a quantitative research in which investigators measure the degree of association (or relation) between two or more variables using the statistical procedure of correlational analysis" (p. 60).

Null hypothesis #2 stated: There will be no difference in fluency scores and speed-recall scores when comparing Algebra student multiplication fact quizzes to Pre-Algebra student multiplication fact quizzes. This researcher decided the *t* test for the difference in means would be the appropriate statistical test. "A *t* test is used to test the difference between means when the two samples are independent and when the samples are taken from two normally or approximately normally distributed populations" (Bluman, 2008, p. 481). This researcher first performed an *F* test to determine whether there was an unequal or equal variance for the two samples compared, and then decided which *t* test to apply.

Null hypothesis #3 stated: There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved 80% or higher on the average of the first and second semester mathematics assessments. This researcher arbitrarily chose an 80% or higher average semester grade as a reflection of the district's proposed target semester grade of an 80% or higher for Algebra I students wishing to continue with honors math classes at the high school level. "The *z* test with some modifications can be used to test the equality of two proportions" (Bluman, 2008, p. 503); therefore, this researcher used a *z* test for difference in proportions to compare the proportions of students with multiplication fact fluency to the proportion of students who achieved 80% or higher on the average of the first and second semester mathematics assessments.

Null hypothesis #4 stated: There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved proficient or advanced on the mathematics MAP test. This researcher used a *z* test for difference in proportions to compare the proportion of students with multiplication fact fluency to the proportion of students who achieved proficient or advanced on the mathematics MAP test.

Null hypothesis #5 stated: There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved a

GMRT grade equivalency at eighth grade or above. This researcher used a *z* test for difference in proportions to compare the proportion of students with multiplication fact fluency to the proportion of students who achieved a GMRT grade equivalency at eighth grade or above.

Null hypothesis #6 stated: There will be no difference in the proportion of students without multiplication fact fluency and the proportion of students who did not achieve a proficient or advanced score on the mathematics MAP test. This researcher used a *z* test for difference in proportions to compare the proportion of students without multiplication fact fluency to the proportion of students who did not achieve proficient or an advanced score on the mathematics MAP test.

Null hypothesis #7 stated: There will be no difference in the proportion of students without multiplication fact fluency and the proportion of students who did not achieve a GMRT grade equivalency at eighth grade or above. This researcher used a *z* test for difference in proportions to compare the proportion of students without multiplication fact fluency to the proportion of students who did not achieve a GMRT grade equivalency at eighth grade or above.

Null hypothesis #8 stated: There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved a proficient or advanced score on the Algebra I EOC test. This researcher used a *z* test for difference in proportions to compare the proportion of students with multiplication fact fluency to the proportion of students who achieved a proficient or advanced score on the Algebra I EOC test.

Demographics of Participants in Study

The participants of this study were eighth grade students enrolled in Pre-Algebra or Algebra I at a Missouri middle school. There were two Algebra I and four Pre-Algebra classes. This researcher worked with participants in the following order: second period – Algebra I, third period – Algebra I, fourth period – Pre-Algebra, fifth period – Pre-Algebra, sixth period – off-period, seventh period – Pre-Algebra, and eighth period – Pre-Algebra.

The general enrollment practice for eighth grade Pre-Algebra and Algebra I primarily depended upon the student's seventh grade placement. Students enrolled in a basic mathematics seven class normally enrolled into eighth grade Pre-Algebra. The exception to this rule required the math seven student to achieve the following mathematics assessment scores: a) a first semester grade of an 80% or higher, b) a mathematics MAP proficient or advanced descriptor, and c) a minimum "Iowa mathematics aptitude test" of a 46 or higher. Seventh grade students enrolled in Pre-Algebra became enrolled into Algebra I if the students achieved a first semester mathematics grade of 85% or higher and a mathematics MAP proficient or advanced descriptor. A seventh grade Pre-Algebra student to be enrolled in Algebra I must have received a teacher recommendation.

The student population of this study included 116 students. Table 11 shows the population breakdown by gender, racial background, and free/reduced count per period. None of the students involved in this study had an individualized education plan (IEP) and the racial backgrounds involved only two: Black and White. Table 12 summarizes the gender, ethnicity, and free/reduced-price lunch population and sample of Algebra I students who participated in this study.

Table 11

Gender, Ethnicity, and Free/Reduced-Price Lunch Count: Population of Algebra I and Pre-Algebra Students Involved in Study

Period	2	3	4	5	6	7	8	Total
Girls	7	3	15	8	n/a	10	8	51
Boys	11	11	10	12	n/a	7	14	65
White	17	14	24	15	n/a	16	19	105
Black	1	0	1	5	n/a	1	3	11
F/R	1	0	9	6	n/a	3	3	22

Note. F/R refers to free/reduced-price lunch.

Table 12

Gender, Ethnicity, and Free/Reduced-Price Lunch Count: Population and Sample of Algebra I Students Involved in Study

	Algebra I Number	Population Percent	Algebra I Number	Sample Percent
Girls	10	31.3%	6	30%
Boys	22	68.7%	14	70%
White	31	96.9%	19	95%
Black	1	3.1%	1	5%
F/R	1	3.1%	1	5%

Note. F/R refers to free/reduced-price lunch.

This researcher worked with students in Algebra I during second and third periods of the school day. The student population in Algebra I totaled 32 students for this study. The student sample in Algebra I totaled 20 students for this study. The percentages of the gender, ethnicity, and free/reduced population are similar to the percentages of the sample size. The population and sample size produced a similar two to one ratio of boys to girls. The White sub-group primarily represented both the Algebra I population and sample size.

The student population in Pre-Algebra totaled 84 students, while the student sample in Pre-Algebra totaled 45 students. This researcher worked with students in Pre-Algebra during the fourth, fifth, seventh, and eighth periods of the day. Table 13 summarizes the gender, ethnicity, and free/reduced- price lunch population and sample of Pre-Algebra students who were involved in this study.

Table 13

Gender, Ethnicity, and Free/Reduced-Price Lunch Count: Population and Sample of Pre-Algebra Students Involved in Study

	Pre-Algebra Population		Pre-Algebra Sample	
	Number	Percent	Number	Percent
Girls	41	48.8%	20	44.4%
Boys	43	51.2%	25	55.6%
White	74	88.1%	37	82.2%
Black	10	11.9%	8	17.8%
F/R	21	25%	16	35.6%

Note. F/R refers to free/reduced-price lunch.

The White percentage, including the percentages of both genders, for the population and sample sizes produced similar results. The percentage sample size for both the Black and free/reduced sub-group was slightly higher than the percentage population size.

Dependent and Independent Variables

Creswell (2008) defined a dependent variable to be “an attribute or characteristic that is dependent on or influenced by the independent variable” (p. 126). This researcher used the following dependent variables for this study: a) each student’s combined average score of first and second semester mathematics assessments, b) 2011 mathematics MAP test scale score, c) 2011 Algebra I EOC raw score, and d) GMRT grade equivalency.

For the first dependent variable, this researcher used the average of first and second semester mathematics assessment scores as a representation of higher level mathematics learning. The questions for each assessment were aligned with the GLEs for students in Pre-Algebra and CLEs for students in Algebra I. Students were allowed to use calculators on all assessments, except for select skill-based topics. This researcher added each student’s assessment score from each quarter and divided by the number of assessments as the quantitative representation for the average first and second semester mathematics assessment score.

For the second dependent variable, this researcher used the 2011 mathematics MAP test as a representation of higher level mathematics learning. The mathematics MAP test was comprised of two types of questions: multiple choice and constructive response. Of the three parts, students had access to a calculator except for multiple-choice questions in part two. Students used a Texas Instrument (TI-73 Explorer) calculator. Part three included some constructive response questions for which the students showed their work and finalized the answer in writing. Students obtained both a scale score and achievement-level descriptor. This researcher used the scale score as the quantitative representation for the 2011 mathematics MAP test.

For the third dependent variable, this researcher used the 2011 Algebra I EOC raw score as a representation for higher level mathematics learning. This assessment applied to Algebra I students only, since Missouri mandated all students who completed Algebra I either in one or two years must complete an EOC test. Missouri implemented the Algebra I EOC test in the fall of 2008. The 2011 Algebra I EOC test was comprised of only multiple-choice questions that included a number of field-test questions. Answering the field-test questions correctly or incorrectly did not affect the students' Algebra I EOC raw scores. The state officials permitted, or allowed, students to use calculators on the entire test. Students used a Texas Instrument (TI-73 Explorer) calculator. Similar to the mathematics MAP test, students obtained a scale score and achievement-level descriptor. Students also obtained a raw score that indicated the number of correct responses out of a 35-point maximum score. This researcher used the raw score as the quantitative representation for the 2011 Algebra I EOC test.

For the fourth dependent variable, this researcher used the 2011 GMRT grade equivalency. The GMRT was comprised of two parts: vocabulary (word decoding) and comprehension. Reading fluency was achieved if the student obtained an assessment score of 8.7, or eighth grade, seventh month, or higher. The assessment score of an 8.7 became the minimum score for reading fluency since the students took the test seven months into the 2010-11 school year while enrolled in eighth grade. This researcher used the grade equivalency as the quantitative representation for the 2011 GMRT.

Creswell (2008) defined an independent variable to be “an attribute or characteristic that influences or affects an outcome or dependent variable” (p. 127). This researcher used the following independent variables for this study: a) multiplication fact speed-recall score and b) multiplication fact fluency score. This researcher calculated the multiplication fact speed-recall score from the number of correct products a student achieved out of 36 problems, with a time-limit for writing the products of 45 seconds. A student achieved multiplication fact fluency with 35 or 36 correct answers, out of 36 problems, within 1 minute and 48 seconds. This researcher allowed one mistake to occur to allow one point for carelessness rather than the lack of knowledge.

Implementation of Multiplication Fact Quizzes

The students took two multiplication fact quizzes during the late part of May 2011. One of the quizzes measured the students’ recall speed while the other quiz determined fluency. The students took each quiz on separate days with a one week break in between. This researcher used an electronic timer from the Smart Board software technology that counted down the total time: 45 seconds for the recall-speed test and 1 minute and 48 seconds for the fluency test. This researcher was unable to implement each quiz for a second time to test for reliability due to other end-of-the-year school activities and functions. This issue caused this researcher to use a different student sample to measure test-reliability.

Reliability of Independent Variables

This researcher separately used both scores from the two multiplication fact assessments as two independent variables for this study. Creswell (2008) defined reliability as the measurement situation in which “individual scores from an instrument should be nearly the same or stable on repeated administrations of the instrument and that they should be free from sources of measurement error and consistent" (p. 646). Due to time constraints, this researcher implemented each type of multiplication assessment one time for this study's sample population of eighth grade students. This researcher was not able to determine whether both multiplication assessments were reliable and valid with this study's sample population of eighth grade students. Therefore, this researcher gave the same two multiplication assessments to a different sample population of middle school students to determine the assessment's reliability.

The new student population chosen to measure test-reliability consisted of 47 eighth grade students who took two multiplication fact fluency quizzes and 52 eighth grade students who took two multiplication fact speed quizzes. Forty-seven seventh grade students took two multiplication fact fluency quizzes and 46 seventh grade students took two multiplication fact speed-recall quizzes. Both the eighth and seventh grade student populations varied in number due to student absences on the day designated for each quiz. For the eighth grade student population, there were seven student absences for at least one multiplication fact fluency quiz and two student absences for at least one multiplication fact speed-recall quiz. For the seventh grade student population, there were five student absences for at least one multiplication fact fluency quiz and six student absences for at least one multiplication fact speed quiz.

This researcher used the test-retest method (Creswell, 2008) or procedure as the means to test reliability. This researcher administered both forms of the multiplication assessments on four different days separated approximately one week from each other. The students consecutively took the same multiplication fact assessment back-to-back.

This researcher, first, arbitrarily implemented the multiplication fact fluency assessment on two different consecutive days. The students had 1 minute and 48 seconds to complete every problem accurately. This researcher followed with the multiplication fact speed-recall assessment on two different consecutive days. The students had 45 seconds to complete accurately as many problems as possible. The non-rejection of the null hypothesis for quiz reliability required no difference in average quiz scores for each class. This researcher discovered a significant difference in the average fluency quiz scores for each class of seventh grade students (t = -4.253; t-critical = ±2.013) and eighth grade students (t= -2.655; t-critical = ±2.013) while no significant difference existed in the average speed-recall quiz scores for each class of seventh grade students (t = -1.472; t-critical = ±2.014) and eighth grade students (t= -1.928; t-critical = ±2.008). Although the fluency quiz statistically resulted in low reliability, this researcher did not account for "practice," or a gain in scores when individuals retested on the same instrument or assessment (Kaufman, 2003), since the second fluency quiz took place one week after first fluency quiz. This researcher reported the reliability calculations in Chapter 4.

Validity of Independent Variables

The validity for both variables provided the integrity of Chapter 4 conclusions of the data. Creswell (2008) stated that validity as the "means that researchers can draw meaningful and justifiable inferences from scores about a sample or population" (p. 649). Creswell (2008) defined the threat to validity stemmed from the statistical and design issues of the study; the "design issues may threaten the experiment so that the conclusions reached from the data may provide a false reading about cause and effect between the treatment and the outcome" (pp. 307 – 308). This study involved both forms of validity, internal and external. Handley (n.d.) defined internal validity as the legitimacy of the results, or the results from the statistical calculations as a function or an extent of the independent and dependent variables measured in this study. The internal validity threats for this study included history and selection. Handley (n.d.) defined external validity as the transferability of results with other groups, or populations.

One of the threats to internal validity included history, which is any occurrence of events that could alter the outcome or results of the study. Previous historical events included timed multiplication fact quizzes and multiplication fact practice worksheets. At the beginning of the year, this researcher implemented periodic timed multiplication fact quizzes to improve multiplication fact recall. Each quiz had 100 problems. This researcher recorded a single quiz grade out of 10 points each quarter. This researcher allowed a maximum of three problems missed for first quarter, two problems missed for second quarter, and one problem missed for third quarter. For fourth quarter this researcher implemented both forms of quizzes used for this study. The number of quiz opportunities varied each quarter. This researcher averaged five multiplication fact quizzes for the first three quarters. This researcher also provided multiplication fact practice worksheets at the start of the second quarter and encouraged, rather than mandated, multiplication fact practice. However, only a few of the Pre-Algebra students utilized the worksheets.

Historical events or testing throughout the day also caused a strong potential threat to internal validity. “History refers to the occurrence of events that could alter the outcome or results of the study” (Indiana University of Pennsylvania [IUP], n.d, p. 1.). A previous historical event occurred when the students took the same quiz the second time which may have affected the scores on the second quiz (IUP, n.d.). A concurrent historical event occurred when the students took each form of quiz throughout the day (IUP, n.d.). Since the eighth grade students shared the same hallway, the earlier or morning participants of this study could have potentially familiarized and reminded some of the other participants scheduled to arrive later in the day.

The selection of students within the middle school involved only the participants enrolled in each of the researcher's six periods, which approximately comprised about 50% of the eighth grade population. The additional selection of students within the middle school may not have only provided a change with group characteristics, but also with the results or findings of this study.

The only viable threat to external validity included the demographics of the following participants involved in this study. This researcher exclusively performed the study with one Missouri middle school throughout the state of Missouri. Although the differences with demographics (percent of students on free/reduced lunch and ethnic percentages) of this middle school compared relatively similarly with the three other middle schools within the school district, other middle schools that are within large metropolitan areas like St. Louis County and City had larger Black and other minority populations. Middle schools that are not within large metropolitan areas like Northwest and Washington School Districts had smaller Black and other minority populations.

Missouri Assessment Program Reliability and Validity

Educators and administrators in MO DESE and throughout the Missouri school districts relied on MAP scores for any preliminary corrective decision-making for school improvement and student achievement. MAP score dependability and meaningfulness depended consecutively on the assessments reliability and validity. Both MO DESE and CTB McGraw-Hill in 2003 wrote the MAP tests in compliance with American Educational Research Association, American Psychological Association, and National Council on Measurement in Education 1999 standards for high-test quality and reliability (MO DESE, 2010). MO DESE evaluated the reliability of the MAP test using Cronbach's coefficient alpha formula. The MAP test's reliability score ranged from 0 to 1. As the value of the coefficient approaches one, the test scores become more dependable or reliable. A coefficient score of one refers to a perfectly consistent or reliable test. "As a rule of thumb, reliability coefficients that are equal to or greater than 0.8 are considered acceptable for tests of moderate lengths" (MO DESE, 2010, p. 133). The reliability coefficient varied from 0.915 to 0.93 from 1997-2003. For the purpose of this study, all of the eighth grade math students took the math portion of the MAP test while only students enrolled in Algebra I also took the state's Algebra I EOC test.

Conceptual and Procedural Difference with Multiplication

The conceptual understanding of multiplication of the same two single-digit factors exemplified two different meanings while the procedural form for both numbers produced the same product. For example, three bags of two loaves in each bag, 3×2, brought a very different picture or meaning if the factors, 2×3, reversed to yield two bags of three loaves in each bag. Although both problems conceptually produced the same answer of six loaves of bread (definition of commutative property of multiplication), the loaves of bread are packaged differently in each instance. This researcher accounted for and applied the commutative property with multiplication, which states that the order between the multiplications of factors procedurally provided the same products with both quizzes; thus, this researcher used a different combination of single-digit factors, 2 through 9 for both quizzes. For both procedural sets of multiplication fact assessments 2 × 3 equals 3 × 2, both problems procedurally resulted in the same product.

Although this researcher intended to use a different combination of factors for every problem, this researcher accidentally applied the commutative property with one set of factors: three and six. Both sets of multiplication fact assessments used the factors three and six in reverse order making the problem 3 × 6 in one question and 6 × 3 in another question. This researcher accidentally eradicated the 6 × 6 multiplication problem. If this researcher used the combination of factors, six and six on each multiplication fact assessment, then each problem would have thoroughly assessed the single-digit multiplication facts, 2 through 9.

For each test, this researcher specifically chose not to test factors 0 and 1. The products for both 0 and 1 each have a rule to explain their products with any other factor. Any factor multiplied by 0 will always produce the same product, which is 0. Any factor multiplied by 1 will always produce a product of the other factor multiplied by 1. This researcher decided the factors 2 through 9 would be the basis for both assessments due to a similar mathematics concept.

The product for factors 2 through 9 shared the similar mathematics conception of repeated addition. This researcher did not want the focus of this study to be how fast a student was able to use repeated addition for the answer to single-digit multiplication problems, but rather how fast the student was able to accurately determine or formulate the answer as the product of two numbers on paper.

Multiplying factors 10 and beyond is defined as a composite skill whereas multiplying factors 2 through 9 is defined as a component skill (Lin & Kubina, 2005). A combination of component skills (basic or foundation skill) form more complex steps called composite skills (Lin & Kubina, 2005). By Lin's and Kubina's definitions of component and composite skills, multiplication of factors 2 through 9 would be considered a component skill while the multiplication of double digit factors would be considered a composite skill. The purpose and focus of this study only involved the component skill, the multiplication factors 2 through 9.

Multiplication Fact Assessments

This researcher was the author of both multiplication fact assessments for this study. Students took two different forms of the multiplication fact assessments on two different days. Each multiplication fact assessment score defined the independent variables for this study. One multiplication fact assessment investigated multiplication fact fluency, while the other multiplication fact assessment investigated the speed-recall from memory. Both multiplication fact assessments had 36 total multiplication fact problems that multiplied single-digit factors of 2 through 9. This researcher randomized the order of multiplication factors.

The primary goal for the first multiplication assessment was to determine what percentage of students achieved multiplication fact fluency for factors 2 through 9. The students took the assessment to complete in a total time of 1 minute and 48 seconds, or less. This researcher specifically chose 1 minute and 48 seconds for this study based on Michalczuk's (2007) research which showed students, for quick recall on single-digit multiplication questions accurately wrote the product in 3 seconds or less. Woodward's study (2006) defined automaticity to be 36 correct multiplication fact problems within a 2-minute time period. This researcher tabulated the number of correct responses in a Microsoft Excel spreadsheet.

The primary goal for the second multiplication assessment was to determine the maximum number of accurate responses in a short period. This researcher arbitrarily chose 45 seconds, which allowed the students slightly less than 1.5 seconds to write the products for each problem. This researcher tabulated the number of correct responses in a Microsoft Excel spreadsheet.

Multiplication Fact Reliability for Speed-Recall and Fluency Quizzes

Due to time constraints at the end of the school year, this researcher measured the reliability of each quiz with student populations different than those used for this study. For testing reliability of the measuring tool, this researcher used a different sample of eighth and seventh grade students. There were inconsistencies in the reliability testing that may or may not have affected the reliability measure. This researcher gave both fluency quizzes first and speed-recall quizzes second. The period between the first and second fluency quiz was 10 days for the eighth grade and seventh grade “challenge” or advanced students; the regular seventh grade math class was only eight days apart. The period between the first and second speed-recall quiz was five days for the eighth grade and seventh grade students in Pre-Algebra; the regular seventh grade math class was only two days apart. Not only the varied time-scale, total number of days implemented between each quiz unto completion of all four quizzes, but also the order this researcher chose to implement each quiz could have affected the check for quiz reliability. Statistical measurements revealed a mean-score increase for each quiz during the second application. The increase of quiz scores during the second application may have resulted from practice effects (Kaufman, 2003).

Kaufman referred practice effects as “gains due to the experience of having taken the test previously; they occur without the examinee being given specific or general feedback on test items, and they do not reflect growth or other improvement on the skills being assessed” (p. 1). Kaufman (2003) suggested examinees are provided the best chance to remember specific items after a short period of time, a few hours or a couple of days later. Except for the second multiplication fact speed-recall quiz with the regular math seven students with a time period between quizzes of two days, every second quiz had a time period of five or more days from the first quiz. This researcher also provided or used no instructional time during the study for multiplication fact improvement. Even with these considerations of time periods between each quiz and no instructional opportunities for multiplication fact improvement in place, the mean-score increase of each quiz still may have resulted from a practice effect. A longer interval between each quiz of the same form may have reduced or eliminated the practice effect, but not necessarily have reduced or eliminated a mean-score increase for each quiz since assessment improvement may be a result of a real growth or a true knowledge of the multiplication facts. This researcher determined the reliability of multiplication fact speed-recall and fluency quizzes in Chapter 4.

Descriptive Statistics of Speed-Recall and Fluency Quiz Scores

The names of each student who participated in the study remained anonymous. This researcher summarized characteristics of both sample and population sizes of students in Algebra I and Pre-Algebra using the descriptive statistics software from

Microsoft Excel. The population size in Algebra I totaled 32 students and the population size in Pre-Algebra totaled 84 students. This researcher created the sample sizes of 20 students for Algebra I and 45 students for Pre-Algebra from each population through use of Research Randomizer (1997). This researcher provided descriptive statistics for both Algebra I and Pre-Algebra students in the following areas: speed-recall score, fluency score, average first and second semester assessment score, mathematics MAP test scale score, and GMRT grade equivalency. A descriptive statistics summary of the Algebra I sample and population students' EOC raw scores were also included.

The score on the speed-recall multiplication fact quiz demonstrated the student's number of correctly computed and written product responses in 45 seconds or less. This researcher attempted to determine if there was a relationship between higher level mathematics learning and mathematics assessment achievement scores. Table 14 summarizes the descriptive statistics results of the multiplication speed-recall quiz.

The descriptive statistics revealed both the mean and median between similar groupings were comparatively higher for the students in Algebra I. The results also revealed students in Algebra I demonstrated a higher mean of multiplication fact product accuracy with the same total time limit of 45 seconds.

The participants who achieved multiplication fact fluency for this study had to achieve either a 35 or a 36 out of a 36-point score. This researcher allowed one incomplete product answer, possibly due to a careless error rather than an unknown math error.

Table 14

Algebra and Pre-Algebra Population and Sample Descriptive Statistics for Multiplication Fact Speed-Recall Scores

Description	2010-11 Algebra I	Population Pre-Algebra	2010-11 Algebra I	Sample Pre-Algebra
Mean	26.15	19.96	24.55	20.33
Median	25	19	23.5	20
Standard Deviation	6.32	5.44	5.93	5.06
Variance	39.94	29.6	35.21	25.64
Skewness	0.038	0.63	0.197	0.34
Minimum	12	8	12	10
Maximum	36	36	36	34

Table 15 summarizes the descriptive statistics results of the multiplication fluency test for students in Algebra I and Pre-Algebra. The score on this test revealed the number of correct product responses that students were able to handwrite in 1 minute and 48 seconds; each problem averaged 3 seconds.

The descriptive statistics for the fluency test revealed similar trends to the descriptive statistics results for the multiplication speed-recall quiz. The central modes of tendency between similar groupings were comparatively higher for the Algebra I students. Students who were enrolled in Algebra I at the eighth grade level were considered to be in a more advanced class than students in Pre-Algebra; thus, a higher percentage of mastery of the basic math skills could be expected from the Algebra I students (Loveless & Coughlan, 2004).

Table 15

Algebra I and Pre-Algebra Population and Sample Descriptive Statistics for Multiplication Fact Fluency Scores

Description	2010-11 Algebra I	Population Pre-Algebra	2010-11 Algebra I	Sample Pre-Algebra
Mean	35.31	32.92	35.2	33.53
Median	36	34.5	36	35
Standard Deviation	1.40	3.61	1.74	3.35
Variance	1.96	13.04	3.01	11.21
Skewness	-3.37	-1.23	-2.83	-1.50
Minimum	29	22	29	24
Maximum	36	36	36	36

The results, or score of the multiplication fact fluency quiz, played an important part in this comparative study of results generated by six of the eight hypotheses.

Descriptive Statistics of First and Second Semester Average Assessment Scores

Each student's school-based mathematics assessment measured the student's understanding of mathematics concepts represented by the Missouri's grade-level expectations, GLEs, for Pre-Algebra and course-level expectations, CLEs, for Algebra I. This researcher took an average assessment grade for each student over the entire school year. Table 16 summarizes the descriptive statistics results of the average first and second semester assessment scores for students in Algebra I and Pre-Algebra.

Table 16

Algebra I and Pre-Algebra Population and Sample Descriptive Statistics for First and Second Semester Average Assessment Scores

Description	2010-11 Population		2010-11 Sample	
	Algebra I	Pre-Algebra	Algebra I	Pre-Algebra
Mean	88.7	76.32	88.53	75.64
Median	89.2	78.15	88.15	76.4
Standard Deviation	6.36	10.37	6.43	10.92
Variance	40.40	107.62	41.3	119.2
Skewness	-0.003	-0.39	0.073	-0.21
Minimum	77.2	51.2	77.2	52.9
Maximum	99.1	95.7	99.1	94.5

An expectation of the school district was the requirement for students in Algebra I to earn an average semester grade of an 80% or higher as a prerequisite to high school regular or honors Geometry. This researcher arbitrarily applied the 80%, an equivalency grade of a B, or higher average expectation as the minimum percentage that characterized higher level mathematics learning. For the purpose of this study, this researcher compared the number of students who earned a fluency score of a 35 or 36 to the number of students who earned an average first and second semester assessment score of 80% or higher.

Descriptive Statistics and Level Descriptor Details of Mathematics MAP Test

The 2011 mathematics MAP test assessed students only in Grades 3 through 8. Each assessment required three to five hours of test administration time for the completion of three types of questions which were selected response (commonly known as multiple-choice) items, constructed response items, and a performance event item. With the constructed response items, students had to show their work for full credit when answering the questions. The performance test provided insight to the student's ability to formulate and apply mathematical understanding to real-life situations. Due to budget cuts or constraints, MO DESE decided to remove the performance event for the 2011 mathematics MAP test. The mathematics MAP test scale scores indicated the individual student's knowledge and understanding of the mathematical concepts described in the GLEs (MO DESE, 2011b).

Each student received an assessment grade through a scale score and achievement level description. The scale score ranged from 450 to 885. Each achievement level descriptor, four total, included a specific scale score range. Table 17 provides a total and percentage summary of students in Algebra I and Pre-Algebra with each mathematics MAP level descriptor.

Table 17

Algebra and Pre-Algebra Population Totals for Mathematics MAP Achievement Level Descriptors

Population	Below Basic (Percent)	Basic (Percent)	Proficient (Percent)	Advanced (Percent)
Algebra I	0 (0%)	0 (0%)	0 (0%)	32 (100%)
Pre-Algebra	5 (6%)	38 (45%)	31 (37%)	10 (12%)

The percentage of mathematics MAP achievement level descriptors revealed 88% of the total population in Pre-Algebra received a lower descriptor score as compared to the population in Algebra I. The data also revealed 51% of students in Pre-Algebra achieved a basic or below basic score while 49% achieved a proficient score or higher.

MO DESE provided the following abbreviated achievement-level descriptors for the eighth grade mathematics MAP test. The lowest achievement level descriptor, below basic, identified the student's minimal mathematical knowledge. Students who received a below basic description mathematically performed the following mathematics concepts (MO DESE, 2011b):

> generalize numeric patterns; generalize relationships between attributes of 2-D shapes; identify the results of subdividing 3-D shapes and 3-D figures using a 2-D representation; solve problems involving area; use scales to estimate distance; interpret graphs; find the mean value of a data set; select graphical representations of data; interpret data; make conjectures based on theoretical probability. (p. 8)

The below basic MAP scale score ranged from 525 to 669.

The next achievement descriptor, basic, described the student's mathematics competence to include the mathematical concepts of below basic. Below basic mathematics skills included the following (MO DESE, 2011b):

> operations with rational numbers; solve and interpret one-step linear equations; extend geometric patterns; generalize patterns to find a specific term; identify relationships in 3-D objects; calculate the theoretical probability of an event; interpret a scatter plot to determine the relationship between two variables. (p. 8)

The basic mathematics MAP scale score ranged from 670 to 709.

The next achievement descriptor, proficient, became the next higher level. Schools within each school district are required each year to have a certain percentage of all student sub-groups proficient or above as outlined by Missouri's AYP. Students who achieved a proficient mathematics MAP score not only met the criteria of the two previous descriptors, below basic and basic but also included the ability to do the following (MO DESE, 2011b):

> identify equivalent representations of a number; identify mental strategies to solve problems; solve multi-step equations; use symbolic algebra; identify transformations; classify angles; create similar polygons; use coordinate geometry; solve problems involving area; identify appropriate units of measure; convert standard units within a system of measurement; interpret graphic organizers; calculate measures of center. (p. 8)

The proficient mathematics MAP scale score ranged from 710 to 740.

The advanced descriptor identified the final and highest achievement descriptor. The advanced descriptor not only included students meeting the criteria of the previous three descriptors of below basic, basic, and proficient, but also the ability to do the following (MO DESE, 2011b):

> estimate the value of square roots; write numbers using scientific notation; solve two-step inequalities; analyze slope and intercept in linear equations; apply the Pythagorean Theorem using coordinate geometry; identify polygons based on their attributes; identify coordinates of vertices of a transformed polygon; use a protractor to measure angles; solve problems involving surface area; select, create, and use appropriate graphical representation of data. (p. 8)

The advanced mathematics MAP scale score ranged from 741 to 885.

Table 18 summarizes the descriptive statistics results of the mathematics MAP test scale scores for students in Algebra I and Pre-Algebra.

Table 18

Algebra I and Pre-Algebra Population and Sample Descriptive Statistics for Mathematics MAP Test Scale Scores

Description	2010-11 Algebra I	Population Pre-Algebra	2010-11 Algebra I	Sample Pre-Algebra
Mean	764.66	711.42	759.1	709.30
Median	761.5	708.5	759	705.5
Standard Deviation	24.71	26.57	11.68	31.59
Variance	610.68	706.03	136.52	998.07
Skewness	3.90	0.26	0.724	0.23
Minimum	741	644	741	644
Maximum	885	786	788	782

The students in Algebra I achieved a higher mean and median score with similar mathematical assessed concepts than the students in Pre-Algebra. The lowest score received from a student in Algebra I with either the population or sample group was a 741 scale score; a 741 score indicated 100% of the students in Algebra I received an advanced achievement descriptor on the mathematics MAP test.

Descriptive Statistics and Level Descriptor Details of Algebra I EOC Test

The Algebra I EOC test became available beginning in the fall of 2008, and replaced the mathematics MAP test at the high school level. The Missouri State Board of

Education identified the following purposes for course level expectations: a) a measurement and reflection of students' readiness towards post-secondary education, b) an identification of students' academic strengths and weaknesses, c) a communication expectation for all students, d) a basis for state and national accountability, and e) program evaluation (MO DESE, 2011a). The 2011 assessments took approximately 110 minutes to complete and included selected response (multiple-choice) items and some constructed response items only. Although a performance event was included in 2010, it was not included in 2011 due to state budget constraints.

Table 19 summarizes the descriptive statistics results of the Algebra I EOC scores for the student population and sample size of this study. The data revealed 100% of the students achieved a proficient level descriptor or higher. The mean score for the Algebra I student population and sample size for this study characterized an advanced descriptor.

Table 19

Algebra I Population and Sample Descriptive Statistics for Algebra I EOC Test Raw Scores

Description	2010-11 Population	2010-11 Sample
Mean	29.88	30.15
Median	30	30
Standard Deviation	3.00	2.58
Variance	9.02	6.66
Skewness	-0.45	0.41
Minimum	22	26
Maximum	35	35

Students' Algebra I EOC scale scores ranged from 100 to 250 based on the number of students' correct responses and points earned on the test. Like the mathematics MAP test, the Algebra I EOC test also created achievement descriptors based from a certain scale score range or raw score of correct responses.

Similar to the mathematics MAP test, below basic identified the first and lowest achievement descriptor. Students, who scored below basic, not only used "very few" strategies, but also "limited understanding" of the essential course mathematical content and concepts to solve problems that included the following mathematical strands: numbers and operations, algebraic relationships, and data and probability (MO DESE, 2011a). An Algebra I EOC scale score range of 100 to 176 or raw score of 0 to 11 categorized students as below basic.

The basic level identified the second highest achievement descriptor. In addition to the mathematical criteria at the below basic level, students, who scored basic, not only used "some" strategies, but also utilized "some understanding" of the essential course mathematical content and concepts to solve problems that included the following mathematical strands: numbers and operations, algebraic relationships, and data and probability (MO DESE, 2011a, p. 9). An Algebra I EOC scale score range of 177 to 199 or a raw score of 12 to 20 categorized students as basic.

The proficient level identified the third highest achievement descriptor. In addition to the mathematical criteria at the basic level, students, who scored proficient, not only used "a range of" strategies, but also utilized "an understanding" of the essential course mathematical content and concepts to solve problems that included the following mathematical strands: numbers and operations, algebraic relationships, and data and probability

probability (MO DESE, 2011a, p. 9). An Algebra I EOC scale score range of 200 to 224 or a raw score of 21 to 27 categorized students as proficient.

The advanced level identified the highest attained achievement descriptor. In addition to the mathematical criteria at the proficient level, students who scored advanced not only used “a wide range of” strategies, but also utilized “a thorough understanding” of the essential course mathematical content and concepts to solve problems that included the following mathematical strands: numbers and operations, algebraic relationships, and data and probability (MO DESE, 2011a, p. 9). An Algebra I EOC scale score range of 225 to 250 or a raw score of 28 to 35 categorized students as advanced.

Descriptive Statistics of GMRT Grade Equivalency

Mathematics MAP test not only required a mathematical ability or competence but also reading ability or comprehension. This researcher compared the GMRT grade equivalencies with students in Algebra I and Pre-Algebra. Table 20 summarizes the descriptive statistics results of the GMRT grade equivalency for students enrolled in Algebra I and Pre-Algebra. The descriptive statistics revealed students in Algebra I achieved a higher central measure of tendency with reading achievement – at least three grade equivalents higher. The GMRT grade equivalent scores with students in Pre-Algebra students resulted in a higher variance, 2 to 3 times higher, than the students in Algebra. A higher variance of the GMRT grade equivalency scores with the students in Pre-Algebra signifies a greater range between data values. The GMRT grade equivalent data values for students in Algebra I were closer to the mean of the data than the GMRT grade equivalent data values for students in Pre-Algebra (Bluman, 2008).

Table 20

Algebra I and Pre-Algebra Population and Sample Descriptive Statistics for GMRT Grade Equivalency Scores

Description	2010-11 Population		2010-11 Sample	
	Algebra I	Pre-Algebra	Algebra I	Pre-Algebra
Mean	12.35	9.92	12.34	9.57
Median	13	9.85	13	9.2
Standard Deviation	1.39	2.69	1.57	2.73
Variance	1.92	7.26	2.47	7.48
Skewness	-2.35	-0.17	-2.45	0.12
Minimum	7.6	5.2	7.6	5.2
Maximum	13	13	13	13

Conclusion

This quantitative study investigated a possible relationship between the multiplication fact speed-score and higher level mathematics learning, and also investigated the GMRT grade equivalency score of eighth grade students in Algebra I and Pre-Algebra. This quantitative study investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and a GMRT equivalency grade at eighth grade or higher.

The population involved 84 eighth grade students enrolled in Pre-Algebra and 32 students enrolled in Algebra I. For perspective, demographics of the Missouri middle school represented the following 2011 total enrollment of 670 students: 0.7% Asian, 6.3% Black, 1.5% Hispanic, and 90.9% White (MO DESE 2011i). For this study, 9.4% Black and 90.6% White determined the students' population.

The analysis of the data involved a quantitative approach for the analysis of eight hypotheses. The first hypothesis determined whether a possible relationship existed between multiplication fact speed-recall quiz and the following four dependent variables: a) 80% or higher first and second semester average assessment grade, b) mathematics MAP test scale score, c) GMRT grade equivalency score, and d) Algebra I EOC raw score. This researcher used a *t* test for difference in means for the second hypothesis to determine whether a difference in fluency and speed-recall scores existed between students in Algebra I and Pre-Algebra. This researcher performed *z* tests for difference in proportion for hypotheses 3, 4, 5, and 8 to test the percentage comparisons of students who have achieved multiplication fact fluency and the students who achieved with each of the following dependent variables: 80% or higher first and second semester average assessment grade, a proficient or advanced descriptor on the mathematics MAP test, a GMRT grade equivalency of eighth grade or higher, and a proficient or advanced descriptor Algebra I EOC score. This researcher also performed *z* tests for difference in proportion for hypotheses 6 and 7 to test the percentage comparisons between students who did not achieve multiplication fact fluency and the students who did not achieve two of the following dependent variables: a proficient or advanced descriptor on the mathematics MAP, and a GMRT grade equivalency of eighth grade or higher. This researcher provided a thorough analysis of the demographics, participants, and instruments that were used to collect the data of this study in Chapter 3. The statistical results of the data will be discussed in Chapter 4.

Chapter 4 will begin to discuss the results and discussion for both multiplication fact speed-recall and fluency quiz reliability. The rest of Chapter 4 includes the discussion and results performed for each hypothesis.

Chapter 4 - Results

Review

This quantitative study investigated a possible relationship between the multiplication fact speed-score and higher level mathematics learning, and also investigated the GMRT grade equivalency score of eighth grade students in Algebra I and Pre-Algebra. This study investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and a GMRT equivalency grade at eighth grade or higher.

NCTM has made it clear in the 2000 Standards that computational fluency with whole numbers should be developed throughout the elementary years in Grades 3 through 5. NCTM defined fluency as a reflection of the students' mathematical knowledge to compute efficiently and accurately mathematics properties and numeric relationships. Algorithms used correctly become mathematical tools problems, rather than rote-procedures of fact memorization to help students generalize the ideas to solve mathematical problems and a preparation for higher level mathematics learning (Wu, 1999). Studies have revealed significant relationships with basic math fact fluency and higher level mathematics learning (Caron, 2007; Clavel, 2003; French, 2005; Wallace & Gurganus, 2005; Wu, 1999). Basic math fact fluency helps free up working memory space and build a mathematical understanding of the higher or more advanced concepts (Hecht, 2002; Tronsky, 2005; Rasmussen & Bisanz, 2005).

Statistical Analysis of Speed-Recall and Fluency Quiz Reliability

The null hypothesis for quiz reliability was that there would be no difference in average quiz scores for each class. This researcher used a *t* test (Paired Two Sample for Means) with a 95% confidence level to analyze quiz reliability. The students had 1 minute and 48 seconds to write the products accurately for as many problems as possible within that timeframe. Fluency quiz Tables 21 and 22 reveal both the seventh and eighth grade *t* test critical value for the two-tailed test were ±2.013. The eighth grade students had a *t* test value of -2.655 and the seventh grade students had a *t* test value of -4.253. Both *t* test values fell inside the critical region; henceforth, this researcher rejected the null hypotheses. This researcher concluded a significant difference in the average fluency quiz scores existed for each class of students and could not support consistent results between the first and second applications of the assessment for accuracy.

Table 21

Eighth Grade Fluency Quiz t Test: Paired Two Sample for Means

Statistics	First Fluency Quiz	Second Fluency Quiz
Population size	47	47
Mean	29.319	30.532
Variance	31.092	29.428
t test value	-2.665	
t critical two tail	±2.013	
α Value	0.05	

Table 22

Seventh Grade Fluency Quiz t Test: Paired Two Sample for Means

Statistics	First Fluency Quiz	Second Fluency Quiz
Population size	47	47
Mean	29.106	31.8722
Variance	44.445	26.766
t test value	-4.253	
t critical two tail	±2.013	
α Value	0.05	

The null hypothesis for quiz reliability stated that there would be no difference in average quiz scores for each class. This researcher used a *t* test (Paired Two Sample for Means) with a 95% confidence level to statistically measure the quiz reliability. The students had 45 seconds to write accurately the products to as many problems as possible. For the speed-recall quiz, Tables 23 and 24 revealed the *t*-test value, -1.928, for the eighth grade and, -1.472, for the seventh grade.

Although the *t* test critical values for the two-tailed test were different, ±2.008 for the eighth grade and ±2.014 for the seventh grade, both *t* test values for each class fell between their critical values; hence, this researcher did not reject the null hypothesis. This researcher concluded no significant difference in the average speed-recall quiz scores existed for each class of students, and could support the reliability of assessment for fluency.

Table 23

Eighth Grade Speed-Recall Quiz t Test: Paired Two Sample for Means

Statistics	First Speed-Recall Quiz	Second Speed-Recall Quiz
Population size	52	52
Mean	20.615	21.481
Variance	33.379	29.078
t test value	-1.928	
t critical two tail	±2.008	
α Value	0.05	

Table 24

Seventh Grade Speed-Recall Quiz t Test: Paired Two Sample for Means

Statistics	First Speed-Recall Quiz	Second Speed-Recall Quiz
Population size	46	46
Mean	21.261	22.217
Variance	41.752	33.552
t test value	-1.472	
t critical two tail	±2.014	
α Value	0.05	

Data Analysis for each Null Hypothesis

Null hypothesis # 1. There will be no relationship between the speed-recall score and 2010 – 11 combined average score of first and second semester mathematics assessment scores, 2011 mathematics MAP test scale score, 2011 Algebra I EOC raw score, and GMRT grade equivalency. This researcher calculated a Pearson Product Moment Correlation Coefficient (PPMC) to determine whether or not there was a relationship between the independent variable of speed-recall score and each of the four dependent variables. After the correlation coefficient was determined, this researcher performed a *t* test for significance of the results at a 95% confidence level for each dependent variable. To test the significance of the calculated correlation coefficient the critical values were -2.101 and 2.101 for the students in Algebra I and -1.96 and 1.96 for the students in Pre-Algebra.

Null Hypothesis # 1a stated there will be no relationship between the speed-recall score and combined average score of first and second semester mathematics assessment scores. The Algebra I correlation coefficient, 0.211, and Pre-Algebra correlation coefficient, 0.280, signified a very weak to no linear relationship between the variables. In testing for significance of the relationships, the results in Table 25 show both *t* values fall in between -2.101 and 2.101 for students in Algebra I and between – 1.96 and 1.96 for students in Pre-Algebra; therefore, this researcher could not reject the null hypothesis for the significance test: There is no difference between the correlation coefficient and zero. Therefore, data does not support a significant relationship between the speed-recall score and the combined first and second semester average assessment score for students in Algebra I and in Pre-Algebra.

Table 25

Hypothesis # 1a: Speed-Recall Score as the Independent Variable with the First and Second Average Assessment Score as the Dependent Variable

Statistical Test	First and Second Semester Average Assessment Scores Algebra I	Pre-Algebra
Correlation coefficients	0.211	0.280
t test value	0.550	0.240
t critical two-tail	±2.101	±1.96

As shown in Figure 1, the scatter plot graph of the 20 student sample in Algebra I represented each student's multiplication fact speed-recall score and his or her first and second semester average assessment score.

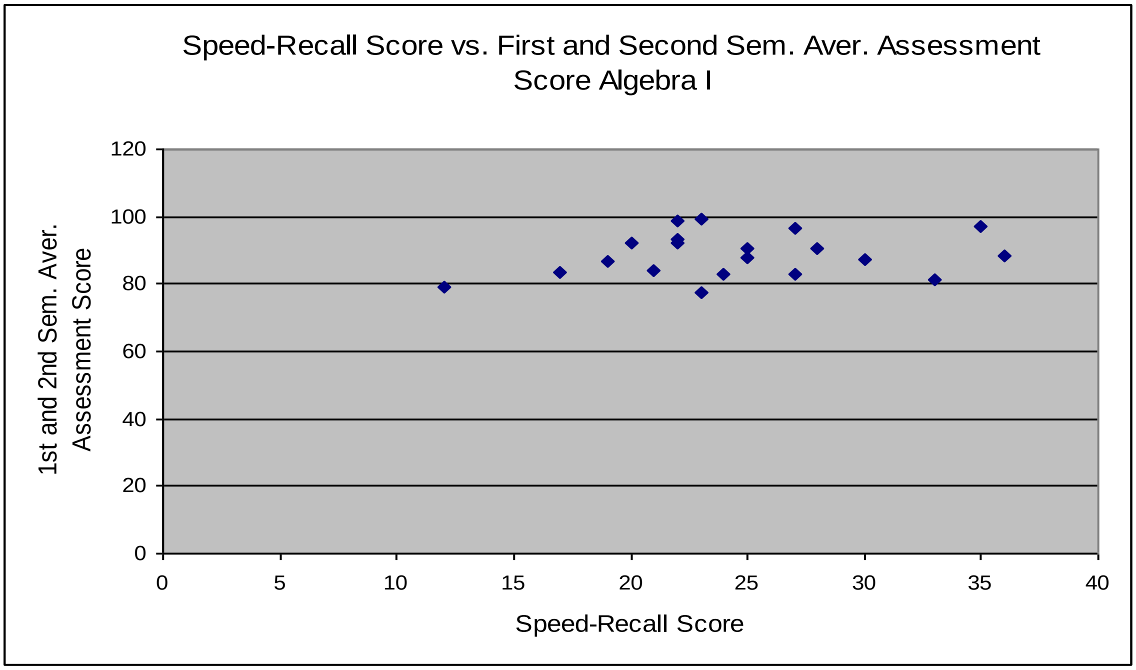

Figure 1. Speed-Recall Score versus First and Second Semester Average Assessment Score for the 20 Student Sample in Algebra I

The 20 dots are clustered somewhat close together in a horizontal fashion. According to Bluman (2008), this representation of dots characterized a no relationship between a student's speed-recall score and his or her first and second semester average assessment score in Algebra.

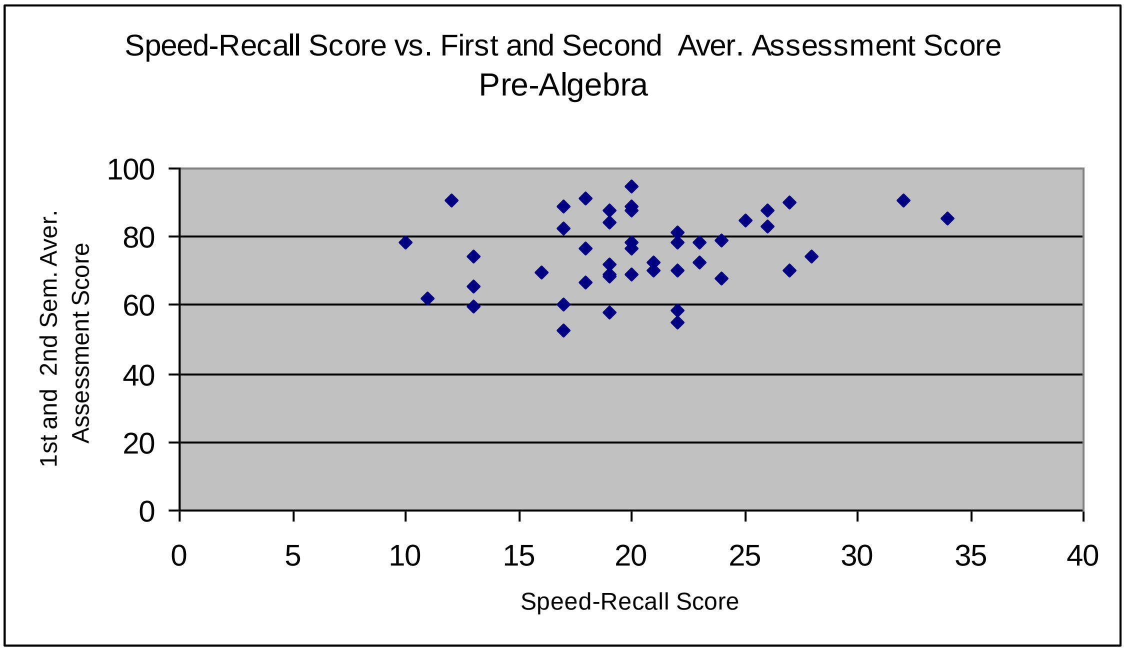

Figure 2. Speed-Recall Score versus First and Second Semester Average Assessment Score for the 45 Student Sample in Pre-Algebra

As shown in Figure 2, the scatter plot of the 45 student sample in Pre-Algebra represented each student's multiplication fact speed-recall score and his or her first and second average assessment score. The 45 dots revealed a weak clustering in a horizontal direction. According to Bluman (2008), this representation of dots characterized no relationship between a student's speed-recall score and his or her first and second semester average assessment score in Pre-Algebra.

Null hypothesis #1b stated there will be no relationship between the speed-recall score and the 2011 mathematics MAP test scale score. The Algebra I correlation coefficient, 0.033, and Pre-Algebra correlation coefficient, 0.331, signified no linear relationship for the students in Algebra I and a very weak to no relationship for the students in Pre-Algebra. In testing the significance of these relationships, the results in Table 26 show both *t* values fall in between -2.101 and 2.101 for Algebra I and -1.96 and 1.96 for Pre-Algebra.

Table 26

Hypothesis #1b: Speed-Recall Score as the Independent Variable with the Mathematics MAP Test Scale Score as the Dependent Variable

Statistical Test	Mathematics MAP Test Scale Scores	
	Algebra I	Pre-Algebra
Correlation coefficients	0.033	0.331
t test value	-0.596	1.31
t critical two-tail	±2.101	±1.96

This researcher could not reject the null hypothesis for the significance test: There is no difference between the correlation coefficient and zero. Therefore, data does not support a significant relationship between the speed-recall score and mathematics MAP test scale score for students in Algebra I and in Pre-Algebra.

As shown in Figure 3, the scatter plot graph of the 20 dots represented each student's multiplication fact speed-recall score and his or her mathematics MAP scale score for the 20 student sample in Algebra I. The 20 dots are not very clustered without any general upward or downward slope. According to Bluman (2008), this representation of dots characterized no relationship between a student's speed-recall score and his/her 2011 mathematics scale score.

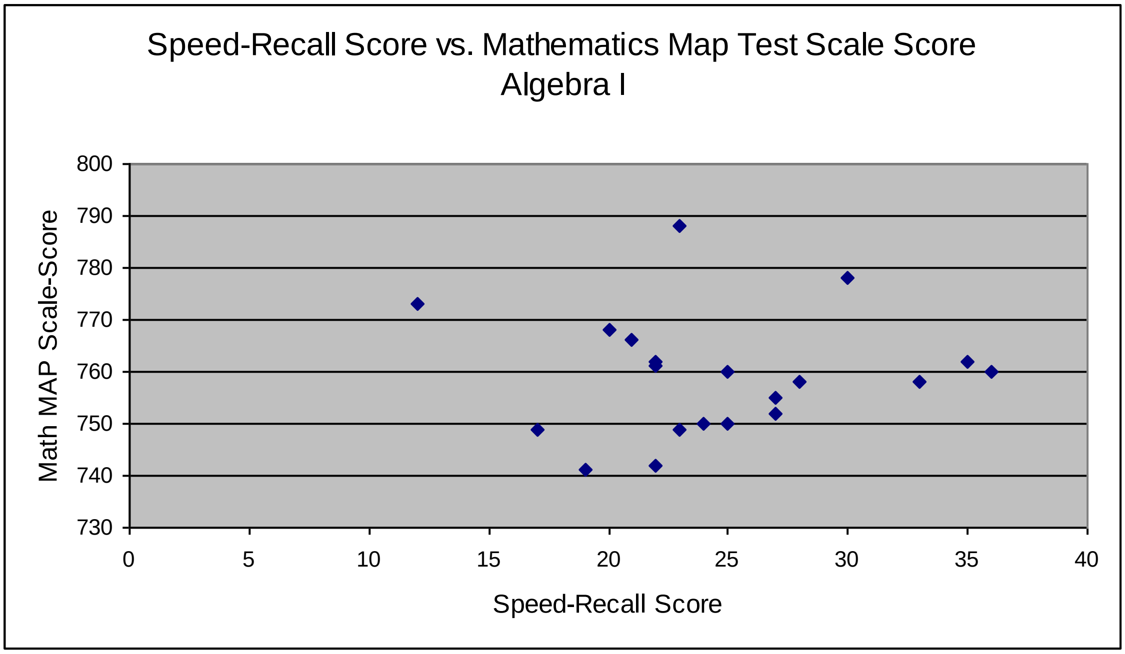

Figure 3. Speed-Recall Score versus the Mathematics MAP Test Scale Score for the 20 Student Sample in Algebra I

As shown in Figure 4, the scatter plot graph of the 45 dots represented each student's multiplication fact speed-recall score and his or her mathematics MAP test scale score for the 45 student sample in Pre-Algebra.

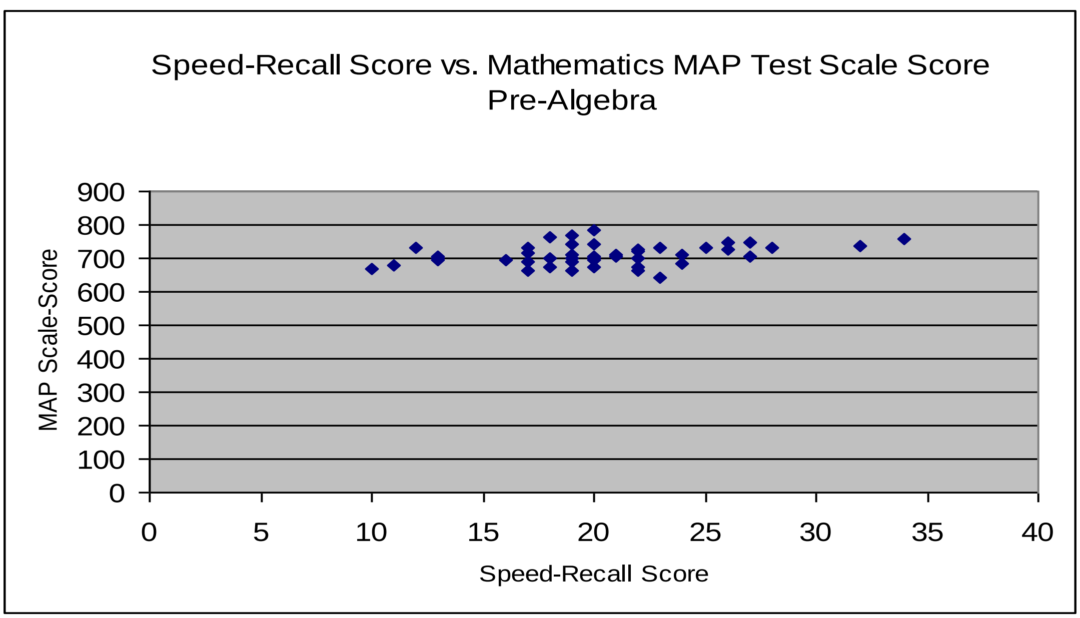

Figure 4. Speed-Recall Score versus Mathematics MAP Test Scale Score for the 45 Student Sample in Pre-Algebra

Although the 45 dots of Figure 4 are clustered close together, the direction of the dots is displayed graphically in a horizontal fashion. According to Bluman (2008), this representation of dots characterized no relationship between a student's speed-recall score and his or her 2011 mathematics MAP test scale score.

Null hypothesis #1c stated there will be no relationship between the speed-recall score and 2011 GMRT grade equivalency score. The Algebra I correlation coefficient, 0.027, and Pre-Algebra correlation coefficient, 0.149, signified no linear relationship for the students in Algebra I and a very weak to no relationship for the students in Pre-Algebra. In testing for significance of these relationships, the results in Table 27 show both *t* values fall in between -2.101 and 2.101 for students in Algebra I and between – 1.96 and 1.96 for students in Pre-Algebra.

Table 27

Hypothesis # 1c: Speed-Recall Score as the Independent Variable with the GMRT Grade Equivalency as the Dependent Variable

Statistical Test	GATES Reading Grade Level Scores	
	Algebra I	Pre-Algebra
Correlation coefficients	0.027	0.149
t test value	0.280	-0.475
t critical two-tail	±2.101	±1.96

This researcher could not reject the null hypothesis for the significance test: There is no difference between the correlation coefficient and zero. Therefore, data does not support a significant relationship between the speed-recall score and GMRT grade equivalency score for students in Algebra I and in Pre-Algebra.

As shown in Figure 5, the scatter plot graph of the 20 dots represented the multiplication fact speed-recall score and his or her GMRT grade equivalency for the 20 student sample in Algebra I. Besides a few students, a majority of the 20 students revealed a horizontal line between 12 and 14.

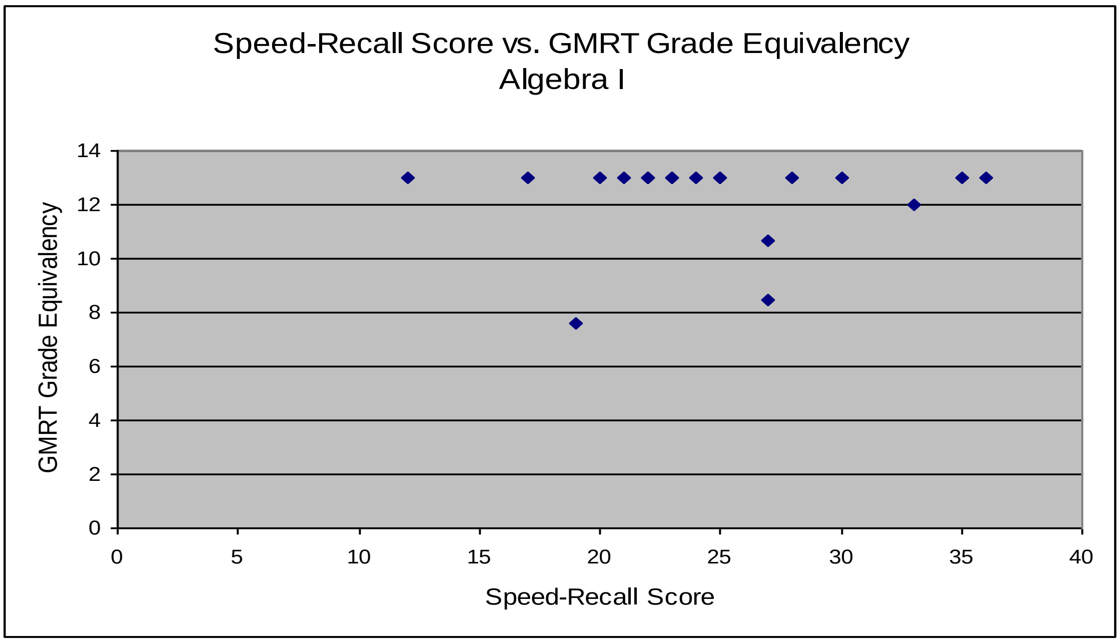

Figure 5. Speed-Recall Score versus the GMRT Grade Equivalency for the 20 Student Sample in Algebra I

According to Bluman (2008), this representation of dots characterizes no relationship between a student's speed-recall score and his/her 2011 GMRT grade equivalency.

As shown in Figure 6, the scatter plot graph of the 45 dots represented each student's multiplication fact speed-recall score and his or her GMRT grade equivalency for 45 student sample in Pre-Algebra. Besides a few students, a majority of the 45 students revealed no sense of clustering or direction. According to Bluman (2008), this representation of dots characterized no relationship between a student's speed-recall score and his/her 2011 GMRT grade equivalency.

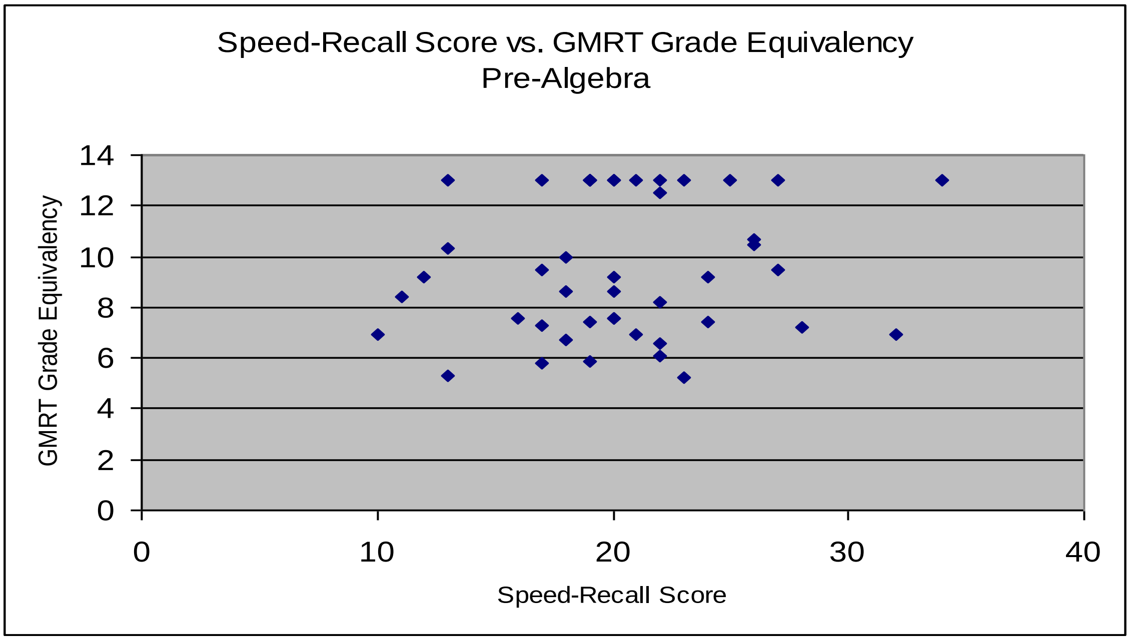

Figure 6. Speed-Recall Score versus the GMRT Grade Equivalency for the 45 Student Sample in Pre-Algebra

Null hypothesis #1d stated there will be no relationship between the speed-recall score and 2011 Algebra I EOC raw score. The Algebra I correlation coefficient, 0.207, signified a very weak to no relationship for the students in Algebra I. In testing for the significance of this relationship, the results in Table 28 show the *t* value falls in between -2.101 and 2.101 for students in Algebra I; therefore, this researcher could not reject the null hypothesis for the significance test: There is no difference between the correlation coefficient and zero. Therefore, data does not support a significant relationship between the speed-recall score and Algebra I EOC raw score.

Table 28

Hypothesis #1d: Speed-Recall Score as the Independent Variable with the Algebra I EOC Test Raw Score as the Dependent Variable

Statistical Test	EOC Raw Scores Algebra I
Correlation coefficients	0.207
t test value	0.809
t critical two-tail	±2.101

As shown in Figure 7, the scatter plot graph of the 20 dots represented each student's multiplication fact speed-recall score and his or her EOC raw score for the 20 student sample in Algebra I. The 20 dots are clustered somewhat close together in a horizontal fashion. According to Bluman (2008), this representation of dots characterizes no relationship between a student's speed-recall score and his or her Algebra I EOC raw score.

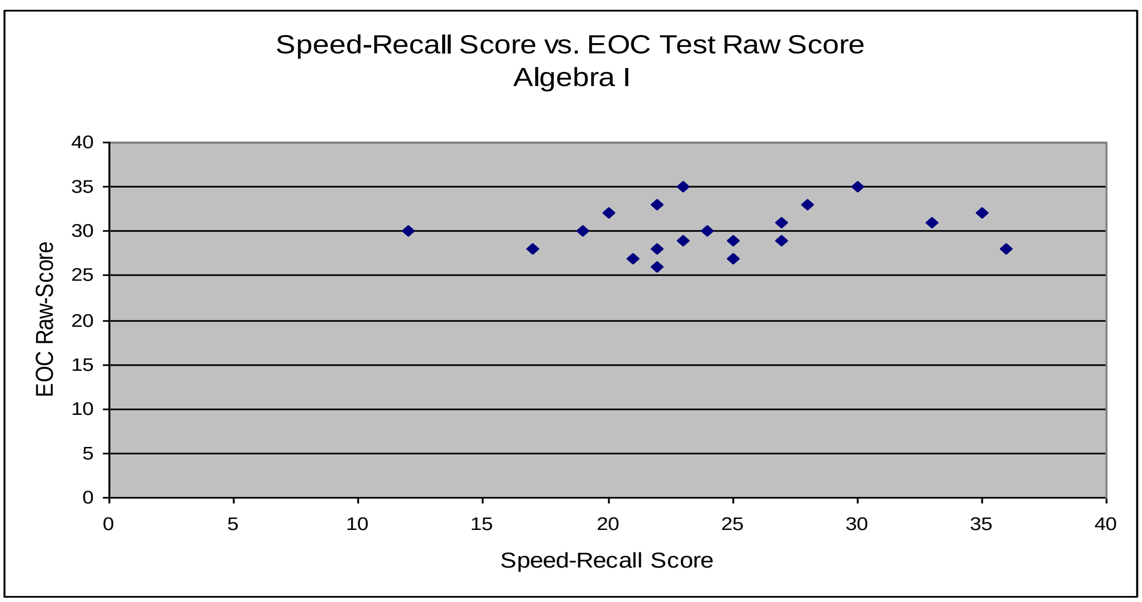

Figure 7. Speed-Recall Score versus the EOC Test Raw Score for the 20 Student Sample in Algebra I

Null hypothesis # 2. There will be no difference in fluency scores and in speed-recall scores when comparing Algebra I student multiplication fact quizzes to Pre-Algebra student multiplication fact quizzes. To test this null hypothesis, this researcher tested the difference between two means. This researcher used a *t* test for the difference in means to perform this test for Algebra I and Pre-Algebra fluency quiz scores. Prior to performing the *t* test, this researcher conducted an *F* test to determine whether or not the two sample variances were statistically equal using the null hypothesis; there is no difference in variance.

The results of the *F* test indicated a no difference in variance since the *F* test value, 0.269, fell outside the critical region (*F* critical = 0.497); therefore, this researcher used a *t* test for difference in means for equal variances. The results in Table 29 revealed the *t* test value, 2.10, was larger than the critical value of 2.00; hence, this researcher rejected the null hypothesis. This data supported a significant difference existed between Algebra I and Pre-Algebra fluency scores; students in Algebra I achieved a higher performance.

As shown in Figure 8, 15% or three out of 20 students in Algebra I failed to achieve multiplication fact fluency, whereas 85% or 17 out of 20 achieved multiplication fact fluency. Students achieved multiplication fact fluency if they missed no more than one problem out of 36 total problems in 1 minute and 48 seconds. The 20 students represented the sample size for the Algebra I population of this study.

Table 29

T test: Two-Sample Assuming Equal Variances: Multiplication Fact Fluency

Statistic	Algebra	Pre-Algebra
Mean	35.200	33.533
Variance	3.011	11.209
Observations	20.000	45.000
Pooled Variance	8.737	
Hypothesized Mean Difference	0.000	
Df	63.000	
t stat	2.098	
P(T<=t) two-tail	0.0399	
t critical two-tail	±1.998	

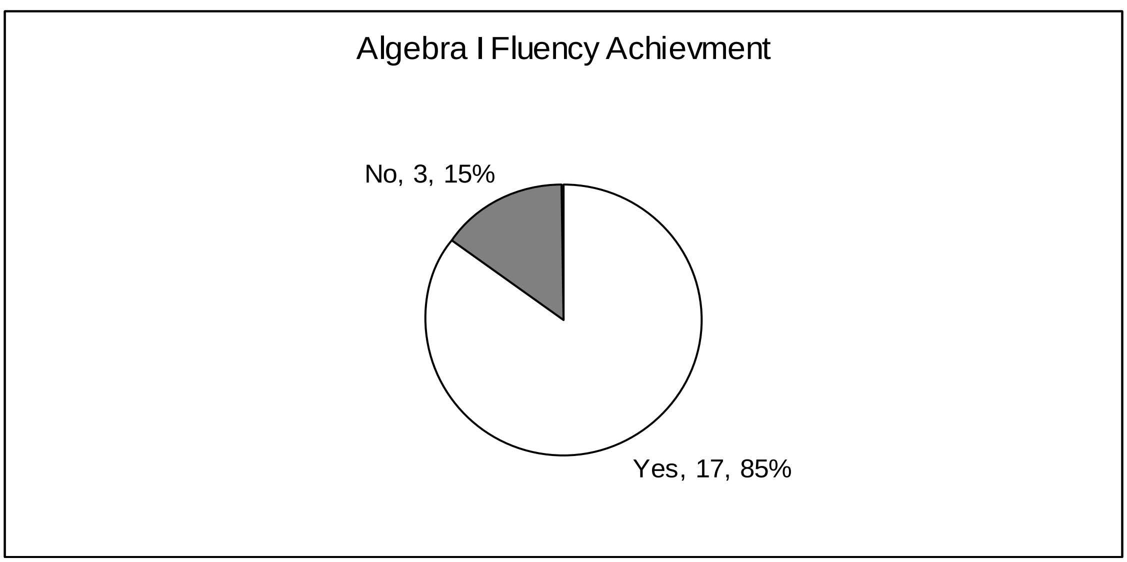

Figure 8. Percentage of Students in Algebra I Who Achieved (Yes) and Not Achieved (No) Multiplication Fact Fluency

As shown in Figure 9, 38% or 17 out of 45 students in Pre-Algebra failed to achieve multiplication fact fluency, whereas 62% or 28 out of 45 achieved multiplication fact fluency. Similar to the students in Algebra I, the students in Pre-Algebra achieved multiplication fact fluency if they missed no more than one problem out of 36 total problems in 1 minute and 48 seconds. The 45 students represented the sample for the Pre-Algebra population of this study.

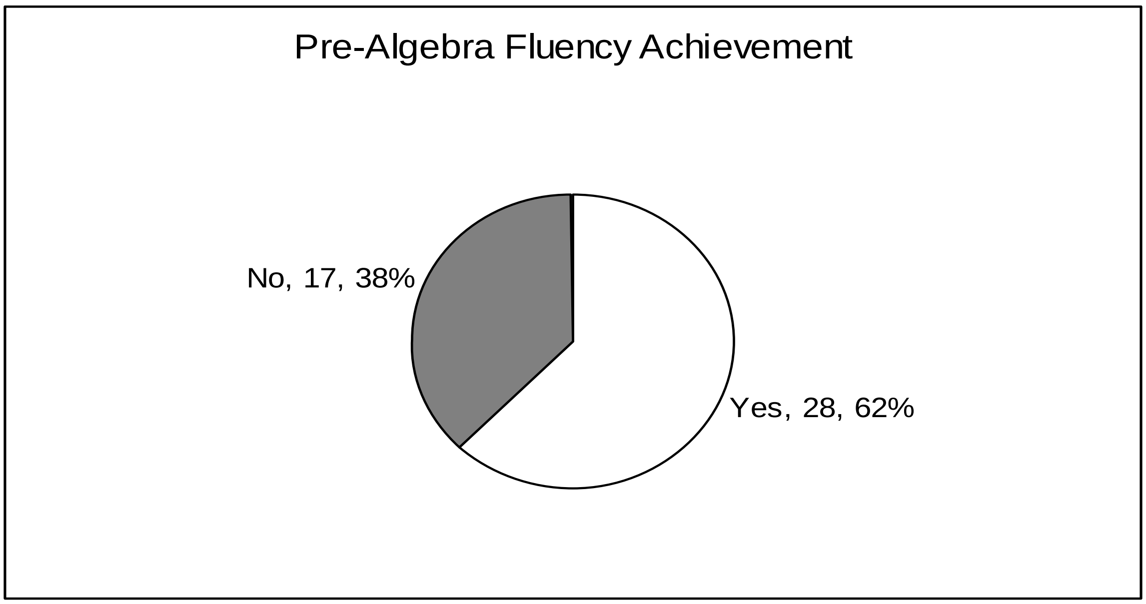

Figure 9. Percentage of Students in Pre-Algebra Who Achieved (Yes) and Not Achieved (No) Multiplication Fact Fluency

This researcher used a *t* test for difference in means to compare the Algebra I and Pre-Algebra speed-recall scores. Prior to performing the *t* test, this researcher conducted an *F* test to determine whether or not the two sample variances were statistically equal, using the null hypothesis: There is no difference in variance. The results of the *F* test indicated that the null hypothesis was not rejected and variances were equal. The *F* test value, 1.373, fell below the critical value, 1.828; therefore, this researcher used a *t* test with variances statistically equaling each other. The results in Table 30 revealed the *t* test value, 2.94, was larger than the critical value of 2.00; hence, this researcher rejected the null hypothesis. This data supported a significant difference existed between the Algebra I and Pre-Algebra speed-recall scores; students in Algebra I achieved a higher performance.

Table 30

T test: Two-Sample Assuming Equal Variances: Multiplication Fact Fluency

Statistic	Algebra	Pre-Algebra
Mean	24.550	20.333
Variance	35.208	25.636
Observations	20.000	45.000
Pooled Variance	28.523	
Hypothesized Mean Difference	0.000	
df	63.000	
t stat	2.938	
P(T<=t) two-tail	0.0046	
t critical two-tail	±1.998	

Null Hypothesis # 3. There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved 80% or higher on the average of the first and second semester mathematics assessments. This researcher tested the null hypothesis with the *z* test for difference in proportions with a 95% confidence level for students in Algebra I and in Pre-Algebra. This researcher used the sample size of 32 students in Algebra I and the sample size of 45 students in Pre-Algebra. For each z test, the critical values at a 95% confidence level were -1.96 and 1.96.

Table 31 shows the results of the *z* test for difference in proportion for students in Algebra I. The *z* test value, 0, fell between -1.96 and +1.96; thus, this researcher could not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Algebra I who have achieved multiplication fact fluency and a first and second average semester assessment grade of 80% or higher, so the proportions are considered to be statistically the same.

Table 31

Algebra I Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and an 80% or Higher with a First and Second Semester Average Assessment Grade

Statistical Variables	Statistical Values
Fluency population proportion	0.906
Average Assessment population proportion	0.906
Critical Value	±1.96
z test	0.000

Table 32 shows the results of the *z* test for students in Pre-Algebra. The *z* test value, 2.53, is greater than 1.96; thus, this researcher rejected the null hypothesis. There is enough evidence to reject the claim that there is no difference in the proportions of students in Pre-Algebra who have achieved multiplication fact fluency and a first and second semester average assessment grade of 80% or higher. The proportion of students fluent in mathematical fact fluency is significantly higher than the proportion of students in Pre-Algebra who have achieved an average first and second semester assessment grade of 80% or higher.

Table 32

Pre-Algebra Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and an 80% or Higher with a First and Second Semester Average Assessment Grade

Statistical Variables	Statistical Values
Fluency population proportion	0.622
Average Assessment population proportion	0.356
Critical Value	±1.96
z test	2.53

Null Hypothesis # 4. There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved proficient or advanced on the mathematics MAP test. Students achieved a mathematics MAP proficient or advanced descriptor with a scale score of 710 or higher. This researcher tested the null hypothesis with the *z* test for difference in proportions with a 95% confidence level for students in Algebra I and in Pre-Algebra. This researcher used the sample size of 32 students in Algebra I and the sample size of 45 students in Pre-Algebra. For each *z* test for difference in the proportions, the critical values at a 95% confidence level were -1.96 and 1.96.

Table 33 shows the results of the *z* test for difference in proportion for students in Algebra I. The *z* test value, -1.84, is between -1.96 and +1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Algebra I who have achieved multiplication fact fluency and a mathematics MAP achievement of proficient or advanced.

Table 33

Algebra I Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and a Mathematics MAP Test Scale Score of 710 or Higher

Statistical Variables	Statistical Values
Fluency population proportion	0.906
$\geq$ 710 mathematics MAP test scale score proportion	1.00
Critical Value	±1.96
z test	-1.84

Table 34 shows the results of the *z* test for difference in proportion for students in Pre-Algebra. The *z* test value, 1.71, is between the critical values of -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. .

Table 34

Pre-Algebra Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and a Mathematics MAP Test Scale Score of 710 or Higher

Statistical Variables	Statistical Values
Fluency population proportion	0.622
$\geq$ 710 mathematics MAP test scale score proportion	0.444
Critical Value	±1.96
z test	1.71

There is not enough evidence to reject the claim that there is no difference in the proportions of students in Pre-Algebra who have achieved multiplication fact fluency and a mathematics MAP achievement of proficient or advanced.

Null Hypothesis # 5. There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved a GMRT grade equivalency of eighth grade or higher. This researcher tested the null hypothesis with the *z* test for difference in proportions with a 95% confidence level for students in Algebra I and in Pre-Algebra. This researcher used the sample size of 32 students in Algebra I and the sample size of 45 students in Pre-Algebra. For each *z* test for difference in proportions, the critical values at a 95% confidence level were -1.96 and 1.96.

Table 35 shows the results of the *z* test for difference in proportion for Algebra I. The *z* test value, -0.44, is between the critical values of -1.96 and 1.96; thus, this researcher did not reject the null hypothesis.

Table 35

Algebra I Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and a GMRT Grade Equivalency of Eighth Grade or Higher

Statistical Variables	Statistical Values
Fluency population proportion	0.906
GMRT grade equivalency of eighth or higher proportion	0.938
Critical Value	±1.96
z test	-0.44

There is not enough evidence to reject the claim that there is no difference in the proportions of students in Algebra I who have achieved multiplication fact fluency and a GMRT grade equivalency of eighth grade or higher.

Table 36 shows the results of the *z* test for difference in proportions for Pre-Algebra. The *z* test value, 1.71, is between the critical values of -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Pre-Algebra who have achieved multiplication fact fluency and a GMRT grade equivalency of eighth grade or higher.

Table 36

Pre-Algebra Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and a GMRT Grade Equivalency of Eighth Grade or Higher

Statistical Variables	Statistical Values
Fluency population proportion	0.622
GMRT grade equivalency of 8th grade or higher proportion	0.489
Critical Value	±1.96
z test	1.24

Null Hypothesis # 6. There will be no difference in the proportion of students without multiplication fact fluency and the proportion of students who did not achieve proficiency or advanced on the mathematics MAP test. This researcher tested the hypothesis with the *z* test for difference in proportions with a 95% confidence level for students in Algebra I and in Pre-Algebra. This researcher used the sample size of 32 students in Algebra I and the sample size of 45 students in Pre-Algebra. For each *z* test for difference in the proportions, the critical values at a 95% confidence level were -1.96 and 1.96.

Table 37 shows the results of the *z* test for difference in proportion for students in Algebra I. The *z* test value, 1.78, is between the critical values of -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Algebra I who have not achieved multiplication fact fluency and the proportion of students who did not achieve a proficient or advanced score on the mathematics MAP test.

Table 37

Algebra I Testing of the Difference between Proportions of Students Who Did Not Achieve Multiplication Fact Fluency and a Proficient or Advanced Score on the Mathematics MAP Test

Statistical Variables	Statistical Values
Non-fluency population proportion	0.094
Non-proficient or advanced mathematics MAP score proportion	0
Critical Value	±1.96
z test	1.78

Table 38 shows the results of the *z* test for difference in proportions for students in Pre-Algebra. The *z* test value, 0.38, is between the critical values of -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Pre-Algebra who have not achieved multiplication fact fluency and the proportion of students who did not achieve a proficient or advanced score on the mathematics MAP test.

Table 38

Pre-Algebra Testing of the Difference between Proportions of Students Who Did Not Achieve Multiplication Fact Fluency and a Proficient or Advanced Score on the Mathematics MAP Test

Statistical Variables	Statistical Values
Non-fluency population proportion	0.600
Non-proficient or advanced mathematics MAP score proportion	0.556
Critical Value	±1.96
z test	0.38

Null hypothesis # 7. There will be no difference in the proportion of students without multiplication fact fluency and the proportion of students who did not achieve a GMRT grade equivalency of eighth grade or higher. This researcher tested the null hypothesis with the *z* test difference in proportions with a 95% confidence level for students in Algebra I and in Pre-Algebra. This researcher used the sample size of 32 students in Algebra I and the sample size of 45 students in Pre-Algebra. For each *z* test for difference in the proportions, the critical values at a 95% confidence level were -1.96 and 1.96.

Table 39 shows the results of the *z* test for difference in proportions for students in Algebra I. The *z* test value, 0.34, is between -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Algebra I who have not achieved multiplication fact fluency and the proportion of students who did not achieve a GMRT grade equivalency of eighth grade or higher.

Table 39

Algebra I Testing of the Difference between Proportions of Students Who Did Not Achieve Multiplication Fact Fluency and a GMRT Grade Equivalency of Eighth Grade or Higher

Statistical Variables	Statistical Values
Non-fluency population proportion	0.094
Non- *GMRT grade equivalency of 8th grade or higher*	.063
Critical Value	±1.96
z test	0.34

Table 40 shows the results of the *z* test for difference in proportions for students in Pre-Algebra.

Table 40

Pre-Algebra Testing of the Difference between Proportions of Students Who Did Not Achieve Multiplication Fact Fluency and a GMRT Grade Equivalency of Eighth Grade or Higher.

Statistical Variables	Statistical Values
Non-fluency population proportion	0.600
Non- GMRT grade equivalency of 8th grade or higher	0.511
Critical Value	±1.96
z test	0.86

The *z* test value, 0.86, is between -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Pre-Algebra who have not achieved multiplication fact fluency and the proportion of students who did not achieve a GMRT grade equivalency of eighth grade or higher.

Null Hypothesis # 8. There will be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved proficient or advanced on the Algebra I EOC test. Students achieved a proficient or an advanced descriptor if they achieved at least a 21-point raw score. This researcher tested the null hypothesis with the *z* test for difference in proportions with a 95% confidence level for the students in Algebra I and in Pre-Algebra. This researcher used the sample size of 32 students in Algebra I. For the *z* test for difference in proportions, the critical values at a 95% confidence level are -1.96 and 1.96.

Table 41

Algebra I Testing of the Difference between Proportions of Students Who Achieved Multiplication Fact Fluency and a Proficient or Advanced Score on the Algebra I EOC Test

Statistical Variables	Statistical Values
Fluency population proportion	0.906
Proficient or advanced Algebra I EOC score proportion	1.00
Critical Value	±1.96
z test	-1.84

Table 41 shows the results of the *z* test for difference in proportions for students in Algebra I. The *z* test value, -1.84, is between -1.96 and 1.96; thus, this researcher did not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the proportions of students in Algebra I who achieved multiplication fact fluency and a proficient or advanced score on the Algebra I EOC test.

Summary

Within the confines of the demographics of this particular middle school, data did not support a strong enough correlation to determine a relationship between the Pre-Algebra and Algebra I students' multiplication fact speed score with the following dependent variables: first and second semester average assessment grades, mathematics MAP test scale score, reading score, and Algebra I EOC raw score.

A comparison of performance between Pre-Algebra students and Algebra I students indicated a dfference in both speed-recall and fact fluency. Students in Algebra I showed no proportional difference between students who achieved multiplication fact fluency and those who achieved an 80% or higher on the average of first and second semester assessment grades. The rest of the data showed no proportional difference between students who achieved multiplication fact fluency and the following assessment scores: a proficient or advanced on the mathematics MAP test, proficient or advanced on the Algebra I EOC test, and a GMRT grade equivalency of eighth grade or higher. This researcher attributed the students' ability to recall multiplication facts quickly and accurately as an influential factor that provided the necessary skills as a means to the advancement in higher level mathematics and reading.

Conclusion

This quantitative study investigated a possible relationship between the multiplication fact speed-score and higher level mathematics learning, and also investigated the GMRT grade equivalency score of eighth grade students in Algebra I and Pre-Algebra. This study investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and the percentage of students with a GMRT equivalency grade at eighth grade or higher.

This researcher conducted a quantitative analysis of the data for all eight null hypotheses. The first null hypothesis investigated whether or not a possible relationship existed between the multiplication fact speed-recall quiz and the following dependent variables: a) Pre-Algebra and Algebra I first and second semester average assessment grade, b) mathematics MAP test scale score, c) GMRT grade equivalency, and, d) Algebra I EOC raw score. This researcher calculated the PPMC and performed a *t* test for significance at a 95% confidence level. This researcher concluded that no relationship existed between multiplication fact speed-recall and each dependent variable.

This researcher used a *t* test for difference in proportion for the second null hypothesis to determine whether or not a difference in fluency and speed-recall scores existed between the students in Algebra I and Pre-Algebra. The null hypothesis stated that there would be no difference in the multiplication fact fluency and speed-recall scores between the students in Algebra I and in Pre-Algebra. The researcher used a *t* test for difference in means to perform this test to compare the Algebra I and Pre-Algebra fluency quiz scores. The researcher concluded a significant difference did exist between the Algebra I and Pre-Algebra fluency and speed-recall scores; students in Algebra I achieved a higher performance.

The null hypotheses of 3, 4, 5, and 8 for each dependent variable stated there would be no difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved each dependent variable. This researcher performed *z* tests for difference in proportion at a 95% confidence level to test the proportional percentage comparisons between students who have achieved multiplication fact fluency and the students who achieved with each of the following dependent variables: 80% or higher on the first and second semester average assessment grade, a proficient or advanced descriptor mathematics MAP score, a GMRT grade equivalency of eighth grade or higher, and a proficient or advanced descriptor Algebra I EOC score. Except for the 80% or higher average of the first and second semester mathematics assessments with students in Pre-Algebra only, there was not enough evidence to reject the claim that there was no difference in the proportions (Algebra I to Algebra I and Pre-Algebra to Pre-Algebra) of students in Algebra I and Pre-Algebra who had achieved multiplication fact fluency and the proportion of students who achieved each of the other three dependent variables.

This researcher also performed *z* tests for difference in proportion to analyze the percentage comparisons between students who did not achieve multiplication fact fluency and the students who did not achieve the two dependent variables of a proficient or advanced descriptor mathematics MAP score and a GMRT grade equivalency of eighth grade or higher. Null hypothesis 6 stated that there would be no difference in the proportion of students who did not achieve multiplication fact fluency and the proportion of students who did not achieve a proficient or advanced on the mathematics MAP test. Null hypothesis 7 stated that there would be no difference in the proportion of students who did not achieve multiplication fact fluency and the proportion of students who did not achieve a GMRT grade equivalency of eighth grade or higher. This researcher tested both hypotheses with a *z* test for difference in proportions with a 95% confidence level for students in Algebra I and Pre-Algebra. This researcher concluded there was not enough evidence to reject the claim that there is no difference in the proportions (Algebra I to Algebra I and Pre-Algebra to Pre-Algebra) of students who have not achieved multiplication fact fluency and the proportion of students who have not achieved a proficient or advanced score on the mathematics MAP test, or the proportion of students who had not achieved GMRT grade equivalency of eighth grade or higher.

Chapter 5 discusses the implications of the results found in each of the statistical analyses conducted as part of this study. Chapter 5 will also include implications for multiplication fact fluency for higher mathematics learning, recommendations for further studies, and a conclusion.

Chapter 5 - Discussion, Interpretations, Implications, and Recommendations

Literature and Investigation Review

The 1989 and 2000 Standards by NCTM ensured an educational change with the mathematics instruction to develop students with a mathematical power for problem solving. Although the Standards, 1989 and 2000, provided an outline to improve mathematics achievement and academics, there is still a large percentage of students not meeting mathematics proficiency, as measured by state assessment; thus a large majority of school districts across the nation are failing to meet Adequate Yearly Progress. The timeline of NCLB for all students to become 100% proficient is by 2014.

Research from multiple sources concluded mastery of the basic facts, which includes single-digit multiplication facts, to be an important piece of subsequent knowledge to achieve higher level math (Caron, 2007; Cavanagh, 2008; Wong & Evans, 2007; Jarema, 2010; Loveless & Coughlan, 2004; Wu, 1999). Cavanagh (2008) reported students without sound knowledge of their multiplication facts are at a profound disadvantage in further mathematics achievement.

This quantitative study investigated a possible relationship between the multiplication fact speed-score and higher level mathematics learning, and also investigated the GMRT grade equivalency score of eighth grade students in Algebra I and Pre-Algebra. This study investigated whether or not a difference existed between the percentage of students who achieved multiplication fact fluency and the percentage of students recognized with higher level mathematics learning and a GMRT equivalency grade of eighth grade or higher.

Review of the Methodology

The population of this study included eighth grade students from a Missouri middle school. There were 116 students, 51 girls and 65 boys, whose data was utilized in the analysis for this study. Of the 51 girls, 10 girls came from Algebra I, while the other 41 came from Pre-Algebra. Of the 65 boys, 22 boys came from Algebra I, while 43 boys came from Pre-Algebra. There were 105 Whites and 11 Blacks who participated in the study.

This researcher used two different multiplication quizzes; one quiz determined the students' speed-recall, while the other quiz determined fluency. Both multiplication fact quizzes entailed 36 single-digit multiplication problems, which only used factors 2 through 9. The difference between each test was the order of the problems and the given amount of time. Although Crawford (2003) recommended a range of 30 to 40 problems per minute for multiplication automaticity, this researcher used a time limit, suggested by Michalczuk (2007), of 3 seconds or less. This researcher maximized the time limit of 3 seconds per problem for the fluency test; therefore, students were given 1 minute and 48 seconds to successfully complete the quiz with one error for multiplication fact fluency. With the multiplication fact speed-recall quiz, this researcher arbitrarily chose 45 seconds to perform as many multiplication problems as possible. The number of problems each student correctly answered on the multiplication fact speed-recall quiz defined the student's speed-recall score.

Both multiplication tests acted as the independent variable for their perspective hypotheses while the dependent variables entailed the following: a) each student's combined average first and second semester mathematics assessment score, b) 2011 mathematics MAP test scale score, c) 2011 Algebra I EOC raw score, and d) GMRT grade equivalency. For the first and second semester mathematics average assessment scores this researcher added each student's assessment score from each quarter and divided by the number of assessments as the quantitative representation for this dependent variable for higher level mathematics learning. For the 2011 mathematics MAP test, this researcher used the scale score as the quantitative representation for this dependent variable for higher level mathematics learning. For the 2011 Algebra I EOC test, this researcher used the raw score as the quantitative representation for this dependent variable for higher level mathematics learning. For the 2011 GMRT, this researcher used the grade equivalency as the dependent variable.

This researcher concluded each dependent variable, except for the GMRT as a limitation. The average first and second semester mathematics assessment score was limited to only the specific school of study. Both the mathematics MAP and Algebra I EOC test was limited only to the students of Missouri. The GMRT test is a national reading test for all schools to have access and measure the students' reading achievement level.

Noted Observations during Multiplication Fact Quiz Implementations

Throughout the implementation of both multiplication fact tests, this researcher mainly noticed a number of Pre-Algebra students using their hands to compute the product for some of the problems; hence, these students could not achieve multiplication fact fluency. The observation supports the study by Steel and Funnel (2001). Steel and Funnel (2001) revealed students who selected the retrieval method (quick and effortless recall), rather than nonretrieval concrete strategies like the usage of fingers, was by far a more effective strategy for the achievement of multiplication fact fluency. Although this researcher did not conduct a survey to determine why some students used their fingers, this researcher suspected students utilized their fingers for certain products that students had not committed to memory. In respect to higher level mathematics learning or complex or multi-step mathematical problems or applications, Hecht (2002) concluded participants who used counting as their primary strategy for computing and solving substantially had difficulty solving or working out such problems, due to an overloaded working memory.

Interpretation of the Results

Hypothesis # 1. There will be a relationship between the speed-recall score and the 2010–11 first and second semester average mathematics assessment score, 2011 mathematics MAP test scale score, 2011 Algebra I EOC test raw score, and GMRT grade equivalency. This researcher used a Pearson Product Moment Correlation Coefficient (PPMCC) statistical test to determine whether a strong correlation or relationship existed between the independent variable (speed-recall score) and each dependent variable. The results of this study revealed no relationship existed between the independent and each dependent variable. The results of this study did not support hypothesis # 1: significant relationship existed between the speed-recall score and each of the dependent variables for higher level mathematics which included reading achievement scores.

Although this study did not find a correlation between mathematics achievement and speed-recall, additional research is needed to determine whether the speed of the student's ability to recall multiplication facts fosters any possible relationship with mathematics achievement. Speed-recall of the multiplication facts is understood to be more of a multiplication fact automaticity that includes a range rather than an exact amount of time for higher mathematics learning. Crawford of Otter Creek Institute recommended a range of 30 to 40 problems per minute as a goal for multiplication fact automaticity. This researcher did not include the writing speed as a differential factor within this study. The time length it takes students to write certain digits may vary with participants. Further research that incorporates the participant's writing speed or the "presentation of a visual stimulus to a keyboard or oral response" (Crawford, 2003, p. 11) would minimize the participant's response time between the problem and answer.

The study also investigated Anderson's (2010) connection between reading and math fluency as functional skills for improved reading and math academic achievements. This study determined whether a relationship existed between the speed of multiplication fact recall and the grade-level reading equivalency through the GMRT. This study did not find a correlation or a relationship between reading achievement and speed-recall: the speed-recall of one functional skill, multiplication facts, did not correlate with the grade-level achievements of the other functional skill, reading. Anderson (2010) stated "fluency is the same principle in reading and math – requiring a functional skill (decoding or algorithmic skills) but not necessarily comprehension" (p. 1). Although there may be a connection between the cognitive ability of symbol processing between reading and mathematics, "the syntax of math and the syntax of running narrative are different and require different strategies for instruction and learning" (Fite, 2002, p. 9). A no correlation between multiplication fact speed-recall and reading ability may be a result of the relational differences of understanding between mathematics and reading text.

Hypothesis # 2. There will be a difference in fluency scores and in speed-recall scores when comparing multiplication fact quiz scores with students in Algebra I to the multiplication fact quiz scores with students in Pre-Algebra. An *F* test was performed to determine whether there was an unequal or equal variance. A *t* test for the difference in means for an equal variance was used to test the second hypothesis. Research relates automaticity of basic math facts, which includes multiplication, positively correlates with a higher successive rate of mathematics achievement (French, 2005). The results of this study supported hypothesis # 2: a significant difference existed between the speed-recall and fluency scores with students in Algebra I and students in Pre-Algebra.

One hundred percent of the Algebra I population from the previous school year achieved a seventh grade mathematics proficient or advanced mathematics MAP descriptor, whereas only about 40% of the Pre-Algebra population achieved a seventh grade mathematics proficient or advanced mathematics MAP descriptor. The percentage differences of students who achieved a proficient or advanced descriptor score from the previous seventh grade mathematics MAP test for students in Algebra I should equate a higher or better average speed-recall score than for students in Pre-Algebra. Mathematics competence and understanding was higher for the students in Algebra I when compared statistically to the students in Pre-Algebra. The students in Algebra I who are of the same age as the students in Pre-Algebra not only revealed a stronger ability to learn or understand higher level mathematics, but also statistically showed faster multiplication fact speed-recall scores. This finding supports Steel and Funnel’s research study (2001) that students who were able to retrieve their multiplication facts with accurate speed performed better on mathematics assessments than students who could not quickly retrieve their multiplication facts. The ability to quickly and accurately recall basic math facts provides a significant advantage for students to free up cognitive capacity to learn higher and more rigorous mathematics just as the data revealed for students enrolled in Algebra I (Caron, 2007).

Hypothesis # 3. There will be a difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved 80% or higher on the average of the first and second semester mathematics assessments. This researcher statistically performed a *z* test for the difference in proportions to test the third hypothesis. The study statistically showed that there was not enough evidence to support the claim that there will be a difference in the proportions of students in Algebra I who have achieved multiplication fact fluency and an average first and second semester assessment grade of 80% or higher. This finding supports the study by Lin and Kubina, Jr. (2005) between fluency and higher or more advanced multiplication problems: multiplication fact fluency, rather than accuracy as a possible or alternate solution to mathematical learning deficits.

In contrast with the students in Algebra I, the results of this study supported the hypothesis for the students in Pre-Algebra. There was enough evidence support the claim that there will be a difference in the proportions of students in Pre-Algebra who have achieved multiplication fact fluency and an average first and second semester assessment grade. Sixty-two percent of the students in Pre-Algebra achieved multiplication fact fluency while only 35.6% achieved an 80% or higher average assessment grade. This significant difference between the percentages of Pre-Algebra students who achieved multiplication fact fluency and an 80% or higher average assessment grade may be a result of mathematics anxiety.

Aschraft (2002) defined math anxiety to be “a feeling of tension, apprehension, or fear that interferes with math performance” (p. 181). Ashcraft (2002) also suggested students with math anxiety also showed disruptions with the cognitive processing that supported working memory for learning and understanding. Students with math anxiety undergo the following toward mathematics: a) avoidance of mathematics, b) negative attitudes, c) negative self-perceptions about their math abilities, and d) end up with lower math competence and achievement (Ashcraft, 2002). A number of students could perceivably fit Ascraft’s (2002) definition of math anxiety, especially on assessments that had problems requiring the students to perform multi-steps to correctly answer the problems.

Hypothesis # 4. There will be a difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved proficient or advanced on the mathematics MAP test. This researcher statistically performed a *z* test for the difference in proportions to test the fourth hypothesis. The statistical results of this study revealed that there was not enough evidence to support the claim that there will be a difference in the proportions for students in Algebra I and Pre-Algebra who achieved multiplication fact fluency and a mathematics MAP test scale score of 710 or higher. The results of this hypothesis supports the understanding that students are able to engage better in higher level mathematics learning if students have mastered basic math operations like multiplication facts (Clavel, 2003).

The percentage of students in Algebra I meeting the 80% or higher average assessment score and achieving a proficient or advanced descriptor score on the mathematics MAP test results were a little more similar than with the students in Pre-Algebra. The population percentage for the students in Pre-Algebra was significantly higher for a proficient or advanced descriptor score on the mathematics MAP test than the population percentage for an 80% or higher average assessment score for the year. While 35.6% of the students in Pre-Algebra achieved an 80% or higher average assessment score for the year, 44.4% of the Pre-Algebra students achieved a proficient or advanced descriptor score on the mathematics MAP test. The 8.8% marginal difference was significant enough when this researcher used a *z* test with a 95% confidence level. The result of this difference, students performing slightly better on the mathematics MAP test, could be possibly understood that students did not have to be tested on a performance event problem due to state budget constraints that would have required the students to use higher order mathematical thinking. This supports Rasmussen and Bisanz (2005) conclusion of their study: the more mathematical processes that occupied the working memory portion, the more difficulty students had in accurate problem solving completion. A higher level of mathematical thinking requires more working memory; thus, this would impact the ability for students that experience math anxiety to mathematically achieve (Ashcraft, 2002). Since the mathematics MAP test did not require the students to complete a performance event, the possible negative effects of this problem did not work against the students' overall mathematics MAP test score.

Hypothesis # 5. There will be a difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved a GMRT grade equivalency of eighth grade or higher. This researcher statistically performed a *z* test for the difference in proportions to test the fifth hypothesis. The statistical results of this study revealed that there is not enough evidence to support the claim that there will be a difference in the proportions of students in Algebra I and Pre-Algebra who have achieved multiplication fact fluency and a GMRT grade equivalency of eighth grade or higher. This supports Anderson's (2010) connection between math and reading fluency: "the automaticity in one skill can then lead to increased speed and understanding in other areas" (p. 2). Students with a multiplication fact fluency skill may lead to a possible increase with reading fluency.

This also supports Fite's (2002) research: "Being able to think mathematically is reflected by the ability to read and comprehend mathematical symbolism in much the same way that we read words" (p. 9). The connection between reading proficiency and mathematics performance stemmed from one's cognitive ability to process symbols (Fite, 2002). "This symbol processing ability is the basis for both language proficiency and math achievement" (Fite, 2002, p. 8). This ability to process symbols is an essential foundation in reading and mathematics. Although good reading skills may not translate into good mathematical solving skills because of various syntax differences, poor reading skills normally will translate into poor mathematical skills (Fite, 2002). "Trying to improve math performance for a student who cannot read will be ineffective" (Fite, 2002, p. 11). Reading is an imperative skill for any aspect of higher learning which includes higher mathematics learning.

Hypothesis # 6. There will be a difference in the proportion of students without multiplication fact fluency and the proportion of students who did not achieve proficiency or advanced proficiency on the mathematics MAP test. This researcher statistically performed a *z* test for the difference in proportions to test the sixth hypothesis. The statistical results of this study revealed that there is not enough evidence to support the claim that there will be a difference in the proportions of students in Algebra I and Pre-Algebra who have not achieved multiplication fact fluency and not achieved a proficient or advanced score on the mathematics MAP test. Although this hypothesis is similar to hypothesis #4, this researcher wanted to see the opposite or negative effect: students who have not achieved multiplication fact fluency would, also not achieve proficiency or advanced descriptor score on the mathematics MAP test. The data statistically revealed students who have not achieved multiplication fact fluency are most likely to experience failure and difficulty with higher level mathematics learning (Greenwald, n.d.; Jarema, 2010).

Dr. Fite’s (2002) literature review on the connection between reading and math found math achievement with word problems required “both reading for comprehension and computational skills” (p. 10). The aspect of this hypothesis reflects the student’s ability to solve mathematics word problems accurately, especially when it comes to the mathematics MAP test. Both reading and basic mathematic fluency are functional skills that serve and assist the student’s ability to successfully complete mathematics word problems.

A large majority of the multiple choice and constructive response items on the mathematics MAP test require reading comprehension and mathematics skill. Zentall and Ferkis (1993) stated both reading comprehension and mathematics computational skills are functional skills that serve and assist the students' cognitive ability to complete mathematics word problems at a greater rate than students with a low reading and mathematics computational skills. Mathematics word problems not only require both procedural skills (a step-by-step mathematical process) and conceptual skills (underlying meaning and understanding of mathematics concepts), but also reading comprehension skills (Fite, 2002).

A study by Fuchs et al. (2004) hypothesized from their statistical results that mathematics difficulties (conceptual and procedural) may contribute more to the deficiency of mathematics problem solving than reading comprehension difficulties. The results of this study cannot determine if the student's mathematics MAP test score contributed more from the student's mathematics or reading ability. Both mathematics conceptual and reading comprehension skills are similar in the following manner: students must understand the meaning of words to accurately process information accurately.

Hypothesis # 7. There will be a difference in the proportion of students without multiplication fact fluency and the proportion of students who did not achieve a GMRT grade equivalency of eighth grade or higher. This researcher statistically performed a *z* test for the difference in proportions to test the seventh hypothesis. The statistical results of this study revealed that there is not enough evidence to support the claim that there will be a difference in the proportions of students in Algebra I and Pre-Algebra who have not achieved multiplication fact fluency and not achieved a GMRT grade equivalency of eighth grade or higher.

The purpose of this hypothesis is similar to the purpose of hypothesis #6: this researcher wanted to see the opposite or negative effect of students who have not achieved multiplication fact fluency would, also, not achieve a GMRT grade equivalency of eighth grade or higher. Thomas (2001) performed a study, “A Model of Mathematics Achievement using Proficiency Scores,” and concluded a student’s reading level had an affect with math proficiency. “Trying to improve math performance for a student who cannot read will be ineffective” (as cited in Fite, 2002, p. 11). Statistical calculations of this study support both Fite’s (2002) and Thomas’s (2001) precepts of reading and math fluency: both reading comprehension and math skill proficiency are important functional skills for higher level mathematics learning that involves mathematical word problems.

Hypothesis # 8. There will be a difference in the proportion of students with multiplication fact fluency and the proportion of students who achieved proficient or advanced on the Algebra I EOC test. This researcher statistically performed a *z* test for the difference in proportions to test the eighth hypothesis. The statistical results of this study revealed that there is enough not evidence to support the claim that there will be a difference in the proportions of students in Algebra I who achieved multiplication fact fluency and a proficient or advanced score on the Algebra I EOC test. The result of this statistical test directly supports and implies students with multiplication fact fluency also were able to achieve a proficient or advanced Algebra I EOC assessment score; this positively connects multiplication fact fluency with mathematics proficiency as it is defined and described by the 2010 CCSS.

Implications for Multiplication Fact Fluency

The results of this study imply that multiplication fact-fluency is an important mathematical tool for higher level mathematics learning or understanding. “If children do not memorize the math facts, they will always struggle with math” (Greenwald, n.d., p. 1). The 2010 CCSS also recognized the importance of multiplication facts and stipulated that students by the end of third grade should know from memory or a quick-recall of all products of two one-digit numbers for higher-grade levels of math. Caron (2007) emphasized the following about the need for multiplication facts:

> Without this seemingly simple set of knowledge, by eighth grade, students are virtually denied anything but minimal growth in any serious use of mathematics or related subjects for the remainder of their school years and, most likely, the rest of their lives. This includes any multiplying, both single and multiple digit, whether on a computation sheet or in a word problem. (p. 279)

Building a mathematics understanding requires the development of foundational math skills like multiplication fact fluency through proper instructional algorithmic techniques rather than just rote learning (Wu, 1999; Johnson, 2001; Wallace & Gurganus, 2005; Cavanagh, 2008).

Wu (1999) emphasized the importance of standard algorithms in the elementary mathematics curriculum. “If there is any so-called harmful effect in learning the algorithms, it could only be because they are not taught properly” (p. 6). He concluded standard algorithms, if taught properly, contained necessary mathematical reasoning skills for a deeper mathematics understanding of our decimal number system through the usage and knowledge of some mathematical properties: commutative, associative, and distributive.

Wallace and Gurganus (2005) not only emphasized the instruction of algorithmic mathematics rules, but also the usage of manipulatives and/or pictorials to help illustrate problem representation and to build conceptual understanding. "Forcing memorization before children have moved through concept development results in acquiring knowledge that has little meaning or usefulness and often creates a dislike of mathematics" (Wallace & Gurganus, 2005, p. 31). They proposed the following as the most effective sequence of multiplication fact instruction: a) introduce concepts through problem situations and link the new concepts with prior knowledge, b) provide concrete (manipulatives) and semi-concrete (pictorials) representations prior to any symbolic or abstract mathematics notations, c) incorporate algorithmic mathematics rules for speed and accuracy, and d) provide mixed practice that include applications and algorithms. Multiplication fact mastery no longer means rote memorization of the facts, but a balanced instructional approach between conceptual understanding and computational fluency (Wallace & Gurganus, 2005). "Rote learning might take place in the context of multidigit multiplication…when the teacher doesn't possess a deep enough understanding of the underlying mathematics to explain it well" (Wu, 1999, p. 6). Zentall and Ferkis (1993) also add that "Less knowledgeable teachers were more likely to explain how to solve problems and to use nonverbal checking and monitoring of student work rather than verbally questioning and listening to students" (p. 15). The works of skilled teachers who understand the subject matter are the educators who must implement the mathematics standards and curriculum in the classroom. The quality of mathematics instruction is not solely, but mostly dependent on the teachers' knowledge of the content (Ball, Bass, & Hill, 2005) and sequence of instruction (Witzel & Riccomini, 2007). "Studies over the past 15 years consistently reveal that the mathematical knowledge of many teachers is dismayingly thin" (Ball et al., 2005, p. 14). The instructional practices of our teachers are not necessarily confined, but definitely related to the implemented curriculum and the professional development in-services provided by the coordinators and administrators of school districts (Ball et al., 2005; Ross et al., 2008).

The study by Ball et al. (2005) created a "mathematical knowledge for teaching" through many close examinations of the actual teachers work with elementary school mathematics. The study found that teachers who successfully implemented the practices of "mathematical knowledge for teaching" also produced higher gains in student achievement. Teachers not only need to have a specialized fluency with the mathematical language of their content, but also the ability "to think from the learner's perspective and to consider what it takes to understand a mathematical idea for someone seeing it for the first time" (Ball et al., 2005, p. 21). A teacher's role in the classroom should include all of the following: a) high expectations, b) asking questions for understanding, and c) encouragement (Kilpatrick et. al, 2001). Effective and quality professional development coordinated by the district's coordinators and administrators truly help develop and maintain high quality instructional leaders to teach proper classroom mathematics like multiplication fact fluency.

Recommendations for Further Studies

The interpretation of the results largely depended on a limited number of participants of one particular school within a timeframe of one year. The limitations of this study involved three areas: specific setting, participants, and timeframe of study. Each of these limitations opens the opportunity for further research on the topics discussed in this study.

The specific setting of this study reflects a percentage of students across the county, state, and country with similar demographics. Future studies should not only include similar middle schools of this setting around this state, but also with similar middle schools across the country. Other studies should reflect other middle schools that have different geographical areas with different demographics.

The participants of this study reflected only a small percent of the larger population of eighth grade middle students within the school, school district, and other school districts within and outside the state of Missouri. The participants of this study largely reflected the White population with a small Black population. Future studies should not only include a larger number of Black participants but also other ethnicities like Hispanics and Asians.

The time frame of this study included only one school-calendar year, August 2010 to May 2011. The time frame for future studies should involve additional years of data where the researcher not only works with the same population of participants in different higher level math classes, but also with different populations of students within the same specific setting. An age level of students for a different population for this study would include fourth grade students who had just “learned” their multiplication facts in the third grade.

Another limitation of this study involved the relationship between reading and mathematics fluency. When it comes to mathematical word problems, does reading or mathematical fluency play a bigger role in the completion and accuracy of the solution? Although this researcher examined a possible relationship between reading and mathematics fluency, this researcher did not determine which aspect of fluency deemed possibly more important with assessments that involved problem solving and applications. This researcher also did not examine the differences and/or similarities that involved the cognitive processes between reading and mathematics fluency.

Conclusion

Basic math facts of addition, subtraction, multiplication, and division are essential math skills for a student's success in math (Basic math facts: A sequence of learning, 2007; Michalczuk, 2007). Learning multiplication facts is a universal and fundamental mathematics skill for all students to master. Multiplication fact fluency is the ability to mentally retrieve and write accurately the products of all the basic multiplication facts, which, for the purpose of this study, reflected only single-digit factors, 2 through 9. Research suggested quick single-digit multiplication fact recall not only acts as an important tool for subsequent mathematics learning, but it also frees up the necessary cognitive capacity and resources to solve more complex or higher level math problems (Carson, 2007; Cavanagh, 2008; Wong & Evans, 2007; Jarema, 2010; Loveless & Coughlan, 2004; Wu, 1999). The present findings of this study suggest multiplication fact fluency with the single-digit factors is an essential mathematics skill for higher level mathematics learning.

Requiring the ability to retrieve the products mentally from single-digit multiplication problems accurately and quickly frees up the necessary cognitive processes in working memory to process other higher learning mathematical concepts (Zentall & Ferkis, 1993; Hecht, 2002; Tronsky, 2005). Automaticity or multiplication fact fluency would allow additional working memory for students to work through other rigorous and higher level mathematical processes for problem completion (Wu, 1999). An overloaded system of working memory would utilize a large part of the necessary cognitive capacity and ability to systematically work out more complex, multi-step mathematics problems very easily. The results of this study support the comments made by Wong and Evans (2007) that basic multiplication facts are considered foundational skills for advancement in higher level mathematics:

> Without this improved recall of basic multiplication facts, working memory is consumed by the most fundamental of problems. Releasing working memory capacity allows students to tack more difficult tasks such as multi-step problems or questions demanding higher order thinking. (p. 103).

The results of this study revealed students who achieved multiplication fact fluency may have freed up the necessary cognitive processes of working memory to learn both the conceptual understanding and procedural processes with higher level mathematics learning.

Finally, this study also examined a possible relationship between multiplication fact fluency and reading achievement scores. Wilson and Robinson (1997) suggested poor reading speed may also adversely affect the proficient recall of multiplication facts. Although this researcher statistically found no relationship between multiplication fact speed-recall and grade equivalent reading scores, this researcher statistically determined students who achieved multiplication fact fluency also achieved an on or above grade-level reading score.

Fite (2002) concluded both reading and math have "similar cognitive skills at the symbol processing level. Symbol processing involves the ability to derive meaning (comprehension) from symbols whether they be letters, words, numbers, or equations" (p. 11). The cognitive processes and practices that develop fluency or automaticity in one skill, whether in reading or math computation, could lead to the automaticity or fluency in another skill (Anderson, 2010). The fluency development of one would support the fluency development of another, which for assessment purposes, both reading and mathematics skills contribute to the comprehension skills necessary for mathematics proficiency under the new 2010 CCSS.

With the implementation of the 1989 Standards over 20 years ago, NCTM de-emphasized and discouraged algorithmic instruction for more discovery learning and social classroom interactions as an attempt to foster a stronger conceptual understanding of mathematics (NCTM, 1989c). Pencil-and-paper computations hindered mathematics understanding to make the necessary connections with higher level mathematics learning (NCTM, 1989c). Even with the 2000 Standards, NCTM emphasized math computation "fluency" for efficiency rather than for "automaticity." A study by Rittle-Johnson et al. (2001) statistically demonstrated that mathematics procedures (knowledge gained by direct instruction of notes and examples) not only strengthened problem completion, but it also supported and increased conceptual understanding for higher level mathematics learning. A study on multiplication fact mastery between instructional techniques, eight years after the implementation of the 1989 Standards by Wilson and Robinson (1997), wrote the following: “The parents claimed that teachers and peers commented on the improvement, including less difficulty with mathematics problems in class, increased self confidence and willingness to attempt more difficult problems” (p. 185). A number of researchers support both instructional components of conceptual understanding and computational fluency as important instructional techniques for higher level mathematics learning (Wu, 1999; Wallace & Gurganus, 2005; Ravitch, 2010).

The results of this study revealed students with a quick retrieval of the multiplication facts 2 through 9, also reflected a higher level mathematics understanding of the concepts. Students need multiplication fact fluency, a quick retrieval of the multiplication facts, as a tool to help students achieve the new 2010 CCSS for higher mathematics learning. Mathematical conceptual understanding and procedures of the basic math facts are equally important for mathematics proficiency (Kostopoulos, 2007; Wu, 1999). “Let us teach our children mathematics the honest way by teaching both skills and understanding” (Wu, 1999, p. 7). With the 2010 CCSS, students by the end of third grade need to recall from “memory” all products of one digit numbers, 0 through 9. Let us make sure our students have mastered their multiplication fact skills with automaticity for higher level mathematics learning.

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Appendix A

Name ___________________________ Date ________

Multiplication Fact Speed Accuracy Quiz

Do <u>NOT</u> turn over until you are told!

Please read the following directions:

The purpose of the quiz is to test your multiplication fact fluency. You are given 36 multiplication problems. You have 45 seconds to complete as many problems as accurately as possible.

If you **FINISH** before the allotted time, **please turn your quiz to this side.**

If you **<u>DO NOT FINISH</u> before the allotted time, <u>PENCILS DOWN IMMEDIATELY</u> for multiplication test validity.**

You will receive <u>5 bonus points</u> for your participation in this study.

Thanks for everything,

Mr. Curry

2	4	8	7	2	4	9	8	2
× 6	× 4	× 2	× 9	× 3	× 5	× 9	× 5	× 2

4	7	2	6	5	9	4	2	6
× 3	× 8	× 5	× 4	× 7	× 3	× 8	× 4	× 3

6	2	8	3	9	4	8	5	7
× 5	× 9	× 8	× 6	× 8	× 7	× 3	× 5	× 3

7	9	3	7	8	5	6	3	9
× 7	× 6	× 5	× 2	× 6	× 9	× 7	× 3	× 4

If you **FINISH** before the allotted time, **please turn your quiz over to other side.**

Appendix B

Name __________________________ Date ________

Multiplication Fact Fluency Quiz

Do NOT turn over until you are told!

Please read the following directions:

The purpose of the quiz is to test your multiplication fact fluency. You are given 36 multiplication problems. You have 1 minute and 48 seconds to complete which is an average of 3 seconds per problem.

If you **FINISH** before the allotted time, **please turn your quiz to this side.**

If you **DO NOT FINISH before the allotted time, PENCILS DOWN IMMEDIATELY for multiplication test validity.**

You will receive 5 bonus points for your participation in this study.

Thanks for everything,

Mr. Curry

6 × 3	2 × 4	4 × 8	9 × 3	5 × 7	6 × 4	2 × 5	7 × 8	4 × 3
2 × 2	8 × 5	9 × 9	4 × 5	2 × 3	7 × 9	8 × 2	4 × 4	2 × 6
9 × 4	3 × 3	6 × 7	5 × 9	8 × 6	7 × 2	3 × 5	9 × 6	7 × 7
7 × 3	5 × 5	8 × 3	4 × 7	9 × 8	3 × 6	8 × 8	2 × 9	6 × 5

If you **FINISH** before the allotted time, **please turn your quiz over to other side.**

Vitae

Steven Curry is currently a mathematics teacher at Hollenbeck Middle School for the Francis Howell School District. Additional experiences at Francis Howell School District include serving as the Missouri Options Coordinator and mathematics teacher at Francis Howell Union High School. Other educational experiences include five years as a science teacher at Marquette High School.

My educational studies have resulted in the anticipation of an Educational Doctorate degree in spring or summer of 2012 from Lindenwood University. A Master's degree in Educational Administration was earned in May of 2004 from Lindenwood University. A Bachelor's degree in education with a chemistry major and mathematics minor was earned from Michigan State University in May of 1995.