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A Comparative Study of Second Grade Students on the Effects of the Use of Manipulative Aids on the Operational and Recall Skills in the Multiplication of Three

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By

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ABSTRACT

The purpose of this experimental study was to investigate the operational and recall ability of second grade children, after instruction and the use of manipulative aids, to understand the multiplication of three. Multiplication of numbers at the early stages, in most mathematics programs, provides for rote memorization and drill instead of concept understanding. Most practices and drills require only answers to the facts, which provide little indication that a student can explain or show how that answer was obtained. Memorizing the multiplication facts is important, but at this early age the operation on those numbers to obtain an answer is as important for retaining the knowledge. It should be learned that an answer can be figured out by several other methods.

The major hypothesis for this study was: Long-term recall of concrete operational concepts of multiplication will be significantly improved because of the daily manipulative experiences. The use of manipulative aids gives the child concrete experiences which can be treated to show the "why" of the fact. The mental growth descriptions, as stated by Jean Piaget, indicate that at

this pre-operational level of understanding a child needs to act upon the objects in order to build an understanding.

The experiment included twenty-three second grade children in one classroom of a suburban public school in St. Louis, Missouri. The class was randomly divided into an experimental and a control group. Before any instruction was begun in the multiplication of <u>three</u> a pretest was administered to the whole class. The following three days the class received instruction on multiplication of <u>three</u>. This instruction included listing and answering the facts, drawing pictures for each fact, completing pages in the mathematics book, quick quizzes, oral game drills, and suggested practice with parents at home. On the fourth day a whole class test was administered to determine operational abilities and understanding.

At this time the class was divided into the previously determined groups. For four days, one-half hour each day, the experimental group remained with the investigator while the control group went to another room for addition/subtraction work with a teach-aide. The experimental group participated in manipulative aid work using concrete, movable objects to further their understanding of the fact and answer. At the end of this

time all work on multiplication of three was completed. After a two week interval a posttest was administered to determine short-term recall. Another posttest was administered at the six week interval to determine long-term recall.

The gain scores of all these tests were compared to decide significant/not significant gain for both the experimental and control groups. Also compared were the gain scores of male and female children. Data was also analyzed to compare the mean gain scores of the children who had an early birthdate in the school year to those who had a late birthdate in the school year.

The results of this experimental study indicate that there were no significant differences in the mean gain scores of the experimental group over the control group. Children in both groups made a moderate gain in correct answers by the end of this study. Females made more consistent gains as compared with the males in this study. The children with late birthdays made more significant gains than the children with early birthdays.

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Chapter I

Learners, including the very young child, are continually being tested, with the results used for placement on educational levels and as indicators of further achievement. Within the educational system of today the majority of learning takes place in a large group situation and usually without regard for the processes that an individual child follows through for learning the prescribed material. One of the major concerns of the learning experiences is the correct application to other learning situations, tests, and everyday problem-solving situations.

Through the previously taught material, recall of the facts, methods, answers and reasons are definitely affected by the learner's level of operation upon the problem. Observation of process and product abilities and errors have led to this inquiry into the need for action-experience learning. After introduction of the operational method, initial learning experiences of that method, and the daily work, some children still cannot show an understanding of the operational concept. Some children complete work papers, tests, and problems by rote example reproduction but do not produce an ability to show the understanding of the operation. The ability to memorize facts, especially in areas such as the multiplication facts and tables, does not show understanding of the idea of using multiplication in place of addition.

In quite a few instances in the educational atmosphere, concept understanding is irrelevant to the memorization of the facts. Sometimes the student is able to memorize these facts quickly, therefore being able to score well on the basis of drills and tests which require only answers, and not methods operations.

In the primary level of elementary school this is especially true in the memorization of the times tables (multiplication facts). Most instructional materials, which include workbooks and teacher-made activities, seem to require only answers to many problems of that certain set. There are few instances that actually teach the procedure as to the understanding of the fact. Because there are many, many multiplication facts to learn, little time is spent on the operation upon the fact or the "why" of those numbers.

Learning the multiplication facts is important, but so, too, is the operation of those numbers. This study was a comparison of groups of children who had been introducted to both the concept of multiplication of the threes and to the operational method used in relation to that fact.

Children in a public school second grade classroom were randomly divided into experiemntal and control groups. Both groups together had been taught the multiplication of two and were introduced, as a whole group, to the multiplication of three. A pre-test of these facts was administered to the whole group before any teaching was started. Then whole group instruction and practice was initiated using matching pictures to the facts, drawing pictures for the facts, story problems, facts, drills, use of manipulative aids to show facts pictures, and some worksheet assignments. As usual the groups were instructed to practice at home with either aids, parents, or both. A whole group post-test was given for comparison to the pre-test and to determine if understanding of the concept of multiplication of the threes was present.

At this time whole group practice and teaching instruction was completed. The following three days the experimental group participated in enrichment activities, using manipulative aids, with the investigator, while the control group was taken to another room for addition and subtraction practice with a teacher aid. Each group worked one-half hour for the four days. At the end of the second week the whole class was again tested for short-term recall of operational and correct answer knowledge of the threes facts. Further testing, for the whole group, to determine long-term recall was administered at the six-week interval.

A comparison of the groups was made using a T-test to analyze the mean gain score. Also compared were male and female scores and scores of the children having early and late birthdates. This testing, and the comparisons of the raw scores was useful in determining the advantage of, or no significant increase, in the operational and understanding level of the children who had been given practice in multiplication and the use of manipulative aids (experimental group), with the children who were not included in the sessions with manipulative aids (control group). This study has given some indication of the necessity of thorough understanding of a concept and the operational method.

As far back as the 1930's children in kindergarten were stimulated by play-action exercises through their natural growth (Schoedler, 1981). Many other educators and psychologists were beginning to become popular in the development of curriculum in the schools. Some, such as Piaget, Skinner, Dewey, Froebel, and Montessori (Copeland, 1979) were advocating active learning through the use of manipulative objects related to the child's level and familiarity. Although each encouraged active learning, there have been some differences of opinion about the usefulness of their theories. Of these theories, Piaget's stages of development and the defined descriptions of the levels of operation by the young child, have been widely accepted by educators.

Piaget has defined these stages of development as: Sensorimotor (birth to one and one-half years of age), Pre-operational (one and one-half years of age until approximately seven years old), Concrete operational (seven to eleven or twelve years of age), and Formal operations, (eleven or twelve years of age to adult (Copeland, 1979). Piaget has determined through extensive studies that action on concrete and manageable objects leads to constructive mental growth and transfer of skills. Also with this theory Piaget stated that the lack of experience with concrete materials leads to inadequate mental operations and inadequate development of the abstractions required for more advanced mathematics (Pulaski, 1980).

Verbal association to mathematics seemed to be an important aspect of teaching and learning the concepts within the generalized instruction of mathematics. In studies of the learning disabled there was an association of failure related to the lack of verbal understanding and a transfer to other situations (Copeland, 1979). As these verbal transfer experiences were changed to include manipulative/action learning, the disabled child applied intelligence to the problem and then related the parts to each other more clearly. Therefore in a normal classroom, as described by Piaget's theory of instruction, teachers should be concerned with this method as another situation where all learners can profit.

Just as early educators placed value on play, action-learning, experience-oriented schooling, there came an era in the 1950's and early 1960's in which educators were concerned with rules, computation, and "new math." The old theories of hands-on situations were replaced with teaching children the structure of mathematics in methods which were not experience oriented (Flener, 1980). As a result, confusion reigned for years over the problematic terms, rules, methods, and concepts. Eventually educators and parents demanded a return to earlier methods such as action-learning and the basics of mathematics. Although, previously, the "basics" were taught by rote drill and not by experiences, another method was begging to be used: that of investigation of the process by which a child learns and its application to learning and everyday experiences. Piaget was coming back into favor.

Today most mathematics texts are based on the theory that children, especially at the preoperational and concrete operational stages, need manipulative work experiences for the understanding, abstracting, and transferring of knowledge. This return to earlier methods is being combined with the tests and use of familiar objects and commercially produced manipulatives.

There has been, recently, this renewed interest in the processes of mental growth and the "why" of the experiences presented. This study, consisting of an experimental and control group, has given a comparison of a normal group of learners in a practice/no practice situation. Concept and operation understanding was taught in multiplication of the threes; the basic facts and the operation upon those facts. Investigation of this type has frequently been studied for the learning disabled as compared to the normal learner, but has not often been studied in any depth as a comparison between groups of normal children. This study can be of value to the teacher for preparation and planning of instruction on different levels of operation that appear in the regular classroom.

Hypothesis: Long-term recall of concrete operational concepts of multiplication will be significantly improved because of the daily manipulative experiences.

Definitions

Knowledge - information that is organized into bodies of meaningful interconnected facts and generalizations.

Cognitive development - internal process by which learners select and modify their ways of attending, learning, remembering, and thinking (Copeland, 1979).

Concept - capability that makes it possible for an individual to identify a stimulus as a member of a class having some characteristic in common (Gagne and Briggs, 1974).

Concrete concept - identification of an object property or object attribute (color, shape, etc.) through human performance (Gagne and Briggs, 1974).

Manipulative aids - objects of varying color, size, and a shape which are handled, moved, grouped, or arranged by the learner for the solution or understanding of a problem.

Operational level - stages of internal mental growth which every person, from birth to adulthood, passes through, as described by Piaget.

Prerequisite - a task which is learned prior to the learning of a target objective and which then "aids" or "enables" that learning to take place (Gagne, 1979).

Long-term recall - the mental process by which prerequisite learning capabilities are brought forth for immediate use (Gagne and Briggs, 1974).

Conservation - the ability to transfer operational methods and understanding to other problems or examples.

Initial learning - the class or individual learning time when the teacher introduces the concept and operation of the method.

Manipulative repetition - a change to use movable objects to enhance the understanding of a concept over a period of time.

Chapter II

Since the time of Jean Piaget's descriptions of mental growth of young children, educators have waivered back and forth in producing curricula for schools that take into account what he has stated about the developmental levels of young children. At first there were many schools who adhered to his theories very strictly, but as the space and technical age approached, the needs of learners changed and so did the curricula. Memorization and computational excellence were needed for the technological work which indicated a change from active-learning to rote drill and practice. But as the technical age progressed, society became aware of the lower scores on national tests and saw the poor understanding of skills that the learners acquired. Another change had to be made because of popular demand.

Again, educators were looking back to Piaget's descriptions of mental growth, but also recognized the fact that active-learning and self-learning should be combined with the memorization and methodology basically needed for everyday living. In the last two decades there has been a trend to this mixture of theories in the hope that it would develop a well-rounded and more thorough student.

Approaches to this mixture of theories are many sided and range from the pure Piagetian school curriculums to the "Back to Basics" (traditional) schools. Techniques are just as varied, but efficiency and effectiveness are usually the two key words in the programs of today.

Penrose (1980) found that logical operations developed very slowly and were not functional before late childhood. Exercise and action on objects trained this logical functioning in the brain. Investigation and discussion had to be used in conjunction with each other, as it was not enough to learn only through verbal association. He felt mathematics was an area where all too frequently teachers relied on passive associations to teach the new skills. Only a small percentage of the learners gained the actual understanding of the skill. This small percentage may have had a mathematical "aptitude," which was an inherited "gift," while others allowed that a percentage of children could be using intelligence to solve mathematical problems (and not aptitude alone). Students who were only mediocre in these skills could figure out ways to solve a problem by using other thought processes.

If the problem was of interest, and a quick solution seemed possible, the child was quick to work it out. Quite often the teacher did not inquire about the method used to solve the problem. Piaget has written that every normal student is capable of good mathematical reasoning if attention is directed to activities of his interest.

As logic develops, experiences encountered by the children will help to shape their interest and motivation. Understanding numbers and processes can only come about as the child gains experiences and has the chance to discover relationships and to create and re-create ideas (Penrose, 1980, p. 57). Logic building is a step by step process beginning in the small child and useful for problem solving about the age of seven. Deductive thinking begins to take place and with these new experiences the true meaning of numbers is shown and also understood.

Approaching numbers slowly, allowing time for relationships to "sink in," giving the child many experiences, and making the work meaningful will help the child develop more thorough thinking skills and satisfaction (Penrose, 1980, p. 57).

Penrose (1980) firmly believed that students failed because teachers began their lessons with language instead of action. This created disinterest right at the start.

Independence was found to be learned through discovering and creating, not through passive learning methods such as were frequently used. Active and spontaneous experiences were sometimes hard for an adult to understand. Independence was developed through guidance <u>by</u> the adult.

Early primary children, ages five to seven years of age, are in the later time of Piaget's preoperational stage of mental growth. This stage is characterized by the concepts of representation or symbolism. These manipulative actions of this stage represent some idea in the real world. Logical thinking is beginning to take place, but at this time is not a reversible process. As the child progresses into the third stage, concrete operational (from age seven to eleven or twelve years of age), real connections in logico-mathematics takes place. A child will be able to operate on objects and ideas. Physical manipulation of ojbects must take place. In this third stage the concept of conservation of numbers begins to be understood (that quantities do not change just because they are rearranged). Manipulation of objects is necessary at the beginning of the concrete operational stage, but the child will progress to less dependence on manipulatives as the logical understanding becomes a part of him (Copeland, 1979).

"Knowing how to" is not the same as "knowing." Babies perform sensorimotor activities, such as crawling, but it is not until about the age of seven or eight that a child will be able to correctly explain the process of "how do you do that." The child is able to break the process into logical steps to provide a clear understanding of the processes. The same idea goes right along with mathematical concepts; children may be able to produce a solution, but not with any understanding. Conceptualization takes place through manipulation of objects and other building-on experiences. Consciousness of the idea is necessary, as is logical identification (Copeland, 1979).

Copeland (1979) recommended that teaching mathematics be done in grouping situations (if whole room teaching was in use). The groups should be determined according to Piaget's stages of mental growth. The first group should include those children not yet ready for formal work and who need many more activities in the early ideas of sets, comparisons, relationships, and areas. Group two would consist of those children who are not able to use reversal (conservation) as yet, but are about to enter the number concept figuration. Group three would include those children who are ready for the more structured ideas of first and second grade work. Symbolism and representation of facts will be more easily understood. Relationships and conservation of numbers is clear. Guidance by the teacher into these areas would allow for the spontaneous discovery by trial and error work.

Jean Piaget, from his descriptions of learning, emphatically stated that a child will develop mathematical concepts by himself, independently and spontaneously (Piaget, 1953). The verbal association used by most educators is not necessarily important. Conservation of numbers is gained from mental growth after age six, as is one-to-one correspondence in rearranged groupings. Children must grasp conservation before number concept is truly understood. Projective relationships, such as in topological understanding, are not fully developed until about age seven when the angle of vision or point of view is independent of outside conditions (Piaget, 1953). Distance and length are not clearly understood in perspective until conservation of space is developed. Logic intervenes to show the child that the line is the same length even if an object is placed on that line. Logical reasoning improves the mental skills through working relationships out by inventing methods that will give solutions to their problems (Piaget, 1953). As the child progresses through the concrete operational stage the build-on of concepts develops and leads to more complex abilities.

"The instructional events of external experiences that influence learning must take place in a sequence. Planning for instruction in a curriculum will include external events that will affect the learner" (Gage, 1977, p. 286). Motivation of the young child is training for interactions, social approval, and more complex learning skills. A young child will be motivated by things that are of interest, in which participation by other children is included, and by a presentation of a challenge. The desire to gain approval of others or to establish a position of social esteem will motivate a youngster to experiment (Gagne, 1977). Further along in mental development, the child will be motivated by the idea of mastery which will enable functioning at a higher level and independently of aids or step-by-step processes.

Achievement motivation follows this mastery level. Self-satisfaction becomes the important task-related motivation. The incentive to learn under new circumstances must be presented to even the youngest learner.

Expected performance must be clearly stated before the learner actually begins the experience. To any teacher the step to actively participating is direction attention-getting techniques. "Alerting learners allows them to become physically prepared for the ensuing process. Then by selective perception of the deliverance does the learner relegate the activity to the short-term memory aid" (Gagne, 1977, p. 287). For long-term memory storage, actual schemes, mental images, communications, and activities must be presented. Transfer of learning follows through recall to use on new situations. Cues are used to stimulate the recall to a transfer. Application of learning in many different situations is important. Learning in these situations is followed by feedback from the experience. In present programs the learner is somehow told if the solution is correct or incorrect. But the knowledge of correctness can also be obtained from the prerequisite knowledge and other experiences, if done correctly by the instructor (Gagne, 1977).

Repetition has long been known to have a marked effect on the remembering of information, and is really providing spaced occasions for the learner to recall the information previously learned (Gagne and Briggs, 1974). Repeating skills is based on earlier sequential experiences leading up to the present point of skill attainment. "If a skill, or some other information, needs to be recalled it must have been previously learned" (Gagne and Briggs, 1974, p. 143). Meaningful activities for concept attainment must have been understood before the material can be reinstated. Relationships must be recognized and transfer present to replicate a concept activity. Activities which are considered repetition differ from those directly administered following the initial learning session. The repetitious activities issue a broader challenge to the learner (Gagne and Briggs, 1974, p. 164).

> There is great difference between being able to discriminate between objects and the concept capability. A discrimination is the ability to make different responses to stimuli which differ from each other in some physical way. A concept is a capability that makes it possible for an individual to identify a stimulus as member of a class having some characteristic in common (Gagne and Briggs, 1974, pp. 163-164).

Identifying concepts is important for more complete learning. Understanding a concept comes about as a child develops this capability of discrimination.

The theory of Piaget, which states that mental development limits the scope of understanding and problem solving, is a basis of much educational curriculum. Gagne enhances that thought to include the reasoning states;

that cognitive strategies develop out of these more specific learned intellectual skills by a process of generalization rather than by simple maturation as the individual gets older (Gagne and Briggs, 1974, p. 171).

The view of learning by the teacher is a very important aspect of curriculum development. Brownell (1944) saw learning as the psychological connection-forming relationships known as the psychological connectionism. This theory states that weakensses are present which include:

1. Attention is directed away from processes of how children learn.
2. Pace of instruction is too rapid and there is a failure to give learners aids.
3. Many wrong kinds of practice are provided.
4. Evaluation of errors and their treatment is superficial (Brownell, 1944, p. 149).

Improving on these would produce a more thorough understanding of mathematical concepts in the young child.

In the process of learning, the use of manipulative aids will encourage the learner to arrive at the answers by themselves and also understand the "why" of the process. Invention is an incentive to be challenged again. Teachers are guides, but can guide only as long as the process is known. Each stage of learning is replaced with more advanced stages through experience and repetition. All experiences should begin with the help of aids.

Practice is necessary in any method of learning, but it should also be on the correct level and encouraged by trial and error learning experiences. Useful abstractions will be easier to understand for the child (Brownell, 1944, p. 157).

At the time Brownell wrote this article (1944) the use of manipulatives and any ideas of psychological connectionism were a new theory. This article raised many questions about the value of educational programs of that time. Since then curriculums have been reconstructed to include manipulative aids.

"Effective mathematics leads to productive living. The instruction is a major influence upon how much is learned, how well it is learned, and the ability to apply what is learned" (Trafton, 1982, p. 4). Comprehensive teaching takes place when material is taught at the appropriate cognitive level and through the most effective methods. "The retention and application is poor when efficiency (the amount of learning compared to the instructional time) is weak" (Trafton, 1982, p. 4). According to Trafton (1982), there are components needed to produce thoughtful, productive learning:

> Instruction needs to be developmental--manipulative objects and symbols relate to skills and rules to be taught.
>
> Instruction needs to be well sequenced--prerequisite skills must be shown and orderly arrangement of components must follow.
>
> Instruction needs to be focused--objectives should be clear, orderly, and related to the end product.
>
> Instruction needs to promote mental growth.
>
> Instruction needs to be cumulative--maintainence programs produce achievement.
>
> Instruction needs to be comprehensive--exploration, manipulation, problem-solving.

All of these components uphold the interaction of the theories and facts (Trafton, 1982, p. 4).

Because this learning influences the learner's attitude and abilities for the rest of his life the learners need more than just book-learning.

Since the age of computer-based education is upon us, the two terms which need to be expanded on within our educational settings are effectiveness and efficiency. The use of learning time is a very important aspect and the teaching of the "why" of mathematics (or any other subject) is now very important. Children of today seek sense in their learning and educators must be able to reach the objective of teaching that "why" (Gibb, 1982).

The definition of efficient is different when speaking about young children. From a child's view learning is active and involved. The use of familiar objects to experiment on concepts creates a very measurable difference in understanding "why" and the motivation to find out (Gibb, 1982).

The efficient use of time does not come right at the beginning of the program. Those involved need time to get adjusted to the program. But through the use of manipulative aids and experiences the children are better able to translate learning to the inevitable workbooks, worksheets, and other record-keeping necessities. This method, using experiences with manipulative objects, provides the ability to gain self confidence, see relationships, gain skills in logic, and enjoy learning while becoming proficient in understanding the original concept. It provides a chance to learn that things in our world can relate to one another and provides adaptability when a new task is presented (Gibb, 1982).

Knowledge of skills necessary for today has changed so much in the last decade that educators must be willing to make changes in the curriculum, too.

Experience is a key word in the developmental readiness for the mathematics curriculum. Exceptional children sometimes lack the readiness skills so necessary and which normal children naturally acquire (Sanders, 1981, p. 54). But that does not say they do not get those experiences. They do not acquire knowledge through them as a normal child will do. Sequence in developmental levels is necessary before understanding takes place.

There are pre-number concepts which must be understood in order to cope with number computation. These begin with the child of six to eight months of age:

1. All gone--contrast between some and none.
2. Object constancy--peek-a-boo, hide and seek.
3. More--expands concept of some and none.
4. Sorting--separate objects as to same and different into categories.
5. Comparing size--fitting lids to cans, toys inside each other, puzzles.
6. Ordering--small to big, long to short.
7. Patterning--patty-cake, hand-feet games.
8. One-to-one correspondence--musical chairs.
9. Number awareness--represented with objects.
10. Number concepts--number words with manipulatives to match.
11. Counting activities--how many up to ten.
12. Conservation of number--number is same even if it is rearranged.

13. Recognition of numerals--concepts in relation to the aids.
14. Writing numbers to ten.

(Sanders, 1981, pp. 55-56).

Equivalence in one-to-one correspondence was achieved by either matching or counting. This ability to conserve numbers came about only when counting was used by the children. Fuson, Secada, Hall, (1983) conducted a study which addressed children's ability to use, spontaneously, matching and counting in making equivalence judgements in conservation-of-number tasks.

Two experiments were conducted using twenty-four 4½ to 5½ year old children. They were assigned randomly to six possible orders of three task conditions. The conditions were counting, matching, and conservation. The procedure was set amid the context of "feeding the animals at the zoo." The children were to see the relationship of peanuts to the animals, then when the animals were moved the conservation questions were asked, such as "Are there more animals, the same, or more peanuts?" and "How do you know?". The arrangement was carried on in the other two methods with conservation and justification asked each time to each child. The analysis showed that more children of this age answered correctly by using the "matching" method.

Verbalization was found to be difficult for this age child in explaining the method (Fuson, Secada, Hall, 1983).

The second experiment consisted of a study in which spontaneous matching and counting were examined. This experiment was based on the ideas that children at the transitional level of operation would respond differently. Also involved was the verbal association with broader range terms. Twenty-eight five year olds were used for this study. They were assigned to either a class or a collection condition. Each child was given six trials of varying degrees which included altering line length of small cars, altering place of, but not length, of a line of small cars, and then moving one car to the other end. In the class condition they were called "blue cars" and "yellow cars," but in the collection condition they were known as "cars in the blue race" and "cars in the yellow race." Conservation questions were then asked and justification, too, was required. After analysis it was shown that counting was the prominent method of equivalence and that the use of verbalization was important in figuring out what to do.

These two experiments confirmed that younger children used gross perceptual features to make judgments without spontaneous use of counting. In the older group of children spontaneous use of counting and matching occurred in task conservation. Conservation did not occur until about the beginning of the concrete operational or late pre-operational level of mental growth (Fuson, Secada, Hall, 1983).

In both the modern and traditional methods of curriculum in mathematics it was deemed necessary to produce quick recall of the basic facts. The pre-requisite instruction for number concept understanding eventually changed over to memorization learning by the child. The measurement of the facts was sometimes difficult because of a lack of understanding of "why" mistakes were made. How the children went about getting the answer was seldom studied in the classroom. The methods were taught and those which they used or in-vented for themselves were found to be necessary tools for correctly answering the problems.

A test was developed at the Learning Improvement Center, School of Education, University of Louisville for use to determine how children learn or answer the facts. A tool called the tachistoscope was made from heavy tagboard. It had a window for flashing the fact. The student had five seconds to answer the fact, then the score (right or wrong) was recorded on a separate sheet. After the test, if the answer was wrong the student was told to "figure" it out by any means necessary. Many "aids" were available on the table. The analysis of the method was recorded on the same sheet as the answers. It gave the abstraction level and understanding level of the learner. This tool gave help in placing the child in the right book, right level, need for manipulatives, confidence, and concentration level. It allowed for grouping facts for learning and showed exactly which facts were missed (Dunlap and Thompson, 1977).

In the current curricula the descriptive levels of mental growth by Piaget have been used as a developmental basis. "The most important message has been that knowledge is not a thing in the outside world. Rather it is an interaction between the person and the outside world consisting of actions between the knower and the known" (O'Brien and Richard, 1971, p. 322). Because today's educational situations are more concerned with the product rather than the process, one should ask if the product is thoroughly understood through relationships and interactions or through recall by repetition of facts until memorization is acquired from that drill.

Assessment of number learning processes can be achieved by interviewing the learner regarding how they obtain solutions to problems. Questioning is an important aspect of gaining insight into thought processes. "How do you know?" leads to an explanation by the learner. Action on objects found in the real world leads to oral classification in order to count, but not necessarily to count in the real sense of the word. Children tend to invent methods and classification groups which is stated as "When you cannot solve the problem, break it into parts and attach the parts" (Polya, 1975). The more complex the problem the more children used that inventive approach of their own voilition. Piaget does not provide ready-made answers, but he does give good solid ideas on which to improve for your own use (O'Brien and Richard, 1971).

During the last twenty years there has been a definite interest in how children process their learning products. "It is known that all children do not think uniformly and are not on the same operational level at any given age"(Weaver, 1955, p. 41). Instruction in arithmetic cannot be effective unless the teacher is also knowledgeable about the level of work that is appropriate. Guiding experiences on that level will lead to understanding of the process that will yield achievement (Weaver, 1955, p. 41).

Weaver (1955) completed an experimental study which was used to develop an understanding of the process and method of problem solving which was used by the children. A teacher named Miss Watkins was used for this study. She began the multiplication instruction with the quiz over previously taught and not-yet introduced facts. The previously taught facts let her know if the child had some previous knowledge of multiplication. The yet-to-be taught fact cards were shown in order to study the different methods the children used to find the products. By interviewing the children she was able to get the methods of their solutions. Each was recorded as nearly verbatim as possible. From these interviews it was shown that she had a wide range of developmental levels within her classroom. Quite a few acquisitional errors from previous learning methods were noticed. Also, some high level thinkers were found who had not been determined before (Weaver, 1955).

To make her teaching more effective she set up a new grouping plan as well as using whole-class instruction. Both methods were frequently used in the classroom. As the groups were formed, flexibility was considered so that students could come and go from the groups accordingly. Her effectiveness in teaching came about from the individual interviews and analyzing the methods of how the children perceived the problem, the process for solution, aids necessary, and speed at doing the work.

"Children are given little instruction in thinking and must create their own ways" (Hendrickson, 1983, p. 42). This sentence was the main idea when Hendrickson, ten years ago, set up ideals for a curriculum for primary age children. Hendrickson based the curriculum on the following premises:

1. Piaget's description of intellectual development is valid.
2. The sequence of stages in the development of operational thinking is particularly important to the planning of mathematics learning activities.
3. Thinking experience is more essential to the learning of mathematics than memorization of symbols and symbol forms and of steps in symbol manipulation.
4. Children should neither write symbol forms nor interpret symbol forms until the concepts that are referents for the symbols are meaningful, and until the use of symbols facilitates thought.
5. Children at the preoperational and concrete operational levels must consistently manipulate concrete materials in order to develop concepts.
6. Mathematics should require as little reading as possible in the primary grades.

(Hendrickson, 1983, p. 43).

A brief summary of each of the primary grades concepts and processes, goals and materials follows, beginning with the kindergarten. At this level the concepts presented are mainly those of comparison, classification, patterns, counting, grouping, and number concepts. Familiar, easy to handle, small objects were used for the experiences. Language is encouraged to develop the verbal-terms knowledge. For grade one, more commercial products were recommended to further the same concepts from kindergarten, but on a higher and more challenging level. In the fourth month a more systematic development begins with concepts of number, place value, and number operations, equality, symbolization of addition and subtraction signs (Hendrickson, 1983).

Beginning in grade two, goals are extended from where the recorded data for each child is compiled. Reinforcement of later first grade skills are encouraged with more complex activities. Number counting in many, many ways is a major activity. Number idea, place value in tens, addition and subtraction in tens and ones, multiplication, fractions, story problems, and measurement are introduced and activities include work with manipulative aids. Grade Three begins to look like a traditional program. Three digit number work, multiplication, division, story problems, area, volumes, and operations with fractions are studied and worksheets are now introduced (Hendrickson, 1983).

This curriculum was based on repetition and reinforcement of concepts through manipulative objects and active experiences on each child's level. Two books were used extensively for gathering information and activities: Mathematics Their Way by Mary Baratta-Lorton and Mathematics: A Way of Thinking by Robert Baratta-Lorton. The teacher takes the part of observer, questioner, suggester, and co-learner.

There is a large difference between setting up mathematics programs using a specific theory related program and one that allows natural experiences to teach the concepts. Through this last program, by the mere existence and interaction of appropriate objects and activities, the child will acquire the necessary skills. Complete dependence on this type of program is in error in the teaching/understanding of concrete concepts (Benham, Hosticka, Payne, and Yeotis, 1982).

These researchers delineated the following to justify their position:

1. Structured interaction between child, teacher, and material is needed.
2. Teacher consciously plans activities using language enforcement.
3. Classroom socialization and structured activities for learning from incorrect and correct responses are present.

As a background for their studies they ascribe to the theories of Piaget and Henry Perkinson, both of whom recommend goals of knowledge which emphasize ideas and materials, not factual information (Benham, Hosticka, Payne, and Yeotis, 1982). Concepts which are lead-ups to the operational level should include observation and interpretation, classification, patterning, measurement, seriation, one-to-one correspondence, equalities, and discrete quantification (Benham, Hosticka, Payne, Yeotis, 1982).

A description of activities which would be included in a program for manipulative experiences follows the discussion. The objects and materials are familiar and easy to handle. Each activity includes some of the concepts mentioned previously.

The researchers warned that the two biggest problems that educators had with this type of problem were the lack of enough observational opportunities and the erroneous information accidentally presented, but not corrected. These, plus others, may lead to anxiety, blocks, and disinterest in mathematics (Benham, Hostica, Payne, and Yeotis, 1982).

"Learning is the interpretation which the child places on his experiences such that it is incorporated into the schemes which he already has available for use"

(Burton, 1980, p. 45). Concepts are the building blocks and are prerequisite in further learning. The nature of mathematical activity consists of three emphasis: "Linguistic ability to verbalize understanding, understanding through environmental-based activities, and process-dominated learning that leads to creativity and effectiveness" (Burton, 1980, p. 43). Mathematics is also skill based in that it teaches the child the necessary skills to perform operations on other work. Understanding takes place when the activities are meaningful and appropriate. Discovery is able to take place more readily when children are able to create on their own. Curiosity is a highly motivating aspect of the young child. Connections and relations come clear when manipulative objects are used. With these tools they gain the ability to assess, order, evaluate, and make judgments. The child feels good about the ability to control experiences and inputs to them (Burton, 1980, p. 44). Internalization must come from a meaningful relationship to the experience. The child is continually learning "to be."

Today's emphasis is on the return to the use of many physical objects in the mathematics curriculum. It is partially founded on the theories of Piaget and Bruner. The new mathematics books are to be used in conjunction with manipulative aids. Commercial products are very accessible and in most schools aids are available. Wiebe (1981) made a list of questions concerning these aids. The most frequently asked question was "Have these materials actually had an impact on instruction or are they largely unused?".

It was decided that a survey questionnaire could answer this question accurately. The first part asked what kind and the number of manipulative sets were available to the teacher and how often they were used. The second part consisted of use questions as to how much time they were actually used, the time available to the children, time for instruction in the use of the aids, time spent with the children, and no-direction-giving-time the child had for manipulation of the aids. Also included were questions concerning the percent of time used for individual time exploration, drill, concept introduction, paper and pencil work, real problem solving, and as an aid for solving calculations (Wiebe, 1981).

This study indicated that even though manipulatives were present they were not as frequently used as thought. More often they were incorporated into the traditional program rather than for concept teaching itself.

Wiebe concluded that the questionnaire was not an effective method of measurement for use in determining the use of aids because: there was a lack of understanding of the terminology, the "please" effect was present, the observations were difficult, and perception of use was higher than reality.

Mathematics programs, as well as mathematics achievement, has a wide range of variance. Accordingly there have been many programs developed for concept attainment in this field. Some of these programs include the use of recreational mathematics to include already and easily available games, puzzles, rhymes, finger plays, and toys which, for small children, teach skills in many areas.

Hollis and Felder (1982) also included other concepts to be taught: classification and sorting; matching and one-to-one; sequencing; counting; patterning and recognition of; numbers, geometry; shapes and spatial orientation; measurement; and related skills such as following directions, concentration; logical decisions and reactions (Hollis and Felder, 1982, p. 71).

Reasoning for this program was developed and expanded through the knowledge that learning takes place when math is enjoyable. It is possible to include many areas of the concept field in this method. "It can be used for instruction of a concept, enrichment, teaching of a strategy, rule following, social interaction, and problem solving techniques" (Hollis and Felder, 1982, pp. 272-273).

One of the difficulties of the technique has been the lack of research as to whether these "fun" skills learned are transferable. Previous reports by Zammarilli and Batton (1977) found it beneficial as did Sparts (1979) and Bright, Harvey, and Montague (1979). These authors all concluded that the recreational mathematics program was valuable as well as enjoyable (Hollis and Felder, 1982, p. 73).

Schoedler (1981) compared the effectiveness of the Houghton-Mifflin program with an active game learning approach for second grade children at one school. Two separate groups were taught fifteen lessons by each method and were given three criterion measures: pre, post, and interval tests. Two control groups were not taught the selected chapter concepts, two control groups were taught by two different teachers directly from the chapter, and two other groups were taught the concepts by the active learning medium. The concepts taught were: lines and curves, points, line segments, linear measurement, one-half, sets, time, and liquid measurement. The book was taught just as presented with manipulative objects or experiences as described in the lesson. The active game method used existing games and author-created experiences. Tests and measurements by Houghton-Mifflin and Daytons's Repeated Measures Analysis were used for whole group analysis.

The results determined by this study showed no great significant differences between the two groups. There was, however, more improvement by the children in the active game groups. Therefore it was concluded that one method was as good as another.

The recent reversal of ideas about mathematics programs has brought about the more basic mathematics program which is accompanied by many manipulative and supplementary activities to enhance the cognitive processes, understanding, and improvement of the computational skills (Schoedler, 1981, p. 365). As an enrichment program active game and movement activities should be included to round out the usage of commercially produced programs.

Smyth (1983) traveled around the world in order to be more able to answer questions and doubts she had about mathematics curriculums in the United States. A total of 125 primary schools were visited. Most were "spur-of-the-moment" visitations, but in some places, and in the larger cities, an appointment was made. Of all the countries visited there seemed to be some dissatisfaction about math achievement. Included in this study were seven American/military schools. Differences in the programs were plentiful, as was class size, instruction style, and materials. One outstanding feature seen in all the schools, except the United States, was a booklet that kept the cumulative work papers of each student. Progress and problems were clearly seen and available to parents as well as teachers. Counting on the fingers was present in all situations and was not discouraged as an aid to solutions to the problems. The schools used brightly colored commercial aids. New Zealand and Australia used many "environmental" aids in their programs. Experiences and exploration were encouraged by time periods for the children to work individually or with a friend. The researcher was impressed with the program in New Zealand, which, for Kindergarten, First, and Second grades was divided into four departments: logic, number, measurement, and shape/space. All kinds of directed and non-directed activities were offered. This was similar to the program devised by Mary Barratta-Lorton called "Mathematics Their Way" used in the United States.

Suggestions the researcher made for our curriculums included: "less pressure on the children, proceding more slowly for each concept, teacher education in manipulative use, smaller class size in the primary grades, educating the parents, teachers, and administrators in developmental growth, and providing mathematics specialists in each school" (Smyth, 1983, p. 20). Instead of turning children away from math we should be turning children on to math.

The literature and studies concerning the need of manipulative aids were theory oriented. Most of that theory came from the studies of Jean Piaget. Several studies were found comparing the learning disabled and the normal child. They all recommended the use of manipulatives for this type of child. It was also assumed that the normal child learned from many different methods while a learning disabled child may not do so. Very little was found that dealt directly with the comparison of two groups of normal children. There were many articles related to the use of manipulatives, especially for primary children, but actual experimental studies had not taken place. Most work relied on the previously stated fact that manipulative aids were necessary and should be, definitely, a part of a curriculum.

Chapter III

During the last few years many educational materials and books have stressed the theories of Piaget as a method of learning in mathematics. These theories are based on the premise that active learning situations lead to understanding, recall, and better application of the concrete concepts, especially in mathematics. In this study it was hypothesized that the long-term recall of concrete mathematical concepts of multiplication would be significantly improved because of daily manipulative experiences.

This study was a controlled experimental study within a second grade classroom of a suburban elementary school. The children were all within the normal range of intelligence (which is defined as having a range from 90 to 110 points on an intelligence test). Most of the students had passed the Minimum Basic Skills tests of the school district that showed their ability to understand, recall, and correctly answer the mathematical problems on a first grade level.

For this study the entire population of a classroom was divided into two groups. The twenty-three children were randomly assigned to a group by having their name pulled from a box and alternately assigned to the control or experimental group. This kind of division gave an equal chance for all to be chosen without regard to ability or performance level and prevented bias by the investigator regarding group assignment.

A pre-test was administered. This test was teacher made for the purpose of determining any previous knowledge of the multiplication of <u>three</u> by the student. It contained exactly the concepts which were to be studied and tested in the experiment.

This pre-test consisted of matching a picture of sets to a fact, drawing a picture for a specific fact, solving a story problem by drawing a picture and writing a fact, correctly answering a list of facts, and writing a fact to match an addition problem (Appendix A). The number of correct responses was determined to obtain a raw score for the pre-test. The scores were recorded on a class record sheet (Appendix B) and on an individual student graph (Appendix C). These record sheets were used for comparisons during the analysis of the scores for the experiment.

Following the pre-test, the class received instruction regarding the multiplication of <u>three</u> concept.

The experiences of the class included (1) specific listing of the three facts with the answers, (2) explanation of the concept through drawing pictures of the problem, (3) arranging groupings of centimeter squares, cubes, and other small objects into pictures of the fact, (4) rote drill practices with the class, (5) quick quizzes for practice, and (6) nightly study with parents at home was encouraged. At the end of this three day study and learning session, another test was administered to the whole group to determine the student's concept understanding and ability to answer the facts correctly. This whole group test contained the same concepts and operations but it was arranged differently on the paper (Appendix D). Analysis and recording of the raw scores was done on the class chart and individual charts. This second test was compared to the pre-test.

The following day, the class was divided into the previously determined experimental and control groups. For one-half hour each day during the next four days, the experimental group remained with the investigator for enrichment experiences taught with the use of manipulative aids. These aids included games, puzzles, partner play, work with small, familiar objects as aids, motorskill activities, worksheets using aids to solve the facts, counting games, and other activities intended to enhance learning the multiplication of <u>three</u> (Appendix E).

The control group, during the same time period, left the classroom to engage in other mathematical activities not related to this multiplication concept. This group was under the supervision of the teacher-aide assigned to the school for the purpose of aiding in extra practice sessions. The investigator prepared the lesson guides for the aide to follow.

At the end of this four day period the children of the control group returned to the regular classroom mathematics program. The grouping part of this experiment was completed. For the next two weeks the class did not study multiplication of <u>three</u>. At the end of the two week interval another test was administered to determine the short-term recall of the students. The post-test contained the same concepts as the pre-test and the whole group test previously administered, however the test items were rearranged on the paper. This was done to prevent order memorization from other tests. The raw scores of this, the third, test indicated the students' short-term recall of the multiplication facts of <u>three</u>. These scores were compared on the class and individual charts to the pre-test and second test.

After a six week interval another post-test, the fourth test, was then administered to determine the long-term recall and understanding of the children in each group. The teacher-made test was constructed using the same concepts and the same kinds of test items used in the previous tests. The raw scores of the class were graphed and graphs of individual student scores were prepared.

After completion of the experiment the data was analyzed by the use of the independent t-test. Data was analyzed by comparing mean gain scores of the experimental group to those of the control group, boys' versus girls' scores, the scores of the children whose birthdays were early in the school year to those whose birthdays were late in the school year. Each of these test score comparisons gave methods of comparison to understand the effect of manipulative aids on short-term recall and long-term recall of multiplication of <u>three</u>.

The research hypothesis of this experiment was: long-term recall of concrete operational concepts of multiplication will be significantly improved because of daily manipulative experiences.

Chapter IV

A chronology of this experimental study indicates that from January 9, 1984 to March 16, 1984 the class was involved in an experimental study on the multiplication of <u>three</u>.

Table 1

CHRONOLOGY OF EXPERIMENT

Date	Experimental	Control
January 9	Pre-test	Pre-test
January 10-12	Introduction and Learning Activities	Introduction and Learning Activities
January 13	<u>Whole Group</u> Test	<u>Whole Group</u> Test
January 16-19	Manipulative Aid Practice	Addition and Practice
January 20 - February 2	No Practice	No Practice
February 3	Posttest at Two Weeks	Posttest at Two Weeks
February 6 - March 15	No Practice	No Practice
March 16	Posttest at Six Weeks	Posttest at Six Weeks

The class was randomly divided into an experimental and a control group. The groups were administered the same tests at the same time. The treatment of the experimental group included work with multiplication in the practice session, while the activities of the control group included only addition and subtraction in the practice sessions.

Reliability of the teacher-made tests was ascertained by the use of the Pearson r Correlations Test.

Table 2

PEARSON r CORRELATIONS BETWEEN TESTS OF THE SERIES

Group	Tests	r	p
Control	1 to 2	.399	.22
	1 to 3	.352	.29
	1 to 4	.082	.81
(N=11)	2 to 3	.687	.019*
	2 to 4	.334	.32
	3 to 4	.474	.138
Experimental	1 to 2	-.107	.74
	1 to 3	-.019	.95
	1 to 4	.118	.71
(N=12)	2 to 3	.019	.95*
	2 to 4	.281	.389
	3 to 4	.466	.124

* significant at $p < .05$.

As a result of this test it was found that the tests were not greatly reliable, but the correlations reveal that they were moderately reliable for the test/ retest conditions of the experimental study.

It should be noted that the correlations between tests varied from a low of .019 to a high of .687. The low correlation in the comparison of the experimental group test 2 and test 3 contained some test score changes which increased from 2 and 3, and some scores in which test 3 was lower than test 2. The control group scores were higher than those in the experimental group. The gain scores of the control group, test 2 to test 3, yielded a correlation of .689. Seven out of the twelve correlations were above .27. All of these correlations were moderately positive.

Included in these correlations were several gain scores which were positive and some which were negative. One score on a pretest in the experimental group was 3 correct out of 30 points. This student continued to make high scores throughout the experiment. Another child responded correctly to 20 out of 30 points on the pre-test, but scored only 15 on the Whole Group test, which was administered after the learning activities. After the six week posttest the gain scores for the whole class ranged from a decrease of -5 to an increase of +10 out of thirty points possible.

Overall, the raw scores of the class, from the pre-test to the six week posttest, ranged from a decrease of -2 to an increase of +13 out of thirty points possible. The control group ranged from -2 to +12 out of a thirty points possible. The experimental group had a wider range of gain scores, from -1 to +26 out of thirty points possible. These scores show a moderate gain by most students from the pre-test to the last posttest in this study.

Independent t tests were used to compare mean gain scores of the experimental and control groups for the series of tests.

Table 3

INDEPENDENT t TEST MEAN GAIN SCORES OF THE EXPERIMENTAL AND CONTROL GROUPS

	Control		Experimental			
Tests	$\bar{x}$	s	$\bar{x}$	s	t	p
1 - 2	3.45	5.63	6.92	7.82	-1.21	-.12
1 - 3	6.09	4.35	6.83	7.69	- .28	.33
1 - 4	6.82	4.92	5.58	7.63	.46	.33
2 - 3	3.00	3.90	.75	.75	1.96	.03*
2 - 4	3.36	4.82	-1.00	2.04	2.87	.00*
3 - 4	0.00	1.84	-1.25	1.60	1.74	.05*

* significant at $p < .05$.

The findings for each comparison were as follows:

1. Pre-test (1) and Whole Group (2): The mean gain score of the control group was 3.45, and the mean score for the experimental group was 6.92. The t test comparison of these mean gain scores was -1.21, not significant at the .05 level. Therefore there was no significant difference in the ability to perform or understand any multiplication of three facts at the beginning of this experimental study.

2. Pre-test (1) and Posttest at two weeks (3): The mean gain score for the control group was 6.09, and the mean gain score for the experimental group was 6.83. The t test comparison of these mean scores was -.28, not significant at the .05 level. Therefore there was no significant difference in the ability to perform or understand multiplication of three after the two week interval between the Pre-test and the Posttest at two weeks.

3. Pre-test (1) and Posttest at six weeks (4): The mean gain score for the control group was 6.82, and the mean gain score for the experimental group was 5.58. The t test comparison of these mean scores was .46, not significant at the .05 level. Therefore there was no significant difference in the ability to perform or understand multiplication of three after the six week

interval between the Pre-test and the Posttest at six weeks.

4. Whole Group test (2) and Posttest at two weeks (3): The mean gain score for the control group was 3.00, and the mean gain score for the experimental group was .75. The *t* test comparison of the mean gain scores was 1.96, a significant difference at the .05 level. Therefore there was a significant difference in the ability to perform and understand multiplication of three between the Whole Group test and the Posttest at two weeks.

5. Whole Group test (2) and Posttest at six weeks (4): The mean gain score for the control group was 3.36, and the mean gain score for the experimental group was -1.00. The *t* test comparison of the mean gain scores was 2.87, significant at the .05 level. Therefore there was a significant difference between the two groups in the ability to perform or understand multiplication of three between the Whole Group test and the Posttest at six weeks.

6. Posttest at two weeks (3) and Posttest at six weeks (4): The mean gain score for the control group was 0.00, and the mean gain score for the experimental group was 1.25. The *t* test comparison of the mean gain scores was 1.74, a significant difference between the two groups in the ability to perform and understand multi-

plication of three between the Posttest at two weeks and the Posttest at six weeks.

The results show a comparable gain between the Pre-test and the Posttest at six weeks for both groups. But, the control group's mean gain scores were higher than the experimental group's mean gain scores from the Whole Group test to the Posttest at six weeks. The experimental group performed more correct responses at the very beginning of this study, from the Pre-test until after the administration of the Whole Group test.

Statistical data was also analyzed, for the experimental and the control groups, in the comparison between the male and female children. There were fourteen female children and nine male children in this study. The results are shown in Table 4:

Table 4

CORRELATIONS OF MEAN GAIN SCORES IN THE SERIES OF TESTS COMPARING MALE RESULTS AND FEMALE RESULTS

	Female		Male			
Tests	$\bar{x}$	s	$\bar{x}$	s	t	p
1 - 2	6.14	7.65	3.89	5.90	.75	.26
1 - 3	7.71	6.68	4.77	5.33	1.11	.14
1 - 4	8.29	6.57	4.22	5.35	1.16	.07
2 - 3	1.57	3.78	- .55	2.07	1.54	.07
2 - 4	1.29	4.71	.33	3.57	.54	.31
3 - 4	- .57	1.74	- .55	2.07	- .02	.49

It should be noted that there was a near significant relationship between the female and male children on the comparison of tests 1 and 4 and on tests 2 and 3. For the tests 1 and 4, the mean gain score for the females was 8.29 and the mean gain score for the males was 4.22. The $\underline{t}$ test comparison of these mean gain scores was 1.55, showing a non-significant difference at the .05 level. Also, the mean gain score for the tests 2 and 3 for the females was 1.57, and the mean gain score for the males was -.55. The $\underline{t}$ test comparison of these mean gain scores was 1.54, showing a non-significant difference at the .05 level. There was an improvement for more female children than male children on tests 1 and 4 and tests 2 and 3. The $\underline{t}$ test comparisons of the mean gain scores for the other tests were more distantly related.

Further statistical data were analyzed for a comparison of mean gain scores between students who had birthdates early in the school year and those students who had birthdates late in the school year. An early birthdate was considered as one occurring between the end of September and the end of February, and those with late birthdates were those occurring from the beginning of March to the end of September. The results are shown in Table 5.

Table 5

CORRELATIONS OF MEAN GAIN SCORES IN THE SERIES OF TESTS COMPARING EARLY BIRTHDATE RESULTS AND LATE BIRTHDATE RESULTS

	Early		Late			
Tests	$\bar{x}$	s	$\bar{x}$	s	t	p
1 - 2	3.79	5.48	7.56	8.59	-1.29	.104
1 - 3	3.93	4.34	10.33	6.82	-2.77	.005*
1 - 4	3.56	4.29	10.55	6.78	-3.13	.002*
2 - 3	.14	1.70	3.11	4.17	-2.39	.012*
2 - 4	- .43	2.95	2.22	3.77	-1.88	.035*
3 - 4	- .57	2.10	- .11	1.54	- .57	.292

* significant at $p < .05$.

In this comparison of mean gain scores there were four test comparisons which showed significant differences at the .05 level.

1. Pre-test (1) and Posttest at two weeks (3): The mean gain score of the Early Birthdate Group was 3.93, and the mean gain score of the Late Birthday Group was 10.33. The t test comparison was -2.77, a significant difference at the .05 level. Therefore the Late Birthdate Group showed a greater and statistically significant gain as compared to the Early Birthdate Group in the ability to perform and understand Multiplication of three.

2. Pre-test (1) and Posttest at six weeks (4): The mean gain score of the Early Birthdate Group was

3.93, and the mean gain score of the Late Birthdate Group was 10.55. The t test comparison was -3.13, a significant difference at the .05 level. It can be concluded that the children in the Late Birthdate Group improved, from the beginning, by a significant number of correct answers.

3. Whole Group test (2) and Posttest at two weeks (3): The mean gain score for the Early Birthdate Group was .14, and the mean gain score for the Late Birthdate Group was 3.11. The t test comparison was 2.39, a significant difference at the .05 level. Therefore the children in the Late Birthdate Group made a significant gain from the instruction and the Whole Group test to the Posttest at two weeks.

4. Whole Group test (2) and Posttest at six weeks (4): The mean gain score of the Early Birthdate Group was -.43, and the mean gain score of the Late Birthdate Group was 2.22. The t test comparison was -1.88, a significant difference at the .05 level. Therefore the children in the Late Birthdate Group made a significant gain from the instruction and Whole Group test to the Posttest at six weeks.

The results of this experimental study indicated that there was improvement by both groups from the Pre-test to the Posttest at six weeks. The control group made a greater gain on the mean gain scores, but the experimental group had a wider range from the Pre-test to the Posttest at six weeks, which showed gains from 3 to 26 points. The treatment of the experimental group did improve the scores, but not significantly over those in the control group.

The long-term recall of the female children was higher than the male children from the Pre-test to the Posttest at six weeks. Seven out of twelve of the female children were in the experimental group which received the experimental treatment.

The statistical analysis showed that the children with late birthdates, those from the beginning of March to the end of September, made a significant gain over those in the early birthdate group, the end of September to the end of February. Five of the nine children in the Late Birthdate Group received the experimental treatment. In the last two correlations there was an indication that those in the experimental group made a more significant gain than those in the control group.

Chapter V

The results of this experimental study on the multiplication of <u>three</u> indicate that there was considerable gain in the raw scores for most students from the Pre-test to the Posttest at six weeks. Improvement came to those from the experimental and the control groups about equally. There was considerable improvement in the raw socres for the experimental group at the end of the study. Improvement ranged from three up to twenty six points on the tests. Overall, the experimental treatment, with the use of manipulative aids, did not seem to have a strong effect on the final scores for the experimental group. Therefore the hypothesis of "long-term recall of concrete operational concepts of multiplication will be significantly improved because of the daily manipulative experiences" has been rejected.

The background information about this study has indicated that there may have been several reasons why the difference between the mean gain scores were not significantly different. This investigator was the classroom teacher of these students. The children were familiar with the investigator and considered this part of the regular classroom mathematics program. The children responded just as they normally did in a classroom mathematics lesson. There was some inattention in their behavior during some of the activities and tests.

The atmosphere of the classroom children, at the beginning, was that of excitement because the multiplication facts were a new and challenging concept to learn. The multiplication of _two_ had been completed quite successfully just prior to the beginning of the experiment. At the beginning of the multiplication unit the children were eager and successful. Multiplication was fun and exciting. The group, as a whole, studied very diligently at home on the multiplication of _two_ facts. For the majority, the facts and the operation of numbers were easy to learn. Understanding of the operations came quickly. The activities of instruction included more active participation than the unit just before the multiplication unit.

After completion of the multiplication of _two_, a Pre-test on the multiplication of _three_ was administered to the whole class. This test was administered to indicate to the investigator the amount of knowledge already present in the multiplication of _three_. The raw scores ranged from a low of three out of thirty problems to a high of thirty out of thirty problems. The majority of the children fell between seventeen and twenty-eight out of thirty problems correct. The child who had only three out of thirty problems correct began working the problems by trying to use addition (3 + 3) instead of multiplication. Her older brother had taught her that for the multiplication of two it could be answered by adding the numbers (4 x 2 is 4 + 4). Therefore, she continued this on the multiplication of three, not understanding the implications. After the whole group instruction and the learning activities, the following three days, this child scored thirty out of thirty problems correct on the second test. Then after the experimental treatment, of which she was a part, she scored thirty out of thirty problems, and then twenty-nine out of thirty problems on the last two tests. This child's results definitely had an effect on the mean gain scores of the experimental group.

Several other conditions and children had an effect on the mean gain scores. One child, because of inattention, fell five points in score after the whole group instruction. This child scored twenty-seven out of thirty in the two posttests. Another child in the control group began receiving help at home by her mother who had just started the day shift at work and had the evening hours to help her child keep up. This child improved her score from seventeen out of thirty correct on the Pre-test to thirty out of thirty on the last Posttest.

As the multiplication unit on three continued, the excitement and challenge dropped considerably. Multiplication now meant many minutes of studying at home, as many parents began working with the children at night to drill them on the facts. By the end of the six-week period the children were more interested in regrouping of subtraction than multiplication. As there was no practice on multiplication during the interval between the two week and six week posttest, interest on this subject had decreased. There were other new mathematical concepts to take its place.

Jean Piaget had defined this age child as being in the Preoperational stage of mental growth. This age child has just begun to be able to explain the reasons for concepts which are taught or have been learned. The skills necessary for these explanations need to be reinforced by different methods and at many different times. Being able to memorize is much different than being able to explain a concept. Some children in the second grade are actually just beginning to be able to explain a concept and understand its value, whereas, some children are mentally at the end of the preoperational stage of mental growth. This might have had an effect on the long-term recall and understanding of the new concept of multiplication. Some second grade children were not ready for this experience. Significant improvement over a lengthy interval might have been affected by the preoperational stage of development as it has been described by Jean Piaget.

The experimental treatment, with the use of the manipulative aids, did have a positive effect on the students of the experimental group. The children were eager to participate and wanted to try each of the activities available. Working in smaller groups, and having only twelve students in the room during the time, was a positive experience. There was more time for individual help, less waste of time, more quiet time, and general overall feeling of accomplishment. The control group, too, worked well and had improved interest. The academic activities of the control group were not new concepts, but the environment was different, the group smaller, the teacher different, and the incentive high.

The class level of mathematical maturity, for the whole, did not seem to be the level that would have been preferable. Perhaps some of the mathematical skills, which should have been mastered, were not fully developed in some of the children. Also, at the time of random assignment to the groups, the three children who were labeled as gifted by the school district (and participated in the gifted program) were assigned to the control group. They are quick to comprehend, have had previous multiplication work, and have the incentive to memorize and understand the concepts ahead of the other children. This might have had an effect on the mean gain scores and the end results of the study.

The previous knowledge of how to find the answer to a multiplication fact, which had included manipulative aids, and the general knowledge of the multiplication of <u>two</u> might have influenced the raw scores on the Pre-test for the multiplication of <u>three</u>. The children might not have had scores as high as they were because, to this investigator's knowledge, they had had little previous experience with multiplication before the introduction of multiplication of <u>two</u>.

The conclusions from the statistical data indicate that from the Pre-test to the Posttest at six weeks there was no statistically significant gain by the experimental group as compared to the control group. The data indicate that both groups showed a moderate gain in understanding and operation on the facts of multiplication of <u>three</u>. Also included in the statistical data were the comparisons of male and female children.

The female children consistently scored higher and had better mean gain scores from the Pre-test to the Posttest at six weeks. Seven of the female children were in the experimental treatment group. Seven of the female children were in the control group. The mean gain score of the female children for the comparison of test 1 - 2 was 6.14. Five of the male children were in the experimental group. Four of the male children were in the control group. The mean gain score for the male children was 3.89. The $\underline{t}$ test value was .75, not significant at the .05 level.

Further data was analyzed for a comparison of early and late school year birthdates. An early birthdate, for this study, occurred from the end of September to the end of February. A late birthdate, for this study, occurred from the beginning of March to the end of September. Fourteen of the twenty-three children had an early birthdate and nine of the twenty-three children had a late birthdate. Five out of nine children who had late birthdates were in the experimental group. Seven out of fourteen children with early birthdates were in the experimental group. Therefore the children who participated in the experimental treatment had an affect on the mean gain score for the late birthdate group.

As a result of this study there are indications that multiplication of <u>three</u>, with practice using the manipulative aids, was beneficial to the children. The experimental group showed no significant difference over the control group.

APPENDIX A

ACTIVITIES FOR EXPERIMENTAL GROUP

EXPERIMENTAL GROUP ACTIVITIES

1. Flash cards (teacher-made) that match pictures and facts.
2. Small objects (beans, Legos, cubes) to be arranged in sets to show facts.
3. Story problem cards to use with the small objects.
4. Individual chalkboard practice.
5. Game called "Red Hots" (teacher-made). It uses candy to solve the problems.
6. Partner Play which encourages students to share ideas and solutions.
7. Songs and rhythms to drill in quick responses.

APPENDIX B

CLASS RECORD SHEET FOR TEST SCORES

Subject	1																		
Birthdate	6-4-76																		
Sex	F																		
Experimental Control	E																		
Pre-test (1)	22/30																		
Whole Group Test (2)	27/30																		
Compare Test 1 and 2	+5																		
Post-test 2 wks (3)	28/30																		
Compare Test 2 and 3	+1																		
Post-test-6 weeks (4)	30/30																		
Compare Test 3 and 4	+2																		
Compare Test 2 and 4	+3																		
Compare Test 1 and 4	+8																		

APPENDIX C

INDIVIDUAL RECORD SHEET

Student Graph

Number Correct	Pre-test (1)	Whole Group Test (2)	Compare Test 1 and 2	Post-test (3) 2 weeks	Compare Test 2 and 3	Post-test (4) 6 weeks	Compare Test 3 and 4	Compare test 1 and 3	Compare Test 1 and 4
30									
29									
28									
27									
26									
25									
24									
23									
22									
21									
20									
19									
18									
17									
16									
15									
14									
13									
12									
11									
10									
9									
8									
7									
6									
5									
4									
3									
2									
1									

APPENDIX D

PRE-TEST

Name ____________________

Test: Pre-test x3 Score: /30

Draw a picture to go with each fact.

4 x 3 = ☐

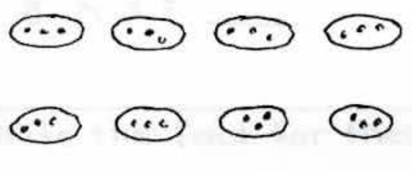
9 x 3 = ☐

7 x 3 = ☐

2 x 3 = ☐

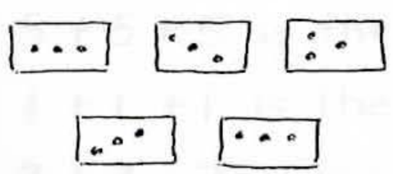
Write the fact for each picture.

___ x ___ = ☐



___ x ___ = ☐

___ x ___ = ☐



___ x ___ = ☐

Complete the story problem. Draw a picture, too.

John bought 5 packs of baseball cards. There were 3 cards in each pack. How many cards did he have in all?

___ ○ ___ = □

Jill has 9 pages of stickers. There are 3 stickers on each page. How many does she have in all?

___ ○ ___ = □

Answer the facts.

2 × 3 = ___	3 × 3 = ___
7 × 3 = ___	6 × 3 = ___
5 × 3 = ___	10 × 3 = ___
9 × 3 = ___	4 × 3 = ___
0 × 3 = ___	1 × 3 = ___
8 × 3 = ___	

Write the fact for these problems.

4 + 4 + 4 is the same as ___ × ___ = ___

8 + 8 + 8 is the same as ___ × ___ = ___

10 + 10 + 10 is the same as ___ × ___ = ___

5 + 5 + 5 is the same as ___ × ___ = ___

1 + 1 + 1 is the same as ___ × ___ = ___

3 + 3 + 3 is the same as ___ × ___ = ___

APPENDIX E

WHOLE GROUP TEST

Name ______________

Test: Whole Group Score ___ /30

Answer the facts.

6 × 3 = ____	0 × 3 = ____	9 × 3 = ____
10 × 3 = ____	7 × 3 = ____	3 × 3 = ____
4 × 3 = ____	1 × 3 = ____	2 × 3 = ____
8 × 3 = ____	5 × 3 = ____	

Complete the story problem. Draw a picture, too.

Pam has 9 packs of balloons. There are 3 balloons in each pack. How many balloons doe she have?

____ ○ ____ = ☐

Tom had 3 packs of gum. There were 3 pieces in each pack. How many pieces did he have?

____ ○ ____ = ☐

Draw a picture to go with each fact.

6 × 3 = ☐

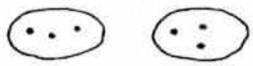
1 × 3 = ☐

10 × 3 = ☐

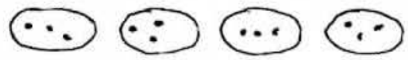
3 × 3 = ☐

Write the fact for each picture.

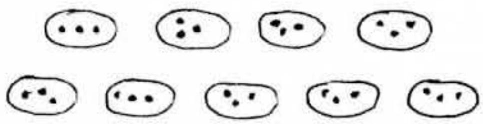
___ × ___ = ☐



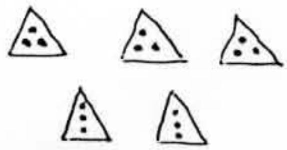
___ × ___ = ☐



___ × ___ = ☐



___ × ___ = ☐



Write the fact for each problem.

7 + 7 + 7 is the same as ___ + ___ = ___

2 + 2 + 2 is the same as ___ + ___ = ___

6 + 6 + 6 is the same as ___ + ___ = ___

9 + 9 + 9 is the same as ___ + ___ = ___

4 + 4 + 4 is the same as ___ + ___ = ___

1 + 1 + 1 is the same as ___ + ___ = ___

5 + 5 + 5 is the same as ___ + ___ = ___

APPENDIX F

POSTTEST AT TWO WEEKS

Name ____________

Test: Posttest-2weeks

Score ____ /30

Answers these facts.

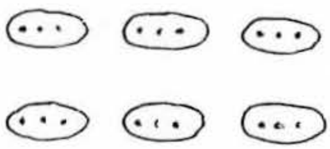
7 × 3 = ____	5 × 3 = ____	4 × 3 = ____
3 × 3 = ____	0 × 3 = ____	
9 × 3 = ____	2 × 3 = ____	
1 × 3 = ____	10 × 3 = ____	
8 × 3 = ____	6 × 3 = ____	

Draw a picture to go with each fact.

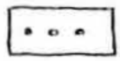
8 × 3 = ☐	2 × 3 = ☐
5 × 3 = ☐	10 × 3 = ☐

Write the fact for each picture.

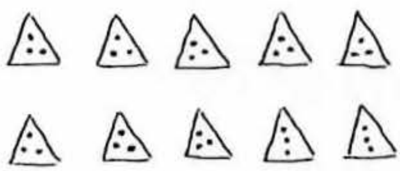
___ X ___ = ☐



___ X ___ = ☐



___ X ___ = ☐



___ X ___ = ☐

Complete the story problem. Draw a picture, too

Don had 3 bags of candy. Each bag had 3 pieces in it. How many pieces did he have? ___ ○ ___ = ☐

Sue had 6 packs of erasers. There were 3 erasers in each pack. How many erasers did she have in all?

___ ○ ___ = ☐

Write the fact for each problem

5+5+5 is the same as ___ x ___ = ___

3+3+3 is the same as ___ x ___ = ___

10+10+10 is the same as ___ x ___ = ___

0+0+0 is the same as ___ x ___ = ___

8+8+8 is the same as ___ x ___ = ___

2+2+2 is the same as ___ x ___ = ___

4+4+4 is the same as ___ x ___ = ___

APPENDIX G

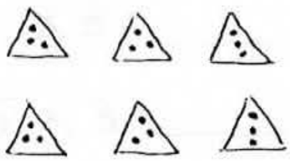
POSTTEST AT SIX WEEKS

Name

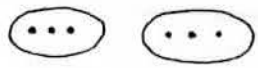
Test: Posttest 6 weeks Score /30

Write a fact for each picture.

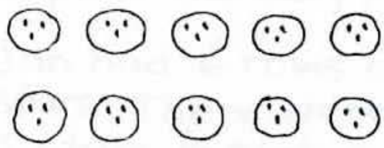
___ X ___ = ___



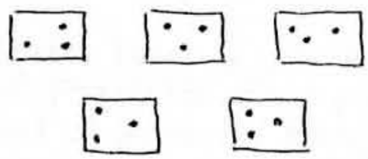
___ X ___ = ___



___ X ___ = ___



___ X ___ = ___



Draw a picture for each fact.

8 X 3 = ☐

5 X 3 = ☐

1 X 3 = ☐

3 X 3 = ☐

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