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The Effect of Manipulatives on Achievement Scores in the Middle School Mathematics Class

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by

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Declaration of Originality

I do hereby declare and attest to the fact that this is an original study based solely upon my own scholarly work here at Lindenwood University and that I have not submitted it for any other college or university course or degree here or elsewhere.

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Abstract

When applied to mathematics education, manipulatives help students to visualize mathematical concepts and apply them to everyday situations. Interest in mathematics instruction has increased dramatically over the past two decades with the introduction of virtual manipulatives, as opposed to the concrete manipulatives that have been employed for centuries. This quasi-experimental study proposed to explain the relationship between concrete and virtual manipulatives when used in a seventh-grade mathematics classroom. Using students' mathematics composite scores on standardized and teacher-created assessments, it compared the effectiveness of using concrete manipulatives alone versus using a combination of concrete and virtual manipulatives. The foundational theory of the study is that when students can visualize a mathematical concept in action, a deeper level of understanding occurs.

The results of this mixed methods study consisting of 44 seventh-grade students (22 in each group) indicated that coupling concrete manipulatives with virtual manipulatives led to a measureable change in mathematics composite scores. One recommendation is that mathematics educators incorporate both concrete manipulatives and virtual manipulatives into their mathematics curriculum. As the results of this year-long study indicated, the combination of these two types of manipulatives enabled the students in this group to accomplish a measureable change in tested mathematical ability. Educators need to offer their students lessons that are authentic and interesting in order to hold students' attention as they attempt to grasp the concepts. The different options also provide students with the needed differentiated instruction to suit their varied learning styles.

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Chapter One: Introduction

Overview of the Study

This study compared the mathematics assessment scores of two groups of seventh-grade students; one group used virtual manipulatives paired with hands-on (concrete) manipulatives, while the other used only hands-on manipulatives. The researcher's primary interest was to gain insight into whether students who used a combination of virtual and concrete manipulatives would outperform students who used only concrete manipulatives. The researcher compared students' composite mathematics scores on both standardized and teacher-created assessments, as well as each of two student groups' written reflections of their learning using both concrete and virtual manipulatives. The research sample consisted of 44 seventh-grade urban public school students divided into two treatment groups. Group A was taught with the use of both virtual and concrete manipulatives, while Group B, the control group, was taught using only the concrete manipulatives. The groups were pre-tested using the Iowa Test of Basic Skills (ITBS) prior to the teacher's use of concrete and virtual manipulatives. Following the use of manipulatives in the mathematics classroom, a post-test using the same standardized instrument, the ITBS, was administered to check for growth in mathematics achievement. The study took place over the period of one school year.

The purpose of this study was to compare the effectiveness of combining the use of concrete manipulatives and virtual manipulatives when teaching mathematics to middle school students. Suh (2005) noted that "Suydam and Higgins published a comprehensive review of research conducted in grades K-8 on the use of physical manipulatives, finding that students who used manipulatives demonstrated greater achievement than those who did not use them" (p. 23). A study published by Parham (1998) showed conclusively that students who had used manipulatives when learning mathematics outperformed students who did not have a history of using mathematics manipulatives on the California Achievement Test (CAT). The theory behind mathematics manipulatives is that when students visualize a mathematical concept in action, a deeper level of understanding occurs; this then increases the motivation of the lower academic achievers, or those students who have a more difficult time grasping mathematics concepts (Raines & Clark, 2011; Moyer, Salkind, & Bolyard, 2008). In addition, increased understanding allows teachers the opportunity to decrease the amount of review material at the beginning of the year, thus allowing substantial new growth. When students retain information, teachers can move forward and teach new material at a faster pace.

Background of the Problem

Teachers today find that they must employ the most effective and efficient instructional methods possible for increasing their students' cognitive thinking so that they can function successfully in the rapidly changing world. Therefore, teachers are searching continuously for instructional ideas and strategies that will assist in this process (Dorwood, 2002).

Theories and research connecting students' interactions with physical objects to mathematical learning have importantly influenced the emergence and use of manipulatives in K-8 classrooms. Manipulatives are both concrete and virtual objects that can be used to represent and give meaning to abstract mathematical ideas. As Moyer (2001) explained, "They have visual and tactile appeal to students and can be manipulated easily through hands-on experiences” (p. 176). Concrete manipulatives encompass any concrete objects that allow students to explore an idea through an active, hands-on approach (Battle, 2007; Anstrom, 2006). Concrete manipulatives include tactile objects, such as pattern blocks, interlocking centimeter cubes, and tangrams. These objects can enable students to recognize patterns. Another manipulative is the number line, which depicts both negative and positive numbers to help students master integer addition and subtraction. Factors of ten can be mastered through the use of colored Cuisenaire rods, which vary in length. Researchers maintain that concrete manipulatives allow students to visualize the math problem(s) and therefore more easily grasp the concepts presented during mathematics instruction (Battle, 2007). “Virtual manipulatives are essentially replicas of physical manipulatives placed on the World Wide Web in the form of computer applets with additional advantageous features” (Reimer & Moyer, 2005, p. 6). They add interest to the lessons taught in the mathematics classroom. They enable students to transcend their everyday mathematical thinking and add the element of higher order thinking. Students are eager to use computers as a part of their math lessons and to use these replicas both dynamically and statically to enhance their learning.

Interest in mathematics instruction has increased dramatically over the past two decades with the introduction of virtual manipulatives. According to Brooks, Lyons and Steen (2006), students experiencing difficulty with mathematics instruction can investigate ideas beyond grade-level expectations when provided with computer-simulated manipulatives. Students who normally would have problems with mathematics instruction can visualize and apply virtual manipulatives into their everyday learning experiences. DeGeorge and Santoro (2004) indicated that virtual manipulatives offer better control and flexibility. Additional research has verified that these hands-on educational experiences, when the virtual manipulative literally is put into the hands of the learners, enable students by giving them opportunities to engage in thinking through the creation of personal expressions (DeGeorge & Santoro, 2004; Clements, 2006). Young (2006) explained, “From an instructional standpoint, virtual manipulatives give students immediate, corrective feedback” (p. 1) Many researchers have asserted that virtual manipulatives are the perfect tool that leads to inquiry-based learning and higher-level problem solving (Clements & McMillen, 1996; Durmus & Karakirik, 2006). Suh and Moyer (2005) found that “low achieving fifth grade students engaged in multiple trial and error interactions when the virtual manipulative was a part of the lesson. They entered multiple wrong answers into the applet and through guidance and feedback provided by the applet…they understood the addition procedure” (p. 17). Virtual manipulatives keep students on task because they do not have to be passed out and collected, and they do not get lost, as a sheet of paper might. Students can stop working on an activity, save it, and return at a later time to resume their work.

Research suggests that students have trouble when attempting to move from concrete to abstract thinking. Heddens (1997) found manipulatives to be useful in assisting students as they move from a concrete to an abstract level of thinking. He added that the use of manipulatives in the mathematics classroom accentuates children’s thought processes, thus causing them to form personal mathematical knowledge.

Statement of the Problem

Manipulatives are progressively paving the way to the future of mathematics instruction; they provide an innovative way to obtain knowledge. Some researchers have speculated about the differences that using manipulatives would make in everyday mathematics instruction. In addition, research exists that questions whether the use of manipulatives leads to a measurable increase in student achievement (Boren & Hartshorn, 1990). This study proposed to explain the relationship and correlation between concrete and virtual manipulatives when used in a seventh-grade mathematics classroom. As Hunt et al. (2011) suggested, “Using concrete, followed by virtual manipulatives is recommended. Once conceptual understanding is effective with concrete manipulatives the subsequent use of virtual manipulatives seems to facilitate bridging to the abstract (Hunt, Nipper, & Nash, 2011, p. 6). Clements and McMillen (1996) determined that base-ten blocks virtual manipulatives actually were easier for children to maneuver.

Importance of the Study

The researcher believes that technology, in the form of virtual mathematics manipulatives, in conjunction with the concrete manipulatives already used commonly, acts as an essential component of enhancing mathematics instruction by ensuring students’ understanding of mathematics concepts. Virtual manipulatives overcome some of the limitations of concrete manipulatives, such as limited materials, but they also come with their own set of challenges (Jones, 2003). While concrete manipulatives are not a mandatory part of the mathematics standards, they are commonly used to assist middle school students. Students having difficulty working on challenging problem-solving tasks have had success when given concrete manipulative to aide them with the challenge (Jones, 2003).

This study compared the effectiveness of using concrete manipulatives alone versus using a combination of concrete and virtual manipulatives when teaching mathematics to seventh-grade students. The foundational theory of the study is that when students can visualize a mathematical concept in action, a deeper level of understanding occurs. Allen (2007) stated that retention in learning, the ability to retain facts in memory, proves measurable when students have the opportunity to visualize mathematical concepts (2007). In addition, better retention allows teachers to decrease the amount of review material incorporated into lessons taught at the beginning of the year, thus allowing substantial new growth. When students are able to understand and thus retain knowledge, teachers can move forward and teach new material more quickly. By giving students concrete ways to view mathematics, students can develop relationships between background knowledge and new knowledge (Goracke, 2009). According to the Common Core State Standards (Maryland Common Core State Curriculum Framework, 2011), mathematically proficient students consider the available tools when solving a mathematical problem. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels can identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They can use technological tools, such as the manipulatives discussed in this study, to explore and deepen their understanding of concepts (Maryland Common Core State Curriculum Framework, 2011, p. 5).

Purpose of the Study

This study utilized a quasi-experimental methodology to determine if adding virtual manipulatives to existing concrete manipulatives in the seventh-grade mathematics curriculum would increase students' mathematics composite scores on standardized and teacher-created assessments. The researcher qualitatively compared the two groups’ written reflections on their own learning using manipulatives. Students in the experimental group completed writing reflections at the end of the lesson on such topics as "How did the virtual manipulatives help you to learn mathematics today?" and “How do you feel about using technology in class today?” Students in the control group completed written reflections on similar questions, such as, “How well did you understand the objective of the lesson today, and is there anything that could have helped you to learn the lesson easier?" Reflective journal writing from both the experimental and control group was qualitatively analyzed in order to gauge students’ motivation, progress, and attitudes toward the use of both concrete and virtual manipulatives. The classroom teacher used the class-assigned journal reflections in the process of improving mathematics instruction.

Hypotheses and Research Questions

The guiding research question for this study was, “How could teacher use of concrete and virtual manipulatives in mathematics instruction improve student achievement in mathematics?”

Null hypothesis (Ho) - Students taught mathematics with virtual manipulatives in addition to concrete manipulatives in a seventh-grade mathematics curriculum will not demonstrate a measureable change in mathematics composite scores on standardized and teacher-made assessments compared to students taught mathematics with only concrete manipulatives.

Hypothesis (H_1) - Students taught mathematics with virtual manipulatives in addition to concrete manipulatives in a seventh-grade mathematics curriculum will demonstrate a measureable change in mathematics composite scores on standardized and teacher-made assessments compared to students taught mathematics with only concrete manipulatives.

The researcher studied the following supporting questions:

1. How do students perceive the effectiveness of their learning/understanding when taught mathematics with both concrete and virtual manipulatives?
2. How does the teacher who has experience using both concrete and virtual manipulatives to teach mathematics perceive her effectiveness when using only concrete manipulatives?
3. How does the combination of virtual and concrete manipulatives affect the academic performance of students in the area of mathematics as opposed to the use of only concrete manipulatives?

Variables

The independent variable in this study was the use of virtual manipulatives when teaching mathematics. The dependent variable was student achievement scores in mathematics.

Limitations

Potential threats to the validity of the study are as follows:

Selection of sample. The researcher focused on one teacher in one classroom who was interviewed in the process of determining the effectiveness of the treatment. The study was quasi-experimental, so the researcher used the existing groups rather than randomly assigning students to the control or experimental group, which would not have been feasible. However, both groups were taught by the same teacher with the same curriculum, activities, and assessments.

Timing of instruction. The time of treatment during the day using both concrete and virtual manipulatives can affect the results. Students taught mathematics in the morning hours might have a different mind-set, attitude, experiences, and motivation towards learning than students who were taught during the afternoon hours. However, both groups had the same teacher, the same curriculum, and the same assessments. Both groups in this study were taught mathematics in the morning, the experimental group had mathematics instruction at 8.30 am and the control group was instructed at 9:30 am.

Demographics. Most of the students in the study school came from environments characterized by income as lower socioeconomic. One hundred percent of the students qualified for free or reduced breakfast and lunch. The teacher involved with the study found ample evidence that these students had little or no home support for academic learning and were school dependent for all anticipated learning. This caused the researcher to discount any significant home support for academic strategies initiated through the school.

Motivation. The researcher reviewed student records to verify that many of the students participating in the study had exhibited relatively little, if any, motivation towards mathematic instruction in previous classes. Therefore, the researcher was aware that existing attitudes towards mathematics may have been reflected in a lack of student motivation towards the use of manipulatives, and some students may have chosen not to be involved in the use of manipulatives.

Student attendance. Irregular student attendance during the study along with issues of truancy and suspension that remove students from the study treatment (the use of concrete and virtual manipulatives) could affect the validity of results from the treatment. This not only pertains to general attendance but also attendance in both the experimental and control groups, which may have been unequal.

Fidelity. Fidelity, or reliability, pertains to the degree of consistency in implementing the treatment program. Based on her observations within the two classrooms, the researcher judged the participating mathematics teacher's delivery of mathematics instruction using both concrete and virtual manipulatives as valid based on the high consistency of application.

Loss of participants. The school and district work daily with a very mobile school population. Student transiency during a school year can reach as high as 25%, thus adversely affecting the cumulative benefits of consistent mathematics instruction using concrete and virtual manipulatives. Thus, not all of the students who began the school year in either the control or experimental group remained at the time of end-of-year assessments.

Test validity. The standardized test(s) utilized could pose a threat to the validity of the study because tests may be unfair to certain groups based on culture, environment, and learning designation (such as gifted, extremely gifted, learning disabled).

Definitions

Attitude toward mathematics. "The general attitude of the class towards mathematics related to the quality of the teaching and to the social-psychological climate of the class" (Hannula, 2000, p. 3).

Beliefs about mathematics. The ways in which an individual cognitively understands the nature of mathematics, as well as the "factors that were found to affect student attitudes toward mathematics: teacher attitudes and beliefs, teaching style and behavior, teaching techniques, achievement, assessment, and parent attitudes and beliefs" (Goodykoontz, 2009, p. 2).

Concrete manipulative. "objects that students can grasp with their hands. This sensory nature ostensibly makes manipulatives 'real,' connected with one's intuitively meaningful personal self, and therefore helpful" (Clements, 1999, p. 2).

Control group. Students not exposed to a special instructional technique, such as the use of virtual manipulatives in the present study; a sample in which a factor whose effect is being observed is not present in order to provide a comparison. "A group in an experimental study that is not given any special treatment" (Bluman, 2008, p. 652). In this study, the control group consisted of 24 seventh-grade students who were taught mathematics using only concrete manipulatives.

Grade equivalent (GE). The University of Iowa (2013) has defined the GE as:

> The score that indicates the grade level at which the student is performing. The grade equivalent is a number that describes a student's location on an achievement continuum. The continuum is a number line that describes the lowest level of knowledge or skill on one end (lowest numbers) and the highest level of development on the other end (highest numbers). The GE is a decimal number that describes performance in terms of grade level and months. For example, if a sixth-grade student obtains a GE of 8.4 on the Vocabulary test, his score is like the one a typical student finishing the fourth month of eighth grade would likely get on the Vocabulary test. The GE of a given raw score on any test indicates the grade level at which the typical student makes this raw score. The digits to the left of the decimal point represent the grade and those to the right represent the month within that grade. (para. 2)

Hands-on activities. Burns (1996) described these as activities that encompass more than one of the senses. These activities involve objects that can be touched, handled, or moved so that exploration and confidence is built as the student continues to engage in reasoning.

Iowa Test of Basic Skills. The Iowa Test of Basic Skills is a group-administered achievement test that comprehensively assesses student progress in major content areas. The test takes 30 minutes or less and provides educational staff the diagnostic data that helps prepare remediation for students at risk of failure. The test provides vital information for each student to help monitor the progress of districts, schools, and students (The University of Iowa, par. 1 2013).

Mathematics. Gilfeather and Regato (1999) defined mathematics as “an area of investigation that logically analyzes ordering, operational, and structural relationships” (p. 2).

Manipulatives. Objects that appeal to the senses and can be physically or mentally moved or touched, such as blocks or computer images (NCTM, 2003).

Mathematics achievement. Measured by comparing the gain in composite scores on tests (The University of Iowa, 2013). “Mathematics achievement is the level of attainment in any or all mathematics skills, usually estimated by performance on a test” (Eluwa, Eluwa, & Abang, 2011, p. 99).

Reflective journals. In reflective journal writing, students reflect on experiences and organize their thoughts and feelings in order to communicate clearly. Students often are given prompts that direct their reflection.

Variables. “A variable is a characteristic or attribute that can assume different values” (Bluman, 2009, p. 3). “A dependent variable is a variable affected or expected to be affected by the independent variable; also called criterion or outcome variable” (Fraenkel & Wallen, 2006, p. G-2). An independent variable is “A variable that affects or is assumed to affect the dependent variable under study and is included in the research design so that its effect can be determined; sometimes called the experimental or treatment variable” (Fraenkel & Wallen, 2006, p. G-4).

Virtual manipulative. “A virtual manipulative is best defined as an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard & Spikell, 2002, p. 372). There are two types of virtual manipulatives, static and dynamic. “Static visual representations are essentially pictures. They are the sorts of visual images ordinarily associated with pictures in books drawings on an overhead projector or even drawings on a chalkboard. Dynamic visual representations can be manipulated in the same way that a concrete manipulative can. …he or she can use a computer mouse to slide, flip, or turn the dynamic visual representation as if it were a three-dimensional object" (Moyer, Bolyard, and Spikell, 2002, pp. 372-373).

Summary

This purpose of this study was to assess the effectiveness of teacher use of a combination of concrete and virtual manipulatives in a mathematics classroom. Some students were taught mathematics with only concrete manipulatives (the control group). Other students were taught mathematics by the same teacher with both concrete and virtual manipulatives (the experimental group). The hypothesis was based on a measurable increase in students' mathematics scores when the teacher used both concrete and virtual manipulatives during instruction. The researcher also designed the study to ascertain how the mathematics teacher perceived the use of both concrete and virtual manipulatives in the classroom based on effectiveness. The significance of this study is determined through the ability of the use of both concrete and virtual manipulatives to effect measurable improvement in student achievement in a mathematics classroom.

Chapter Two: Literature Review

In this study, the researcher investigated the effect of using computer-simulated (virtual) manipulatives and hands-on (concrete) manipulatives on seventh grade students' learning skills during mathematics instruction. The researcher's primary objective was to determine whether students who were taught by a teacher using virtual manipulatives coupled with concrete manipulatives would show greater measurable achievement in mathematics than students who were taught by the same teacher using only concrete manipulatives. This review of the literature contains evidence of previous research on the use of manipulatives and the corresponding effect on student achievement in mathematics.

Significance of Manipulatives

Researchers tend to believe that manipulatives are everywhere, from street signs to the money we carry in our pockets. Hayes and Fagella (1988) stated, "Our role, as adults, is to help each child recognize mathematics situations in their activities and encourage the children to apply their knowledge and experiences to any problems that occur" (p. 9). These manipulatives serve as tools to help students solve the given situation or problem as though it were a real-life experience.

Cognitive psychologist Jerome Bruner (1960) asserted that individuals learn by recognizing symbols and patterns; we "remember a formula, a vivid detail that carries the meaning of an event" (p. 25). Grasping symbolic notation is thus the first step in figuring out mathematical concepts. As children continue to absorb the given concept more profoundly, these layers of meaning open up, moving from the concrete to the abstract and ultimately to a symbol. Bruner (1960) viewed learning as a graduated process that,

> Requires a continual deepening of understanding of ideas that comes from learning to use them in progressively more complex forms. Authentic access to a body of learning is crucial, regardless of the learners' age and prior experiences. We must teach at the learner's level of comprehension and continuously offer them chances of deepening their understanding. (p.13)

In order to accomplish this task, educators need to explore a variety of instructional methods. Herrington, Oliver, and Reeves (2003) stated,

> Influenced by constructivist philosophy and with new advances in information technology, there is increasing interest among education faculty in authentic activities as a basis for learning in web-based courses. Whereas traditionally, activities have primarily served as vehicles for the practice of discrete skills or processes taught in courses using traditional instructional methods such as lecture and readings, a more radical approach being explored by innovative instructors is to build a whole course of study around a large-scale authentic activity. (p. 59)

According to educational psychologist Howard Gardner (1991), many students indicate that they do not grasp the concept they are expected to learn because lessons are nothing more than instruction and then a test. Students do not understand why they are being taught a particular lesson because it holds no relevance to them (Gardner, 1991). Learning that is structured to complement the child's natural learning styles allows for more inquiry. There is no one size fits all when teaching for learning. Children need a variety of learning experiences to hook their interest, and not all children learn in the same manner. Some children are visual learners, others kinesthetic, and still others thrive with a combination of learning styles (Gardner, 1991). For learning to be relevant and lasting, children have to feel a connection to the material they are required to understand (Gardner, 1991).

Authentic learning enables students to examine, uncover, and collaborate on problems that mimic real-life situations. The students then can take these concepts and apply them to their everyday lives, thus binding concepts and relationships in contexts that involve real-world problems and projects that are relevant to the learner (Donovan, Bransford, & Pellegrino, 1999). The learning environments cross over into multiple content areas so that the concept can be taught as a real-life situation. Lessons are not designed to teach a designated skill but rather to teach a real-life skill. The teacher may teach an addition and subtraction lesson, but the overall concept taught is how to balance a checkbook. These lessons are authentic as the students can take them and apply them to their real lives. Authentic learning encourages students to operate within a team structure to work through the complex challenges presented to them.

Students collaborate with one another to determine the best ways to resolve the challenge presented. The resolution does not always materialize quickly; sometimes it may take students a few class sessions to reach a consensus. Through authentic learning, students attack a challenge rather than becoming frustrated at the very sight of one. They become accustomed to searching for their resolution and collaborating with classmates to reach a real-life outcome (Herrington, Oliver & Reeves, 2003). Instructional feedback that guides students and enables them to question the task using the most appropriate plan to reach a real-life resolution is more effective than simply supplying the answer. However, feedback must be administered in a timely fashion in order to lend value to the learning environment. Virtual manipulatives are one way to provide feedback to students immediately upon rendering their response. Virtual manipulatives are considered real-life learning tools because they are objects that students associate with on a daily basis (Crompton, 2011). Students are comfortable using virtual manipulatives; they do not see it as a threat, but rather more of a challenge in the same manner they would view a game (Crompton, 2011). After receiving immediate feedback, students can rethink their course of action and collaborate with classmates on an alternative process to reach a resolution (Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009). Another method used to provide immediate feedback is for teachers to display the problem on an interactive whiteboard and turn it into a whole class learning experience in which all students respond using one of a variety of tools or methods (Gardner, 1991; Uttal et al., 2009).

Educational researchers have concluded that when the learning environment is constructed around real-world situations, students systematically assume real-life roles, whether at work, at play, or working cooperatively in a true, authentic learning activity (Herrington, Reeves, Oliver, & Woo, 2002; Lombardi, 2007; Reeves, 2006). When the learning is authentic, students can connect this new knowledge directly to their lives, combine it with their existing knowledge, and form strong inferences to store for future use (Herrington et al., 2002).

Authentic learning, in turn, leads to authentic assessment, which focuses more on the thought behind the process that the learner utilized than on the actual outcome (Wiggins, 1990). Grant Wiggins, a researcher and consultant on school reform issues, proposed in his article "The Case for Authentic Assessment" that,

> Authentic assessments present the student with the full array of tasks. These tasks then mirror the priorities and challenges found in the best instructional activities:

> conducting research; writing, revising and discussing papers; providing an engaging oral analysis of a recent political event and collaborating with others on a debate. Through authentic assessment, students are more engaged in the task and a teacher can be more confident that the assessment she/he gives is meaningful and relevant. (p. 1)

Authentic assessment, also referred to as performance assessment, requires students to explain or demonstrate their thinking, strategies, and knowledge by constructing a response or project through a variety of assessment options (Wiggins, 1990). Authentic assessment should provide a variety of responses, short performance assessments balanced with longer performance assessments. Some assessments should be more complex than others and lead to students engaging in higher-level thinking skills (Wiggins, 1990). Manipulatives provide an avenue for students to demonstrate their thinking as part of these assessments.

Several scholarly articles have presented reviews of the use of mathematics manipulatives, most supporting their use in the classroom. Some of these articles contained suggestions that students be encouraged to make a personal connection with lessons through hands-on activities containing manipulatives (Burns, 1996; Allen, 2007; Clements, 1999). There are two types of manipulatives: concrete and virtual. Some controversy persists regarding the use of virtual manipulatives. Most existing historical research has promoted the effectiveness of concrete over virtual manipulatives because the former can be touched and held, while the latter, existing on a computer screen, cannot be touched or held. Lappan and Ferrini-Mundy (1993) attributed the effectiveness of manipulatives to active touching by the student. However, advances in technology have enhanced the quality of virtual manipulatives and their use in the classroom.

According to Heddens (1996), “manipulative materials must be selected that are appropriate for the concept being developed and appropriate for the developmental level of the students” (p. 47). Manipulative usage motivates and holds the interest of children far longer than direct instruction. Children need the opportunity to increase their ability to channel energy to something that is relevant to them. Manipulatives provide students with opportunities to become actively engaged in meaningful learning experiences. Because they become actively engaged in the learning process, students take ownership of their learning and then can make the transfer from concrete to symbolic and to real-life problems (Battle, 2007; Blair, 2012; Heddens, 1997). Other researchers (Steen et al., 2006; Burns, 1996) have maintained that students would look forward to mathematics instruction if the experience were engaging and pleasurable, thus lessening the effects of a stigma that often is associated with mathematics as being both difficult and boring. Their desire to investigate new topics would lead to unique shared experiences. Burns (1996) claimed that children who were able to use manipulatives to explain the process they applied to solving mathematics problems to their peers and their teacher felt less frustration. Confidence in mathematical understanding leads to math literacy (Steen et al., 2006; Burns, 1996).

Mathematic literacy is the ability to see beyond the process of mathematics and apply the concepts learned to the activities of everyday life. Most researchers (Burns, 1996; Heddens, 1996) agree that students struggle with mathematical concepts due to a lack of interest in the subject and confidence in their ability to understand the abstractness of mathematics as it relates to their personal experiences. The use of manipulatives provides a method for instruction and fosters an environment conducive to learning the concepts. Picciotto (1995) suggested that the use of manipulatives assists all students, but particularly the weaker ones. By providing a visual and kinesthetic avenue for understanding, a deeper level of comprehension takes place and encourages both low and high achieving students to be more motivated and engaged in their learning. Additionally, manipulatives allow the teacher to demonstrate in a concrete manner different methods for solving the same problem. As Picciotto (1995) explained,

> Working with well-designed manipulatives can help build the necessary foundation to facilitate the leaps to abstraction that are embedded and embodied in the notation of algebra. For some students, manipulatives provide an important tool, for others, they provide a mathematics context where they can broaden and deepen their understanding, which is often only mechanical mastery. (p. 112)

Some factors must be considered when investigating the use and effectiveness of manipulatives in the classroom. While a considerable amount of research has indicated that manipulatives allow children the advanced ability to reach higher levels in their abstract mathematics thinking, Remer and Moyer (2005) believed that the teacher's role is to lay the foundation for success with manipulatives. In order for manipulatives to improve student learning, the teacher must be knowledgeable of the many types on the market and must be able to choose the appropriate tool for the students to be able to grasp the concept. When used properly, manipulatives enhance understanding, retention and problem-solving. In order for this meaning to take place, however, students must have teachers who can help them reflect on their representation of mathematical ideas and help them develop an increasingly sophisticated understanding of mathematical functions (Allsopp, 1997; Remer & Moyer, 2005; Picciotto, 1995). Furthermore, Allsopp (1997) contended that while much research has been conducted on the benefits of using manipulatives with elementary students, the extent of their use with older students learning more abstract concepts has not been examined as thoroughly. Remer and Moyer (2005) asserted that initial signs indicate that even older students can benefit from the use of carefully selected manipulatives during well-planned lessons. Even the use of simple manipulatives can enhance the learning and greater understanding of algebraic equations (Allsopp, 1997). After students learn to solve basic equations through direct instruction with manipulatives, they can begin to progress toward an abstract level of comprehension by transferring to symbolic representations of the problem through either drawing or providing written descriptions of their work.

Suydam and Higgins (1977) stressed that the use of manipulatives be kept consistent with the goals of a mathematics program. They further stated that teachers also should encourage children to record results, which can promote the development of higher-level thinking skills and deeper peer interaction. Children also can question their own procedures, as well as those of their peers, thus instigating the cooperative problem-solving that will bring real-life situations and their solutions into the classroom. Children then will discover the importance of verbalizing their mathematics thinking and concepts. Teachers recognize that the use of manipulatives in mathematics instruction allows students to experience different ways to solve problems other than just following teachers' directions. As DeGeorge and Santoro (2004) reported,

> 46 percent of teachers viewed hands-on projects as an effective learning technique for all students. Another 54 percent said that this approach was particularly well-suited for students who learn more effectively in non-traditional approaches, particularly visual or kinesthetic learners, slow readers, and students with limited English-language skills. (p. 1)

DeGeorge and Santoro (2004) also noted that teachers noticed differences in the behavior of their students, as well. The teachers believed that the children were more inclined to ask questions, were engaged in discussion, completed assigned tasks and were motivated and eager (DeGeorge & Santoro, 2004).

Manipulatives also allow for creative memorization. Research has shown that if the brain does not make a connection with the material being taught, retention will suffer (Bellonio, 2001; DeGeorge & Santoro 2004; Suh & Moyer, 2007). When children seek to recall information with which they have not connected, they will have much more difficulty retrieving it. The brain learns from patterns and searches for those patterns that make learning easy. Manipulatives allow students to make those critical connections and to form patterns that are most relevant to their learning styles (Bellonio, 2001; DeGeorge & Santoro 2004; Suh & Moyer, 2007).

Balka (1993) described the benefit of using manipulatives:

> The use of manipulatives allows students to make the important linkages between conceptual and procedural knowledge, to recognize relationships among different areas of mathematics, to see mathematics as an integrated whole, to explore problems using physical models, and to relate procedures in an equivalent representation. (p. 22)

If not used properly, connections between the use of manipulatives and an abstract concept may not be made.

In conclusion, when students are given the opportunity to connect the mathematics lessons to events occurring in their everyday lives the brain will in turn store that memory and enable students to retrieve the information when the cause arises.

History of Manipulatives

As manipulatives can encompass a wide variety of physical objects, they naturally have been present in societies for many years. Historically, many individuals have relied indirectly on manipulatives in the teaching of mathematics. Many of the early manipulatives were types of counting boards (About, 2005). The Southwestern civilizations used wood or clay trays that were covered with sand in which they would draw symbols so they could tally items that resembled an inventory. Another early version of a manipulative, the abacus, dates back to 300 B.C. (About, 2005). This abacus, known as the Salamis Tablet and used by Babylonians, was discovered in 1846 and was believed to have been perfected by the Chinese (About, 2005). Sheepherders used an instrument closely resembling this to count sheep, placing a knot in a rope for every ten sheep. This manipulative is much like the abacus that children use, which many researchers believe encourages abstract thinking and leads to higher-level thinking skills (Hoffman, 2007). Manipulatives have developed greatly from these early counting devices.

Manipulative blocks proved valuable in the teaching of early mathematics and have served as educational tools for over 200 years. According to Meredith Portsmore (2007), “The evolution of the block has been driven by two forces, the need to represent more complex ideas and the world of children's toys" (p. 1). In his research, John Locke, a 17-century English educator, cited various toys used for educational purposes. In the 17th century, a new way of thinking emerged in which children were not viewed as young adults but rather as developing individuals. This thinking initiated the production of new toys that could be categorized as concrete manipulatives. The first type of blocks used as manipulatives were alphabet blocks (Read, 1992). By the time blocks emerged as an important learning tool, the Industrial Revolution had begun. Germany was credited with the appearance of toys as learning objects (Read, 1992). The Industrial Revolution lowered the prices of toys that previously had been constructed by hand. Toys began being produced at a faster pace, opening the door for educators to examine their use as learning tools (Read, 1992).

Jean Jacques Rousseau, an influential 17-century philosopher from France, posited that individual freedom is more important than the structure imposed by the government. His written work on the subject of education asserted that children learn best by intermingling without restraint in their environment. Rousseau's thoughts on education foreshadowed the educational reforms of the 20th and 21st centuries. Doyle and Smith (2007) discussed and expanded upon Rousseau's work:

> People must be encouraged to reason their way through to their own conclusions - they should not rely on the authority of the teacher. Thus, instead of being taught other people's ideas, Émile is encouraged to draw his own conclusions from his own experience. What we know today as 'discovery learning'. One example Rousseau gives is of Émile breaking a window - only to find he gets cold because it is left unrepaired. (p. 1)

The research and use of manipulatives continued to evolve throughout the next century (Doyle & Smith, 2007).

In the early part of the 19th century, new manipulatives emerged in the form of concrete, movable objects. Due mainly to the efforts of Swiss educator Johann Pestalozzi, a believer in the teachings of Rousseau, manipulatives made small inroads in mathematics teaching. Pestalozzi (1951) believed that all learning should focus on the child rather than on the curriculum because all people learn in their own way. He contended that all people are born with their own innate, unique abilities and stressed that educators need to unlock those natural abilities and afford children the time to explore on their own. Pestalozzi believed that children require sensory stimulation to grasp the intended concept being presented. Children could better envision the result and apply it to their own natural surroundings if the concept was tied to their real lives (Barlow, 1977). However, not until the 1930s did manipulatives become part of the mathematics curriculum (Sowell, 1989). Similar to Locke and Rousseau, Pestalozzi (1951) believed that a child has thought processes that begin with imagery, as best achieved by studying objects and processes as they naturally occur. To engage this imagery, teaching should employ the senses. His written work on the subject of education argued that children learn best by interacting freely with their environment. These thoughts on education anticipated the educational reforms of the future.

Friedrich Froebel, a German educator and avid follower of Pestalozzi, was the first educator to consider how these blocks could be used to educate children. Froebel’s attempts in the 1800s were the most organized and had the most long-term results (Hayward, 1979). His kindergarten employed games that encouraged children to work within groups to learn cultural values and to cultivate their social and physical skills. These games involved objects of various shapes, as well as other materials, such as sand and cardboard, that facilitated learning and comprehension. Froebel's techniques opened the door for further exploration by future educators (Hayward, 1979).

In the early part of the 20th century, Maria Montessori, an Italian physician and educator, argued that teachers should be trained to use Froebel's methods. Montessori founded schools and acquired a multitude of followers who believed in and stressed the importance of concrete, authentic learning experiences. She believed that children actualize their innate desire to learn through self-directed exploration of developmentally appropriate manipulatives (Ward, 1971). She affirmed that children demonstrate greater success when they are able to judge their own progress. According to Montessori, there are three stages of learning. The first stage involves introducing the concept, presenting it to the child without hiding any details. The result is revealed to the children so that they can uncover the necessary process to reach their solution. The children then develop a process, thus demonstrating their understanding of the concept. Montessori claimed that children should not focus their attention on teachers, but rather that teachers should focus their attention on the children. The children's experiences enable them to self-correct their lesson, so they can identify incorrect solutions and figure out alternative solutions to the challenge. Montessori had the opportunity to implement her methods to treat and educate the mentally retarded. Some of her eight-year-old mentally handicapped students scored above average on the state examinations for reading and writing, an accomplishment considered the first Montessori phenomenon. Her response to their success was, "if mentally disabled children could be brought to the level of normal children then (she) wanted to study the potential of 'normal' children” (Cohen, 1990, p. 65). Using her methods, teachers acted as the facilitators and encouraged children to try different approaches to a challenge. They saw children who once struggled start to bloom and show confidence. The children also gained inner respect that was not present prior to the preparation of the lesson (Cohen, 1990). Montessori’s discoveries led to more advanced studies of student learning.

In the second half of the 20th century, new theories began to emerge as to how students learn best. One of these theories was known as constructivism. John Dewey, an American psychologist and educational reformer, believed that all children benefit from active learning (Lane, 2010). Thanasoulas (2000) explained the theory:

> The constructivism theory takes an interdisciplinary perspective, inasmuch as it draws upon a diversity of psychological, sociological, philosophical, and critical educational theories. In view of this, constructivism is an overarching theory that does not intend to demolish but to reconstruct past and present teaching and learning theories, its focus with shedding light on the learner as an important agent in the learning process, rather than in wresting the power from the teacher. (p.1)

Constructivism strayed from the philosophy proposed through behaviorism. Behaviorists believed in lessons that were teacher centered while students sat passively. The constructivist paradigm holds that children can learn by constructing their own knowledge and thoughts on concepts if given autonomy. The child then takes this newly discovered concept and attaches it to previously stored knowledge so that it can be transferred to real-life situations (Lane, 2010). Swiss cognitive psychologist Jean Piaget agreed with constructivist views that inquiry learning promoted children to use discovery learning to form a schema for new material (Lane, 2010). When children form a schema to represent the information they are dealing with, then they are constructing a process for problem-solving. They then can organize the information so that it is easier for them to interpret when needed. Piaget asserted that children use two strategies when organizing new material, assimilation and accommodation (Atherton, 2010). Assimilation transpires when students take new knowledge and combine it with their existing knowledge. Accommodation occurs when students alter their perceptions of the material and show understanding as they apply it to real-world situations (Ginesi, 2008). Russian educational psychologist Lev Vygotsky also believed in the constructivist views. He maintained that in addition to inquiry-based learning, students also need social interaction, collaboration with their peers, to master a concept and obtain the full range of their learning. Vygotsky introduced the term Zone of Proximal Development (ZPD), the range within which students can no longer work on a task without guidance. He asserted that when students are encouraged to discuss their obstacles with peers and their teacher, discovery learning is more apt to occur. He also stated that children may start a task with very different views on the task, but when they are encouraged to discuss their findings, they will eventually reach a shared consensus, otherwise referred to as subjectivity. The ZPD is associated with the term *scaffolding*. Scaffolding uses the principle that the amount of assistance provided to a child will vary based on that child's proficiency with the task. Vygotsky believed that if more dialogue was encouraged during the task, children would add this to their pre-existing knowledge and form new ideas (Vygotsky, 1978; Breaux, 2009).

Further research led to the understanding of the neurological underpinnings of learning. In the book *Brain Matters: Translating Research into Classroom Practice*, Wolfe (2001) contended that authentic learning takes place when connectors form between neural networks. The author suggested that three levels of learning take place when strengthening learning through these connections. The first is the concrete level, followed by the representational, symbolic level, and finally true understanding being demonstrated at the abstract level. Without the use of concrete experiences, the student cannot move through the stages of learning in order to gain meaning. Therefore, educators should seek to develop lessons that involve multiple approaches and tools to improve the chances of reaching all students. This approach gives the students opportunities to see the problem in a way that will allow them to acquire full comprehension of the concept. The manipulative allows the students to see the abstract in various solutions, and they then begin to recognize the common thread between operations. This learning then transfers into a greater understanding of the symbolic form of mathematical functions. At this point, abstract functions can be performed on symbolic representations of the problem without the use of the concrete. Through reflection, students are able to connect the new knowledge with previously learned information (Wolfe, 2001).

In the 1960s, manipulatives focused on the use of concrete objects and pictorial representations to help children better understand abstract mathematical ideas (Sowell, 1989). Now, manipulatives are available in most classrooms around the world. The research of Hungarian educator Zoltan Dienes is important to the understanding of manipulatives in classroom instruction; he is renowned for his dream of teaching math through activities. According to Dienes, students will not achieve success in math until they realize that information and skills are arranged around familiarity and abstractions. Several of Dienes' inventions became standard equipment in the mathematics laboratory (Dienes, 1961). As Hirstein (2007) explained, Dienes'

> Multi-base Arithmetic Blocks gave a concrete representation for number bases. The principles of the base ten-numeration system took for granted that most students did not grasp the value of a base system. Dienes' Blocks allowed students to explore the numeration system and then students determined how the operations on numbers were addressed by the system. (p. 169)

Dienes maintained that manipulatives were important in increasing students' understanding of mathematical concepts. He stressed that children learn based on prior knowledge connected to new knowledge. They then use these inferencing skills and make the connection to the new concept. He stated that the manipulatives should be used as a reference to build upon several concepts as opposed to one abstract idea (Dienes, 1961). By the 1970s, microprocessors made the first electronic calculators possible, and educational toys using microprocessor technology became popular.

The reform math era was a time of restructuring in math education toward autonomous student learning. Lessons were student-guided, utilizing manipulatives through games and learning centers. Students collaborated more frequently and relied upon experience more so than rote memorization. From the 1970s through the 1990s, the overall math scores for U.S. students decreased to a point that educators became alarmed (National Commission on Excellence in Education, 1983). The 1983 report commissioned by Terrell Bell, the U.S. Secretary of Education, was titled *A Nation at*

Risk, and it highlighted shortcomings in U.S. education, including math education. The report indicated that other nations were surpassing the educational practices in the U.S. and outperforming U.S. students in mathematics. The report emphasized that the percentage of students electing to enroll in a mathematics class had declined significantly, and 35 states only required one year of mathematics during high school (National Commission on Excellence in Education, 1983). At the beginning of the 21st century, numerous states implemented stricter basic math standards for their school districts. Calculators became standard equipment in kindergarten through high school classrooms. Advanced graphing calculators were available to students who enrolled in advanced math courses, such as calculus and statistics. Computers presented a new type of manipulative in the form of a virtual manipulative. These virtual manipulatives proposed games as a type of learning tool, along with greater algorithm practice. Elementary schools around the country were turning to a new series that focused on a circular learning pattern, meaning that students would return to a specific topic multiple times within a school year. This new series entitled "Everyday Math" used eye-catching illustrations and a variety of math manipulatives (Allen, 2007).

One contrast throughout the history of education has been the benefit associated with the use of manipulatives. Currently, they are highly promoted by the National Council of Teachers of Mathematics (NCTM) because manipulatives serve as hands-on tools that help students construct an understanding of mathematical concepts. The *Principles of the NCTM* (2000) stated,

> Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. The use of manipulatives also

> provides equity in the classroom. Not all students benefit from the same type of instruction. Many students profit from this hands-on collaborative learning that manipulatives afford. (p. 20)

This educational opportunity helps students cultivate a deeper understanding of mathematics when combining multiple teaching strategies.

Concrete Manipulatives

Mathematics literacy refers to the ability to see beyond the process of mathematics and apply the concepts learned to the activities of everyday life. Most researchers agree (Burns, 2006; NCTM, 2000) that students struggle with mathematical concepts due to a lack of interest and confidence in their ability to understand the abstractness of mathematics as it relates to their everyday lives. Concrete manipulatives can be objects such as building blocks, color sticks, counters or other physical items that can be used mathematically to build the connection between concepts and reality. Researchers maintain that if children use concrete manipulatives, they can form visualizations in their heads and more easily grasp the concepts presented during mathematics instruction (Battle, 2007; Burns, 2006; NCTM, 2000).

Children learn and retain information best when they can manipulate objects with their own hands, as Montessori espoused, and they desire this type of contact. “Movement, or physical activity, is thus an essential factor in intellectual growth, which depends upon the impressions received from outside and through movement we come in contact with external reality and it is through these contacts that we eventually acquire even abstract ideas” (Montessori, 1966, p. 97). She also maintained that children need to interact with each other. “This then is the first duty of an educator: to stir up life but to leave it free to develop" (Montessori, 1967, p. 111). The NCTM (1989) challenged teachers to revamp the way "they provide students with a lasting sense of number and number relationships, learning should be grounded in experience related to aspects of everyday life or to the use of concrete materials designed to reflect underlying mathematical ideas" (p. 87). They further encouraged, "A shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving" (NCTM, 1989, p. 125). Using concrete resources does not always mean that students fully comprehend the idea. Clements (1999) advanced his belief that students need concrete resources to construct preliminary meaning, and they must nurture their actions with the manipulatives to do so. The idea was based on a need for students to connect manipulative use with the formation of the concept presented.

Clements (1996) and Heddens (1997) proposed that if students do not come to understand the concept being taught with the use of manipulatives, then those manipulatives simply function as toys with which to play. Piaget (1952) found that students progress in the way they think, beginning with a recognition of only the concrete before advancing to pictorial and then to abstract thinking. Once educators are able to view the overall picture, they will understand that children do not necessarily see the same picture that they see. Holt (1964) said that he and his fellow teachers

> were excited about using the rods for math because we could see strong connections between the world of rods and the world of numbers. We therefore assumed that children, looking at the rods and doing things with them, could *see*

> how the world of numbers and numerical operations worked. The trouble with this theory is that [my colleague] and I *already* knew how the numbers worked. (pp. 138-39)

Fick, O'Donnell, Puchner, and Taylor (2008) advanced their research that effective lessons using manipulatives do not merely happen. Rather, these lessons are the product of teacher thinking and preparation based on years of teacher training and support for the use and interpretation of manipulatives into their curriculum (2008).

A number of researchers have reported experimental results pertaining to the use of concrete manipulatives in the teaching of mathematics. Battle (2007) completed a study of first-grade students learning to add and subtract whole numbers. She administered a pretest and post-test in her study to determine the change in student success. The experimental group was provided counters (a type of manipulative) to use while adding and subtracting whole numbers. The control group was not provided with manipulatives. The post-test scores revealed that the experimental group experienced a higher degree of success in adding and subtracting whole numbers.

Suydam and Higgins (1977) published "a comprehensive review of research conducted in grades K-8 on the uses of physical manipulatives," finding that "students using manipulatives demonstrated greater achievement than those not using them" (p. 92). The key to their findings was that "physical manipulatives would yield positive results if the manipulative was used well" (p. 92). Similarly, a study conducted by Parham (1998) demonstrated that students using manipulatives outperformed students who had not experienced the use of manipulatives on the California Achievement Test. The test results showed that students with access to the manipulatives scored in the 80th percentile, while students who were not exposed to the manipulatives scored in the 50th percentile.

A comprehensive meta-analysis conducted from 1953 to 1987 confirmed the effectiveness of concrete manipulatives in kindergarten through college. Sixty studies were conducted, and they demonstrated evidence of increased achievement in mathematics when concrete manipulatives were used throughout instruction. The study also showed a positive increase in the students' attitudes toward the manipulatives. The most noticeable increases occurred when the teacher evidenced comfort with and knowledge of the effective use of the manipulative based on the experience of others. This study helped to verify the role of the long-term use of manipulatives in increasing mathematics achievement (Sowell, 1989).

Hiebert (1997) affirmed,

> Mathematical tools should be seen as supports for learning. However, using tools as supports does not happen automatically. Students must construct meaning for them. This requires more than watching demonstrations; it requires working with tools over extended periods, trying them out, and watching what happens. Meaning does not reside in tools; students construct it as they use tools. (p. 10)

If not used properly, connections between the use of manipulatives and an abstract concept may not be made.

The potential for disengagement between a concrete manipulative and its actual symbolic representation is evident in the following research conducted by Uttal, Scudder, and DeLoache (1997). In a study of three- and four-year-old children, Uttal validated the idea that children have difficulty linking models to their physical counterparts. The researchers built a replica of a room, but on a much smaller scale than its actual size. When children were shown the replica with the exact location of a hidden toy, children could not enter the original room and find the hidden toy. After hearing an explanation of what the replication indicated, a few could find the toy, while others remained confused. This result shows that symbolic representations that may be direct for adults can be confusing to children (Uttal et al., 1997).

Aligning manipulatives with curriculum standards can help overcome the disconnect created by stagnant classroom learning. Burns (1996) proposed that the best way for children to learn is through abundant hands-on learning with the use of models to solve problems. Although there has been significant support for the use of manipulatives in the mathematics classroom, challenges also have surfaced, some in relation to classroom management. Teachers have reported difficulty in monitoring and assessing students' use of manipulatives. Another dilemma arose from the realization that many school districts lacked the necessary funding to purchase sufficient manipulatives. Another important issue directly affecting teachers’ use of manipulatives has been the lack of professional development coupled with the necessary follow through and support that would permit teachers to use manipulatives effectively (Crawford & Brown, 2003).

Ball (1992) cautioned against the unrealistic expectations that many teachers have about manipulatives, concluding that, "Manipulatives and the underlying notion that understanding comes through the finger tips have become part of educational dogma" (p. 17). She acknowledged several studies that showed failure in student achievement due to limited teacher knowledge of the effective use of manipulatives. A year-long study conducted using interviews and observations in 10 different schools provided evidence of the challenges associated with manipulatives. Moyer found that some pitfalls in the use of manipulatives became evident when teachers were unaware of the mathematical concept and did not know how or why they were using the manipulatives (2001). Additional research confirmed that unless students can transfer the mathematical concept from the concrete manipulative to a conceptual and procedural understanding, the manipulative was not effective (Suh, 2005). Another important idea contained in the literature on physical manipulatives centers on Clements and McMillan’s (1996) new interpretation of the word *concrete*. They proposed that students do not necessarily need the help of physical objects in order to assimilate concrete understanding. Virtual manipulatives have the substance to support students’ integrated concrete experiences, the difference being that those images are viewed on a computer screen. According to Clements (1999), a concrete activity’s effectiveness depends on its level of mental stimulation; therefore, virtual manipulates have the opportunity to serve as equally meaningful learning tools.

Virtual Manipulatives

Moyer, Bolyard and Spikell (2002) stated that the growth of technology is always present and that everything humans encounter in the world is affected by technology. Virtual manipulatives, which represent the usage of technology in mathematics, are defined as “interactive, web-based representations of a dynamic object that present opportunities for constructing mathematical knowledge” (Moyer, Bolyard, & Spikell, 2002, p. 373). These manipulatives are considered concrete although they are not physical (Clements & McMillen, 1996). Clements and McMillan (1996) noted that while children cannot touch the virtual manipulatives, they can move the objects on the computer screen and interact with them. They further commented that "Teachers can integrate these representations into their classrooms because they can become more manageable, clean, flexible, and extensible" (p. 271).

> In an early study of different types of educational software, teachers looked at both narrowly directed drill-and-practice software and at software that purports to open up opportunities for students to ask their own questions. They found not only that different approaches to software design implied radically different models of learning and teaching, but also that in the process of examining software critically the teachers became more aware of their own values. Teachers saw the enormous pedagogical difference between solving problems and formulating them and between answering someone else's question and generating their own question. (Olds, Schwartz, & Willie, 1980, p. 40)

As asserted in the quotation, the type of virtual manipulative presented in class directly affects student learning. Manipulatives that perform in a rote manner are less captivating than those that compel students to dramatically modify their thinking to accommodate this mode of learning. This transformation in thought encourages students to evolve from their prior experiences and to accept the challenges presented. These types of compelling virtual manipulatives also sanction collaboration among peers, which is encouraged so that students can share mathematical concepts as they analyze their problem-solving results. Students then can connect their results to real-life situations and understand that more than one solution exists to a given challenge. Students then collaborate with peers and teachers to share outcomes and procedures used to reach their intended solutions (Moyer, 2008).

Later, in the 1990s, researchers at Utah State University began a project to produce a computer program offering interactive tools for teaching and learning mathematics. Hartshorn and Boren (1991) focused on organizing the various tools into learning benchmarks and grade levels. Explanations of how to use the software and activities for implementation were features that accompanied the Java software. The images initially were basic representations of Base 10 blocks, geo boards, and pattern blocks. This early research tried to mimic the use of concrete manipulatives, and the results varied depending upon a variety of factors. As more tools were developed and implemented, teacher comfort with strategies for using these tools to enhance the academic success of students continued to improve (Hartshorn & Boren, 1990). Students who used the tools became actively engaged in learning, which enhanced both their understanding and confidence. However, they found that for the learning to be useful and for the student to be able to move from the concrete to the abstract, the teachers must have carefully selected the activities and manipulatives used. The progression from concrete to abstract occurs by moving through the semi-concrete stage of learning. At this point, the student can make meaning of the visual representation of the problem and transfer that meaning into real situations (Hartshorn & Boren, 1990). The achievement of the abstract level forms the bridge to real understanding. At this point, students can explain the processes and results of their investigations in both oral and written form. In order to ensure that students retain their understanding, the use of manipulatives must be ongoing throughout the lesson (Hartshorn & Boren, 1990).

Sowell (1989) suggested that long-term use of manipulatives is more effective in maintaining and even increasing learning than short-term use, which indicates that use of manipulatives should be continuous throughout middle school and high school. However, implementation on all three levels has been limited. As a result, information on their effectiveness at the elementary level is limited. Jones (2009) asserted, “it is more likely that manipulatives would increase their value in later grades, in teaching more complicated skills, as children mature and become mentally able to develop understanding of operations” (p. 5). Jones (2009) maintained that the use of manipulatives at the elementary level would allow the students to bridge the gap between what they are doing and the meaning it represents, thus increasing understanding and not just computation. As Bellonio (2009) emphasized,

> Experiential education is based on the idea that active involvement enhances students’ learning. Applying this idea to mathematics is difficult, in part, because mathematics is so abstract. One way of bringing experience to bear on students’ mathematical understanding, however, is the use of manipulatives. (p. 1)

Children who actively engage in their learning gain a greater understanding of the material than passive learners. While virtual manipulatives do not adhere to the traditional definition of concrete manipulatives, they may provide even more meaningful representations of objects and concepts than those that can be touched. Gardner (1993) stated that employing virtual manipulatives increases the chances of students engaging as active participants in the acquisition of a skill. Not all students learn in an identical fashion, so it benefits students and teachers to address as many of Gardener’s eight intelligences as possible. Moving from concrete to virtual manipulatives provides visual, auditory, and kinesthetic modes of instruction, allowing the students to gain an understanding of the material more easily (Gardner, 1993).

Takahashi (2000) conducted a study of 18 fourth-grade students as they explored ways to cover an equilateral triangle using green and blue pattern blocks. The students were divided into two groups; the first nine children worked alone on computers with virtual manipulatives to solve the challenge, while the others used concrete manipulatives. The results demonstrated that the children working with the virtual manipulatives stayed on task and successfully met all 18 intended outcomes. Takahashi observed that one reason the virtual manipulatives proved more successful was due to the immediate computer-generated response. The children working with the concrete manipulatives had to draw a replication of each equilateral triangle as it was completed, and then take it apart and start over (Takahashi, 2000).

In a study of two classes, one that employed problem-based learning with a focus on technology, and one that took a traditional approach using a textbook followed by assessments, Boaler (1999) found that significant gains were displayed in the classroom that employed problem-based learning. The study also showed that students exposed to problem-based learning significantly outperformed their counterparts on the state test. These students also saw no difference between their mathematics lessons and real-world mathematics. Students in the traditional math class did not hold the same views and were not as apt at translating the school math into real-world math (Boaler, 1999).

Virtual manipulatives add another dimension to mathematics lessons for children with special instructional needs. Children who are not proficient with the English language are often at a disadvantage expressing their understanding of mathematical concepts. Virtual manipulatives allow these children the opportunity to demonstrate their understanding of the concepts (Moyer, Salkind & Bolyard, 2008). The NCTM (2000) has endorsed that

> Technology can help students develop number sense, and it may be especially helpful for those with special needs. For example, students who may be uncomfortable interacting with groups or who may not be physically able to represent numbers and display corresponding symbols can use computer manipulatives. (p. 80)

Therefore, students with special instructional needs benefit when given the opportunity to interact with virtual manipulatives in their lessons. Virtual manipulatives cause less apprehension as the students need only to contend with the computer image. They have less fear of making mistakes and will take higher levels of challenges.

Virtual manipulatives provide feedback and hints as the children work on the mathematical concept. This is not the case with traditional concrete manipulatives because assistance from the teacher is needed if difficulty arises. Children then will be able to see the whole picture and have an easier time relating the visual and symbolic representations, which assist in reaching higher levels of learning (Suh & Moyer, 2007).

In addition to immediate feedback, researchers have found that children achieve a greater connection to virtual manipulatives because they can manipulate them while working on a computer monitor. Therefore, students actually see the outcomes of their manipulations instantaneously. Students often are driven by their own competitiveness to achieve success using virtual manipulatives (Moyer, Bolyard & Spikell, 2002). In the mathematics classroom, technology plays a key role in helping children to form relationships with numerical reasoning. Children who use appropriate technology persevere over a longer period of time despite problems or difficulties they may encounter while learning. Researchers have observed that those children actually showed eagerness and enjoyment, and exhibited growth in mathematics performance (Ainsa, 1999; Bellonio, 2001; Bolyard & Moyer-Packenham, 2006).

Virtual manipulatives can be both static and dynamic. A static manipulative is simply a visual representation on the computer screen. Children cannot manipulate the representation, but it may change according to the program. While children can still receive immediate feedback from their interaction with the static image, it is not as engaging as its counterpart, the dynamic manipulative (Clements & McMillen, 1996; Moyer, Bolyard & Spikell, 2002). Dynamic manipulatives allow children the ability to move and change the image on the screen, thus forming a deeper understanding of the concept based on their own actions. Burns (2006) maintained that these dynamic manipulatives are thought provoking and lead children to focus on the lesson, causing scaffolding in their individual learning. Today's classrooms embody such diversity in learning styles that the dynamic manipulative has become a necessity in advancing mathematics instruction. Many levels of the same lesson can be presented to children and then differentiated according to their academic levels. With most students now having access to a computer and the internet at school and home, it only makes sense to include virtual manipulatives in the curriculum. Students now have such heightened exposure to digital media that it has become an acceptable and even expected medium for enhancing instruction. Mathematics educators now can access digital manipulatives via the internet as an innovative and interesting way to enrich their curriculum. Technology adds to the instruction and creates a more student-centered learning environment by allowing the student to work at his/her own level and pace. Dynamic, interactive media make it possible for students to view objects and concepts in more than one dimension, expanding their conceptual framework of understanding. The ability to link representations to previous learning facilitates the transition from the concrete to the more abstract (Durmus & Karakirk, 2006; Moyer, 2005).

Moyer (2005) found that when using virtual manipulatives to reinforce a concept, it was beneficial to provide direct instruction in the use of the program and concept to improve learning. The students then could spend more time understanding the concept while learning the program “The transformative nature of many virtual tools simply allows students to explore ideas flexibly, modeling the fluidity of the brain’s activity and human thinking in ways that cannot be done in a physical space” (Moyer, Salkind, and Bolyard, 2008, p. 216). Multiple applications of a technological manipulative provided an additional advantage over a traditional manipulative with a single application. The use of computers in the classroom also allows students who may have difficulty with motor skills or written expression to achieve more success because the virtual manipulatives do not require physical movement or responses in written form (Crawford & Brown, 2003; Steen, Brooks, and Lyon, 2006).

One concern with using computers for instruction is that they do not replace the interaction between student and instructor and cannot supply the kinesthetic element present when using concrete manipulatives. Others have posited that children may not be capable of understanding the symbolic representations in computer instruction and instead will need a hands-on approach achieved only through the use of concrete materials (Brown, 2007; Fueyo & Bushnell, 1998). Taylor contended that another shortcoming is that children often view manipulatives on a computer as just a game and not a connection to real-life situations (2001).

Young children often do not use fractions in real-life situations, but this is an area where the use of manipulatives can assist greatly. Fractions, though important, are not used as commonly as whole numbers. Children need help expanding their pertinent experiences with fractions so that their informal understanding of fractions and connections to procedural knowledge adhere to a conceptual understanding (National Research Council, 2001). In one study, Suh, Moyer, and Heo (2005) investigated three fifth-grade classrooms in which students were applying fractions in order to demonstrate these connections. They also investigated the use of virtual applets to aide in this concept. The three classes were categorized as consisting of students considered low, medium and high achievers. The students did not seem to realize that they were applying previously learned knowledge to the new concepts, as evidenced by interviews with the researchers. All students in the study were introduced to and taught the lessons by the same teacher. All three groups showed discovery learning, higher levels of conjectures, connections to previous learning, and greater levels of peer interaction. However, the results of the study showed that the lowest achievers demonstrated the greatest gains (Suh, Moyer, & Heo, 2005).

Bellonio (2001) importantly noted that manipulatives are viewed differently by different users. Virtual manipulatives can be just as important to some as concrete manipulatives are to others. Sometimes, the virtual manipulatives were easier to manipulate than their concrete counterparts. One example of the power of virtual manipulatives occurred with children learning the concept of number sense. One group used a dynamic virtual manipulative to assemble beans, sticks, and number symbols to conceptualize number sense; the other group used concrete bean sticks. The results showed that the virtual bean sticks were easier for the children to manipulate than the concrete bean sticks. However, the findings did not show a noticeable difference in assessment scores between the two groups (Bellonio, 2001).

According to Rhodes (2008), manipulatives hold students' attention and challenge them to solve problems and develop an understanding of higher-level mathematics concepts. Manipulatives feature lively, bright colors; they contain sounds and incorporate games that talk to students, all of which are features not available with textbooks. Students are taught how to use concrete manipulatives, which can guide them to a level of abstract thinking. Students engaging in abstract thinking can transfer their understanding of mathematics concepts to authentic learning situations and engage in problem solving as active learners (Rhodes, 2008).

Clements and McMillen (1996) determined that virtual base-ten blocks were easier for children to maneuver than concrete blocks. The virtual base-ten blocks were essentially in line with the students' own mental actions pertaining to the intended learning outcome. As the children continued to utilize the virtual manipulative, the intended outcome became clearer. Children could break the blocks apart to form ones or fasten them together to form tens. This activity proved to be natural for the children and contributed to building their inference skills. Children received immediate feedback because every time they changed the block, the number shown on the computer screen changed also. As Clements and McMillen (1996) explained,

> Actual base-ten blocks can be so clumsy and the manipulations so disconnected one from the other that students see only the trees -- manipulations of many pieces -- and miss the forest -- place-value ideas. The computer blocks can be more manageable and clean (p. 3).

Clements and McMillen (1996) stated that virtual manipulatives also offer less distraction than their concrete counterparts. When using concrete manipulatives, teachers run the risk of students using them inappropriately. Some of the manipulatives also may become broken or lost (1996). Virtual manipulatives offer more versatility than concrete manipulatives, allowing students to change the data representation with a simple keystroke. The students then are able to connect the different representations to all of the possible outcomes for a given problem. They can deduce that there is more than one way to reach a resolution to their challenge. They then can apply this reasoning to real-life situations (Durmus & Karakirik, 2006; Yong, 2010; Blair, 2012). Students can save what they are working on and return later to both review their earlier work and continue their learning. Students then can revise their strategies through true mathematical exploration. Additionally, computer games provide the same type of exploration and reflection as lessons (Crompton, 2011; Bellonio, 2001). Clements and McMillen (1996) stated that,

> Computer manipulatives link the concrete and the symbolic by means of feedback. For example, a major advantage of the computer is the ability to associate active experience with manipulatives to symbolic representations. The computer connects manipulatives that students make, move, and change with numbers and words. Many students fail to relate their actions on manipulatives with the notation system used to describe these actions. The computer links these

> two actions, and students are then able to associate the concrete and symbolic easier. (p. 3)

Clements and Battista (1989) proposed research supporting their belief that after students draw shapes using Logo, their ideas regarding shapes are more exact and mathematical. In their study, when students drew rectangles on paper, they did just that, drew a rectangle. When drawing the rectangles by hand, children often did not connect them to math. When using Logo on the computer, students had to enter a series of commands and procedures to draw the rectangle. Part of deciding how to create their rectangles involved analyzing the shape of the rectangle in order to construct the necessary commands. Because students construct these rectangles themselves, they become more aware of the shape as having opposite sides with equal lengths. If the wrong numbers are entered, feedback regarding the connection between the numbers and symbol is immediate (Clements & Battista, 1989).

The fact that virtual manipulatives require computers may not be problematic in the home or with small classes, but when there are many students and few computers, this could be considered a disadvantage (Rhodes, 2008). Similarly, while using computers helps to overcome the problem of storing so many concrete manipulatives, it also creates the problem of ensuring the security of the computers. Another potential problem lies in giving students easy access to the Internet, which, without constant supervision, may result in some students straying to sites that are distracting, inappropriate for school, and even dangerous. Rhodes (2008) also commented on some additional potential issues concerning virtual manipulatives, stating,

> The activities one may do with virtual manipulatives are limited to the confines of the program and may stifle creativity. For example, you might have trouble creating three- dimensional figures using tangrams, or finding a specific virtual manipulative for your favorite manipulative activity. Also, by having a list of virtual manipulatives at your fingertips, teachers may become content with the activities on a given site, without spending the time to think up new, creative uses for manipulatives. (p. 1)

While using virtual versus concrete manipulatives can present some problems, there are ways to overcome those problems. The teacher needs to be versed on students' limits that the virtual manipulatives may cause them to reach and push students to work out of their comfort zones.

Suh and Moyer's (2005) research on three fifth-grade mathematics classrooms separated by achievement level further indicated that,

> One characteristic afforded by the virtual manipulative concept tutorials used in this project was their design that combined both visual and symbolic images in a linked format. This may have encouraged students to make connections between these modes of representation and, thereby, developed students' representational fluency, particularly for visual learners. (p. 9)

They also asserted that the class identified as having the lowest achieving students benefited the most from working with virtual concept tutorials (Suh & Moyer, 2005). Suh and Moyer (2005) affirmed through interviews that students believed that the visual representation afforded by the virtual manipulative made it easier to understand the connections between the fractions and the applets with which they were working. They were able to connect symbolically through scaffolding and thus build upon previous lessons. The immediate feedback provided to the students also motivated them to work through the challenging fraction lessons. Students liked how the feedback prompted them to find various solutions to their problems. They collaborated with one another, which led them to justify their solutions and to explain why a solution was mathematically accurate (Suh & Moyer, 2005).

Virtual learning creates a situation in which the students are prevented from continually making the same or a similar error because they receive immediate feedback on their errors, which can be overlooked in a traditional setting. This immediate feedback leads to the students working with more precision and exactness. In addition to the learning advantages, the use of digital media is more practical in that they are more cost effective, require less space for use, and are less time consuming than concrete manipulatives. While most virtual manipulatives require access to the Internet, some can be downloaded and used offline, still allowing students to be exposed to and participate in the learning process (Crawford & Brown, 2003; Suh & Moyer, 2005). Virtual manipulatives are more than just electronic replications of their physical counterparts. According to Crawford and Brown (2003), they lift the text off the pages by creating visual images of the concept, raising student confidence, and making learning more enjoyable. Computer programs and digital media augment children’s learning experiences by providing visual connections to concepts and facilitating their cognitive development while encouraging them to investigate ideas beyond their current level of understanding. Virtual manipulatives are improving and changing constantly, offering the learner an interactive environment that is self-paced and that provides immediate feedback for self-correction or teacher intervention. This mirrors Suh's (2005) observation that computers used to improve and supplement instruction have proven to be successful teaching tools when informed teachers who utilize appropriate methods and materials employ them.

According to the Illinois State Board of Education (ISBE) (2001),

> Technology provides a means to carry out operations with speed and accuracy; to display, store and retrieve information and results; and to explore and extend knowledge. The technology of paper and pencil is appropriate in many mathematical situations. In many other situations, calculators or computers are required to find answers or create images. Specialized technology may be required to make measurements, determine results or create images. Students must be able to use the technology of calculators and computers including spreadsheets, dynamical geometry systems, computer algebra systems, and data analysis and graphing software to represent information, form conjectures, solve problems and communicate results. (p 1)

This mathematical comprehension, along with the computer skills necessary to complete the task, will prove invaluable in the workplace.

Student Attitudes Toward Math

Children enter preschool with a carefree attitude toward mathematics. Many times, children think of math as a game and find the challenge interesting. It is only as they advance through school that the struggles and consequent dislike for mathematics begin to emerge (Burns, 2006). When children were surveyed as to whether they liked or disliked mathematics, the most common reason for their dislike was that teachers moved too fast or taught in an abstract manner, thus not ensuring understanding of the presented concept. Once a lack of understanding and math helplessness becomes embedded, students tend to lose motivation to deal with the challenges within mathematics. Discovery learning is no longer fun, and the fuel that propelled their motivation disappears. If a strong foundation in mathematics is not embedded early, students will lack the necessary confidence to persevere when presented with more challenging problem-solving mathematics lessons (Kilpatrick, Swafford, & Findell, 2001). According to Burns (2006), it is necessary to build that strong foundation in math at an early age so that children will continue to possess a natural desire to enjoy learning through guided discovery.

According to Kilpatrick, Swafford, and Findell (2001),

> As students build strategic competence in solving non-routine problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes… similarly, when students see themselves as capable of learning mathematics and using it to solve problems, they become able to develop further their procedural fluency or their adaptive reasoning abilities. (p. 131)

When teachers offer students various methods with which to solve mathematics problems, they are encouraging students to utilize their varied learning styles, which fosters a learning community that encourages and promotes individualized learning. Teachers can specifically note progress in students' problem-solving abilities and also observe their deeper understanding of mathematical concepts. Teachers who encourage students to use diverse approaches to solving problems further develop confidence in their students' abilities to succeed (Burns, 2006; Kilpatrick, Swafford, & Findell, 2001).

From a meta-analysis of 79 studies that investigated the use of non-graphing calculators, Hembree and Dessart (1996) contended that student learning increased when they used hand-held calculators. The researchers specifically noted progress in students' problem-solving abilities and observed a deeper understanding of mathematical concepts. "Their analysis also showed that students using calculators tended to have better attitudes towards mathematics and much better self-concepts in mathematics than their counterparts who did not use calculators" (Hembree & Dessart, 1996, p.86). The researchers also found that students continued to show adeptness at performing computations with paper and pencil during mathematics instruction. They believed that this was because the students had garnered a deeper understanding of the concept and were able to apply it to their real-life experiences (Hembree & Dessart, 1996).

Toward manipulatives. Manipulatives permit students to construct their own knowledge, which in turn encourages deeper understanding of a mathematical concept. Students become more involved in the lesson and are able to form their own solutions. When students feel this connected to the lesson, they develop a deep understanding of its content (Goracke, 2009, Wiggins, 1990). Steen, Brooks, and Lyons (2006) maintained when students form ownership for their learning through the use of manipulatives, the fear is removed from learning mathematical concepts, and they are intrinsically rewarded for their efforts. They then can build on their positive experiences by engaging in work with more thought-provoking concepts. Then, students are empowered to take these learned concepts and apply them successfully in their daily lives. According to Steen, Brooks, and Lyons (2006), when children have an opportunity to visualize a mathematical concept, there is less confusion, thus allowing deeper student understanding to occur. When there is less confusion, students feel more confident in their mathematical abilities, and valuable groundwork is laid for future mathematical endeavors. Children also are able to sort through important mathematical concepts and store those that hold deeper meaning instead of trying to memorize unnecessary items (Steen et al., 2006).

Toward concrete manipulatives. Goracke (2009) concluded from an action research study of her eighth-grade mathematics classroom that the use of manipulatives had a positive impact on students' attitudes and their overall understanding of mathematical concepts. She found that the students took pleasure in engaging with the manipulatives not necessarily due to the academic benefit but rather from the hands-on, active participation. Her students did, however, show significant gains when given assessments that involved the use of the manipulatives. Goracke (2009) asserted that students also displayed a more optimistic attitude towards mathematics. They were more confident and sought multiple solutions to challenging problems. They also felt comfortable enough to share their assumptions and discuss alternative outcomes with their classmates (Goracke, 2009).

Clements and McMillen (1996) observed one boy who wrote a procedure for drawing a rectangle:

> He created a different variable for the length of each of the four sides. He gradually saw that he needed only two variables because the lengths of the opposite sides are equal. In this way, he recognized that the variables could represent *values* rather than specific sides of the rectangle. No teacher intervened;

> Logo supplied the scaffolding by requiring a symbolic representation and by allowing the boy to link the symbols to the figure. (p. 274)

In this way, the boy was able to build on prior knowledge stored through his interactions with Logo.

Toward technology. Brown (2007) and Steen, Brooks, and Lyons (2006) concluded from their research that as technology continues to grow as a force in the daily lives of our students, there is a developing need to provide varied instruction in order to gain and maintain students' attention while they are engaged in the process of learning. This presents challenges for educators. One strategy for educators lies with using a variety of instructional techniques, including manipulatives, to increase students' on-task behavior, encouraging higher-level classroom thinking, and addressing differences in learning styles. The use of technology to provide students with learning experiences that incorporate virtual manipulatives can assist in developing a learning continuum that leads students from a phase of simple awareness to actual utilization of a skill. Technology also increases positive student attitudes toward mathematics through the use of manipulatives in the classroom.

According to researchers, (Suh & Moyer, 2005; Allen, 2007) students perceive the computer as a tool for obtaining an abundance of information. They concluded from their research that the computer offers unlimited access to information and allows students, with guidance from their teachers, to guide themselves through lessons and ultimately solve problems on their own (Suh & Moyer, 2005; Allen, 2007). They also receive immediate feedback on their ideas. In their study of fifth-graders using a fraction applet, Suh and Moyer (2005) found that "the applets allowed students to experiment and test hypotheses in a safe environment. The guided format features of the applets allowed guessing and trial-and-error, and at the same time, would not accept an incorrect response" (p. 10). Student interviews and attitude surveys indicated that the applet's refusal of incorrect responses caused them to problem solve in ways they would not have considered otherwise. They also enjoyed the collaboration that the virtual lessons encouraged. Students also worked faster than when using traditional paper-and-pencil methods. They believed that through collaborative efforts with their peers, they were able to retain the information and retrieve it later. Some students felt that the lesson seemed more realistic to them. As a majority of the students were frequent computer users, this experience was more authentic to them. The students also noted that the fraction applet allowed greater manipulation of fractions than their concrete counterpart (Suh & Moyer, 2005). Students indicated that the web-based lessons were closely linked to their everyday needs. They stated that when collaborating with their peers and teachers, they achieved a feeling of ownership over the lesson and were able to both retain the information and apply it when needed to solve a problem (Suh, 2005; Allen, 2007).

Other researchers have cited additional benefits of incorporating technology into teaching. Young (2006) noted that "Another pedagogical benefit of virtual manipulatives is that they have the ability to provide multiple representations of a single concept at the same time" (p.1). When students are problem solving with virtual manipulatives, they are using the computer graphics, words that appear on the computer screen, and numbers. Students indicated in the survey that all of these representations, when simultaneously presented, added to their deeper understanding of the proposed concept (Reimer & Moyer, 2005). Reimer and Moyer (2005) argued that this benefit of virtual representations provides an advantage over physical manipulatives: “Unlike physical manipulatives, electronic tools connect the iconic with the symbolic mode” (p. 7). They also asserted that transfer, or application, of knowledge that was once limited and specific could be increased through virtual representations to a more general knowledge base (Reimer & Moyer, 2005).

Ainsa (1999) introduced one group of preschool/kindergarten-aged students to the concept of subtraction using M&Ms as concrete manipulatives and another group to a software program that was designed to allow them to explore the concept. Correct responses were recorded through observation as the students worked through the activities. The study's findings revealed that there were no significant differences in success rates between students who participated in concrete activities and those who participated in computer-based activities. However, the students using the computer-based activities indicated that the experience was fun and enjoyable learning. In another study conducted by Allen (2007), a group of students participated in a program entitled Everyday Math, which incorporates manipulatives, games, cooperative learning, and other tools with pre and post-tests administered with each unit. Allen found that the manipulatives contributed to students’ significant improvement in skill development, higher levels of understanding, and positive attitudes towards learning mathematical concepts. Throughout the lesson, the students demonstrated an interest in the lesson and enjoyment of learning while engaged with the manipulatives. Allen (2007) asserted, “The students were visibly more active in class and developed more self-confidence in their math skills” (p. 14).

Technology is at its best when it is paired with the mathematics curriculum and aligned with assessment. When schools take the time to incorporate technology into their lesson the learning becomes seamless. Students are able to succeed in their virtual learning and move back and forth with greater ease. In a review of studies, the CEO Forum concluded, "Technology can have the greatest impact when integrated into the curriculum to achieve clear, measurable educational objectives" (p. 1). Educators need to align mathematics lessons so that the virtual learning coincide with the learning objective. Therefore, students will gain a deeper understanding of what is expected of them and when assessment is given there is no question of what is asked of them.

Toward virtual manipulatives. Virtual manipulatives enable students to think in a more fluid manner, one that is closely structured with cognitive processes. According to Suh and Moyer (2005), students learn and retain more when they, while engaged in learning experiences, receive immediate feedback and are encouraged to use strategies that, if wrong, will elicit that immediate corrective feedback. When students are encouraged to collaborate with their peers to test their assumptions and to manipulate a virtual representation, they retain more of what they learn because they build ownership of the solutions generated. Students indicated through interviews that they felt more confident with their findings when they were able to visualize the symbolic representation and turn it in to their own solution. When collaborating with peers, they felt secure in their efforts to explain their procedures in mathematical terms (Suh & Moyer, 2005; Young, 2006, Reimer & Moyer 2005).

According to Steen, Brooks, and Lyon (2006), a group of students exposed to virtual manipulatives as part of their daily instruction showed an increase in motivation and challenged themselves to higher levels. The third graders in this study showed significant gains in conceptual knowledge and expressed a greater satisfaction with the tools' specific feedback and ease of use. The students also expressed a greater enjoyment of learning throughout the unit on fractions.

Another advantage cited by Clements and McMillen (1996) was that students enjoyed the fact that they were able to spend a considerable amount of time on the problem and actually focus on understanding the learning objective. The students also felt that by understanding what they were learning, they required less practice on the concept. Many students were able to retain the information and did not forget the process involved when computing the problems on assessments. Students did not have to force themselves to remember the concept; instead, they developed a true understanding of the concept and were able to apply it to many different challenges (Clements & McMillen, 1996).

Toward the teacher. Student attitudes towards their teachers improved greatly when manipulatives were incorporated in the mathematics classrooms. In a meta-analysis of 18 classrooms, Ellington (2003) found that students using graphing calculators became much more interested in and had much better attitudes toward the subject itself. Observations and surveys revealed that students were more motivated and eager to work with their teacher on challenging problems. Ellington (2003) asserted, however, that presenting a calculator to students does not automatically ensure that they will learn to solve problems. The teachers who Ellington observed demonstrated excellent strategies and knew when to take advantage of their students' positive attitudes, thus challenging them with additional problems. The observed students demonstrated greater collaboration and dynamic learning with peers and teachers when presented with these higher-level problems. Ellington (2003) summarized some of the benefits of calculators, stating,

> In giving students a graphing calculator, teachers can also give students more responsibility for their own learning. Students can examine multiple representations interactively and examine meanings of representations and their relationships. They can work on interactive explorations, real-world data collection, and investigations. Furthermore, they can assess their work and discover errors on their own. (p. 448)

Students use graphing calculators to collect data and in turn realize that there is more than one way to represent the problem and form relationships with their findings. Moreover, research conducted over the past decade showed that when students displayed a positive attitude toward the lesson and their teachers, they had a tendency to excel in math and math-related careers (Ellington, 2003; Trusty, 2002). Trusty (2002) asserted the importance of middle school teachers providing a positive mathematics environment during their students' mathematics lessons. He believed that when students experience confidence, they display determination, which urges them to challenge themselves to work through problems that are more difficult at a time when interest may wane.

When students use technology in the mathematics classroom, they are further inspired to tackle difficult mathematical challenges. Using problem-based learning in mathematics, the lesson begins with a challenge that requires deep problem-solving skills. The teacher should facilitate learning by supporting, guiding, and monitoring the learning process. The teacher must build students' confidence as they work through the challenge. Problem-based learning transitions from the traditional learning style of working through problems in isolated steps given in a lecture to a style in which the teacher guides the students as they actively learn. Students are presented with a challenge for which they have no former procedures available for recall. This, in turn, causes students to question, form hypotheses, and then test those hypotheses and communicate with peers and teachers as to the accuracy of their answers. Teachers who enable students to gain autonomy also are fostering a life-long interest in mathematics. When teachers encourage students to use graphing calculators, for example, they are stepping back and letting the students take ownership of the lesson**.**

Teacher Attitude

Toward technology. In order for students to have the opportunity to use technology in the classroom, their teachers must be aware of, know how to use, and embrace that same technology. Kilpatrick and Swafford (2002) stated that, “As with any instructional tool, calculators and computers can be used effectively or not so effectively. Teachers need to learn how to use these tools—and teach students to use them—in ways that support and integrate the strands of proficiency” (p. 13). According to some researchers, the manner in which teachers view technology makes a significant difference in the way it is integrated into their classrooms (Crawford & Brown, 2003; Duffin, 2010). Teachers who perceive technology as a stepping-stone to collaboration and higher-level thinking are more apt to incorporate technology in their curriculum. The researchers made these statements based on observations and surveys conducted with classroom teachers. They verified that when students were more engaged in the lesson, more interaction occurred between them, their peers, and their teachers. The use of web-based lessons encourages students to share information in new and innovative ways. The researcher (Duffin, 2010) observed students challenging each other as they moved towards attaining the lesson objectives. Teachers were free to move away from the center of the classroom stage and observe their students as they interacted and challenged one another, often guiding each other to demonstrate higher-order thinking skills. Duffin (2010) and Crawford and Brown (2003) found that teachers believe that the use of technology in mathematics lessons encourages students to expend more effort to succeed because they receive ongoing feedback. Student efficacy, a belief that their efforts are aiding their understanding, is enhanced through the use of technology as a teaching tool in the classroom.

Teachers stated that incorporating virtual lessons into their classrooms did not pose a problem. Many indicated that online lessons were readily available, of high quality, and free. Teachers also agreed that the use of virtual manipulatives offers a vast array of educational opportunities for students and introduces an abundance of learning tools. When concrete manipulatives were used along with worksheets, teachers frequently found that supplies of these materials were limited. However, this was not the case with virtual manipulatives because more materials could be produced with just the click of a button on the computer (Rhodes, 2008; Moyer, Bolyard & Spikell, 2002). Other teachers asserted that if they had an interactive whiteboard, they would not have to print copies and would never run short of materials. Another advantage of virtual manipulatives is that they provide teachers with a variety of instructional strategies to ensure student learning (Moyer, Bolyard & Spikell, 2002).

In another study, Crawford and Brown (2003) contended that teachers who felt comfortable using computers in their personal lives would also view them as important and useful in the classroom. Other teachers showed concern about using technology in the classroom, stating that they felt that students might become too dependent on the web-based lessons and fail to learn basic mathematics concepts (Duffin, 2010). Another concern was the lack of knowledge and technical support provided by the school districts. Teachers indicated that when their districts did provide technical support, there was often no follow-up. Teachers were sent to one-day seminars at which a plethora of information was presented. When the teachers left the professional development session and returned to their classrooms, they remembered very little of what had been presented. Many maintained that if they had been able to meet with colleagues and collaborate when working on the materials presented in the workshops, the results would have been more beneficial. Teachers also indicated that professional development, if focused on specific skills and followed by time for teachers to become accustomed to technology strategies, as well as the support to practice, review, and revise these strategies, would have alleviated some of their concerns regarding the use of technology and even encouraged its use (Crawford & Brown, 2003). Teachers indicated that they did have computers available for student use but were not comfortable incorporating them into their curriculum. Some teachers cited an insufficient number of available computers for student use, which resulted in considerable off-task behavior that interfered with the concentration of those students working on their web-based lessons (Puchner et al., 2008).

According to Roschelle, Pea, Hoadley, Gordin, and Means (2000), many teachers who participated in the survey reported an enhancement of children's learning “because many of the best uses of technology supported four fundamental foundations of learning as defined by cognitive science: active engagement, participation in groups, frequent interaction and feedback, and connections to real-world contexts" (p. 79). Teachers verified that technology expanded the amount of learning children experienced in the classroom. They observed that through the use of technology, children were able to visualize, model, and simulate situations not readily available in the real world. The technological representations enabled students to "see dynamic graphical representations of concepts linked to algebraic and other symbolic notation" (Roschelle et al., 2000, p. 88). Results from the survey verified that the amount of technology used by teachers significantly affected student achievement in mathematics. In a study involving six schools, researchers' assessments showed that "students whose teachers relied heavily on technology scored significantly better than students whose teachers rarely used technology" (Roschelle et al., 2000, p. 91). The study found significant achievement in classrooms in which teachers who were high-level users of technology, according to the degree of computer usage, displayed a positive attitude toward the value of technology for teaching (Roschelle et al., 2000).

Toward the learning objective. Kilpatrick and Swafford (2002) found that when teachers were knowledgeable in mathematics and comfortable with teaching to the learning objective, they possessed a confident attitude that they were able to pass on to their students. Students in these classes were more apt to achieve success and show increased motivation toward mathematics. Students also developed positive attitudes towards learning the subject if their teachers used creative teaching strategies (Kilpatrick & Swafford, 2002). Given these findings, Kilpatrick and Swafford (2002) concluded that teachers who used varied and diverse strategies in teaching mathematics enhanced their students' success in the subject.

According to Burns (2006), teachers should not be satisfied with just writing lesson objectives in their plan books; they also should search for strategies to allow them to diversify instruction for each student. They should present the lesson objectives clearly to students so that those students will understand exactly what they will be expected to demonstrate at the culmination of the lesson. Kilpatrick and Swafford (2002) asserted that if students do not know and understand the learning objective, then the subject will not grab and hold their attention, and their opportunities to achieve the overall goal will be limited severely. If students are presented with overarching questions that refer to the enduring understandings expected, then they can refer to these for direction as the lesson progresses. Teachers who present clear lesson objectives in the form of overarching questions to students can also provide students with ongoing assessment of their progress towards the learning objectives, thus contributing to student motivation, positive attitudes, and attainment of the learning goal(s) (Kilpatrick & Swafford, 2002; Wiggins, 1990).

Toward manipulatives. Teachers voice concern about the amount of time they will have to allocate to the planning of manipulative lessons without a guarantee that students will understand the underlying concept. The "mere presence of manipulatives does not assure that a connection will be made; they are not magic" (Moyer, 2001, p. 176). Lappan and Ferrini-Mundy (1993) stated, "The mathematics must be embedded in the task, and the task must be mathematically connected to students' prior learning and to what the teacher wants them to learn" (p. 627). If teachers cannot make students understand the true meaning of the mathematics lesson so that they see it as authentic or meaningful to their lives, the students then will view the lesson as a game, and no enduring understanding of the concept will be realized (Lappan et al., 1993; Goracke, 2009; Wiggins, 1990).

Durmus and Karakirik (2006) noted that,

> Usage of manipulatives not only increase students' conceptual understanding and problem solving skills but also promotes their positive attitudes towards mathematics since they supposedly provide "concrete experiences" that focus attention and increase motivation. A concrete experience in mathematics context is defined not by its physical or real-world characteristics but rather by how many meaningful connections it could make with other mathematical ideas and situations. (p. 1)

According to Clements and McMillen (1996), students who are visual learners and have difficulty sitting through a teacher's lecture tend to behave appropriately, absorb the necessary information, and demonstrate understanding when given the opportunity to use manipulatives. Moyer and Jones' (2004) research reinforced the use of manipulatives as a positive force in student learning. When interviewed, students stated that instead of trying to construct in their minds the image the teacher was trying to convey, they had a much easier time seeing it visually, whether it was on the computer screen or through the use of concrete manipulatives.

Implementing digital technology in the classroom requires that the teacher effectively plan for each lesson by choosing the correct manipulatives, games, or software, which requires research and time. Because of the necessary effort, many teachers may not feel confident using current technology without appropriate and thorough training. However, Crawford and Brown (2003) found that when training and associated materials were selected carefully, teachers reported high levels of satisfaction with the use of technology in their classrooms. In a study conducted by Steen, Brooks, and Lyon (2006), teacher participants found that using computers in the classroom allowed them to quickly assess the students' understanding by continuously monitoring the screens as they moved about the room assisting students when and where necessary. Students also were able to move through a lesson effectively because there was no waiting time for materials or feedback. The computer provided each student the same level and quality of instruction, with the teacher assuming the role of facilitator. Teachers expressed their satisfaction resulting from a decrease in the amount of preparation time they needed because they primarily were distributing and collecting manipulatives. As such, both the teachers and the students spent more time on the task, which allowed students to engage in more practice problems during the class period. Students also could explore important concepts in depth while working at their own pace of learning and understanding.

Another concern was that manipulatives require more time to set up than traditional lessons, as well as more planning time. Some teachers expressed that it takes more time to explain how the manipulatives work and the importance of each piece to the overall concept of the lesson: "In addition to time there are the students who are not abstract learners, so I go back and teach the concept with numbers and variables" (Goracke, 2009, p. 1).

Additional research found that some teachers use manipulatives in the classroom strictly as rewards for students who display appropriate classroom behavior, which the researchers interpreted as not using the computer programs as a specifically organized sequence of instructional activities with a purpose. Moyer (2004) stated, "Teachers who view manipulatives as time wasting or as instructional materials secondary to the serious work of learning mathematics will inadvertently encourage their students to use these materials for play rather than for mathematics learning or understanding” (p. 29). Moyer (2004) conducted a year-long study involving two middle school teachers who were using manipulatives in their mathematics instruction. The researchers used interviews, observations, and self-reporting to investigate how these teachers used manipulatives in a classroom setting. Their findings showed that many of the teachers used manipulatives more to entertain students than to advance the students' mathematical knowledge. While some teachers used manipulatives to assist students in understanding a new concept, others used them for diversion or fun when they thought the classroom needed a change of pace. Still others expressed that they did not understand how manipulatives could replace traditional teaching using paper and pencils when teaching to the curriculum mandated by state standards.

Challenges of Virtual Manipulatives

Teachers and students can enjoy free, easy accessibility to virtual manipulatives via the Internet, with the disadvantage being that not all schools can afford well-equipped computer labs with consistent Internet connections. Rhodes (2008); Herrington, Oliver, Reeves, and Woo (2002); and Durmus and Karakirk (2006) advanced research indicating that teachers also face a number of challenges when incorporating technology in the classroom, particularly in the amount of time needed to teach one lesson. Many teachers consider their skills limited and fear that teaching lessons with virtual manipulatives will not help students develop either proficiency in the use of technology or an understanding of mathematical concepts. Others do not know how to integrate computer-generated lessons into their existing curriculum (Rhodes, 2008; Duffin, 2010). These researchers found that other challenges with virtual manipulatives occur when students and teachers have limited knowledge of the workings of the Web and lack experience with maneuvering through search engines to locate needed resources. Another formidable challenge was with the Internet server that hosted the program being susceptible to overload and thus crashing, causing participants to lose their work towards particular learning objectives. Internet connections can fail and school buildings can lose their connection to the World Wide Web, resulting in students losing their motivation to continue towards achieving the learning objective. Internet connections also may become congested with many users that they will either operate slowly or cease to function. This can result in student frustration and loss of interest in achieving the learning objective. Technology is not flawless and can lack dependability, thus reducing the quality of the learning experience (Rhodes, 2008; Herington, Oliver, & Reeves, 2003; Dumas & Karakirk, 2006).

Another concern of teachers, according to researchers, (Moyer, 2001; Rhodes, 2008; Duffin, 2010) is that only a few mathematical problems may be completed in one class period, which reveals their assumption that more is better when teaching material. The results from surveys administered to teachers suggested that if children in their classes did not complete at least 20 problems plus assigned worksheets, then they were not experiencing the necessary activities associated with learning the lesson objective. Many of the teachers surveyed tended to judge student learning strictly by the volume of material covered and thus were concerned with completing all details of the assigned curriculum (Moyer, 2001; Rhodes, 2008; Duffin, 2010).

Teachers also noted that when students become frustrated with the virtual lesson, they tend to venture out on the Web to sites that are not pertinent (Rhodes, 2008). They believed that students need teachers to monitor their on-task behavior and to provide the necessary assessment feedback to work through challenges. Teachers also expressed concern with the issue of students' possible dependence on virtual manipulatives. Meyers (2001) and Rhodes (2008) found that teachers believe strongly that students need to memorize their multiplication facts, and if they depend solely on a computer or calculator to solve mathematics problems, they might not have the foundation to problem solve when placed in a realistic setting without these tools at their fingertips.

School districts frequently deal with financial problems when attempting to reduce deficiencies in the area of technology to support the operations of computer labs. Many districts do not have the funds to provide maintenance services to computer labs when the need arises. Others do not have the resources to establish and maintain the necessary connections to the Internet or to upgrade their systems in a timely manner.

Summary

This review of the literature sought to address how virtual manipulatives are complementary to concrete manipulatives when teaching mathematics, as well as how students perceive the effectiveness of their learning/understanding when taught mathematics with both concrete and virtual manipulatives. The literature review also addressed how teachers directly involved in using both concrete and virtual manipulatives when teaching mathematics can determine their effectiveness in improving the academic performance of students.

Chapter 3 contains details of the methodology used by the researcher. This study utilized a quasi-experimental methodology to determine if adding virtual manipulatives to existing concrete manipulatives in the seventh-grade mathematics curriculum would increase students' mathematics composite scores on standardized and teacher-created assessments. The chapter includes details of the selection of the 44 participants and their involvement, the instruments used, the setting of the study, and the quantitative and qualitative methods employed.

Chapter Three: Methodology

Research Overview

Chapter 3 contains an explanation of the methods used by the researcher to gather and analyze data during the study, as well as a description of the participants, controls, methods, and procedures employed in the study. The issues of bias and internal validity are considered in this chapter, as well as a justification of the methodology used. The researcher's primary interest was whether students who used virtual manipulatives coupled with concrete manipulatives in the mathematics classroom outperformed students who used only concrete manipulatives.

Rationale

The researcher wanted to determine the importance of using technology in the mathematics classroom but could not find ample quantitative or qualitative evidence in the literature to support a measurable difference in student achievement when incorporating technology. This study compared the mathematics assessment scores of two groups of seventh-grade students; one group used virtual manipulatives paired with hands-on (concrete) manipulatives, while the other used only hands-on manipulatives. The researcher's primary interest was to gain insight into whether students who used a combination of virtual and concrete manipulatives would outperform students who used only concrete manipulatives.

Research Hypotheses

Null hypothesis (Ho) - Students taught mathematics with virtual manipulatives in addition to concrete manipulatives in a seventh-grade mathematics curriculum will not demonstrate a measureable change in mathematics composite scores on standardized and teacher-made assessments compared to students taught mathematics with only concrete manipulatives.

Hypothesis (H_1) Students taught mathematics with virtual manipulatives in addition to concrete manipulatives in a seventh-grade mathematics curriculum will demonstrate a measureable change in mathematics scores on standardized and teacher-made assessments compared to students taught mathematics with only concrete manipulatives.

The researcher will answer the following question(s):

1. How do students perceive the effectiveness of their learning/understanding when taught mathematics with both concrete and virtual manipulatives?
2. How does the teacher who has experience using both concrete and virtual manipulatives to teach mathematics perceive her effectiveness when using only concrete manipulatives?
3. How does the combination of virtual and concrete manipulatives affect the academic performance of students in the area of mathematics as opposed to the use of only concrete manipulatives?

The Nature of Manipulatives

Manipulatives help students to develop the skills necessary to solve specific problems in mathematics. Heddens (1997) believed that students are introduced to manipulatives in mathematics without a consistent set of instructional strategies and questioned if employing manipulatives in mathematics instruction actually leads to increases in student achievement. Heddens maintained that employing manipulatives in mathematics helps students to develop a greater understanding of the skills necessary to solve problems successfully. Researchers have not ignored the potential negative effects of using manipulatives to teach mathematics, but the positive effects on student learning feature more prominently in the literature (Reimer & Moyer, 2005; Ainsa, 1999; Uttal, Scudder & DeLoache, 1997). The intent of the researcher conducting the study presented here was to provide an explanation of the relationship between student achievement in mathematics and the use of manipulatives to teach mathematics in the elementary school classroom. This mixed methods study proposed to determine if the joint use of concrete and virtual manipulatives in teaching mathematics can result in a measurable change in achievement among elementary school students.

Picciotto (1993) maintained that manipulatives are extraordinary tools that can help all students, but particularly low-achieving students. Jones (1986) found that when students were able to visualize a mathematical concept in action, they developed a deeper level of comprehension, which contributed to increased motivation to continue learning among both high- and low-achieving students.

This study utilized a mixed methods design. Johnson and Onwuegbuzie (2004) defined mixed methods research as “the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods and approaches, concepts or language into a single study” (p. 17). Fraenkel and Wallen (2006) concluded that mixed methods studies provide a more extensive perception of the research. This type of research enables the researcher to provide both qualitative and quantitative data, thereby, expanding the understanding of what was studied. This mixed methods research, referred to as a triangulation design, was employed throughout the study. According to Fraenkel and Wallen (2006), “Triangulation design is when the researcher simultaneously collects both quantitative and qualitative data” (p. 443). The purpose of this study was to determine if adding virtual manipulatives to existing concrete manipulatives in a seventh-grade mathematics curriculum resulted in a measureable change in the students' mathematics composite scores on standardized and teacher-created assessments. The researcher also compared the two research groups' written reflections on the impact of concrete and virtual manipulatives on their own learning. She believes that using a mixed methods research design will clarify the importance of virtual manipulatives during mathematics instruction and produce more dependable findings (Frankel & Wallen, 2006; Johnson & Onwuegbuzie, 2003).

The researcher believes that technology, in the form of virtual mathematics manipulatives, in conjunction with concrete manipulatives already utilized, is essential for enhancing mathematics instruction by ensuring student understanding of mathematics concepts in a seventh-grade mathematics class. The virtual manipulatives overcame some of the limitations of concrete manipulatives, such as limited materials and storage space.

This study compared the effectiveness of using concrete manipulatives alone versus in conjunction with virtual manipulatives while teaching mathematics to seventh-grade students. When students had the opportunity to visualize a mathematical concept in action, a deeper level of understanding was observed. In addition, it was expected that better retention would allow teachers the opportunity to decrease the amount of review material incorporated into lessons at the beginning of the year, thus allowing substantial new growth. If students could retain more information, teachers could move forward at a faster pace and cover new material.

Manipulatives are concrete or virtual objects that can be used to represent abstract mathematical ideas. Moyer (2001) found that manipulatives have visual and tactile appeal to students and can be manipulated easily through hands-on experiences. Manipulatives serve as tools for teachers to give meaning to abstract mathematical ideas.

Participants

The school district in which this study was conducted is located in the Midwest. The district is organized into seven buildings, which include kindergarten through eighth grade, one early childhood center, a freshman academy, a sophomore academy, one building for eleventh- and twelfth-grade students, and another for seventh through twelfth grade. Ninety percent of the district's schools have been identified as not making adequate yearly progress as mandated by the state and currently are receiving federal money to improve the academic achievement of their students. During the 2010-2011 school year, the school district's population reached 4,237.

This is an urban school district in which 90.2% of students are classified economically as living at or below the federally designated poverty level. In 2010, all schools qualified for federal Title I remedial instruction services, with one school designated for full federal assistance. Families within the school attendance area are classified as economically disadvantaged and have limited means of support; therefore, 100% of the school's population receives free breakfast and lunch during the school week. The ethnic background of the district's school population at the time of this study is detailed in Table 1.

In the school year during which the study was conducted, 2010-2011, 20% of the students were receiving special education services. There was a 20 to 1 student-to-teacher ratio at the elementary school level, and a 17 to 1 student-to-teacher ratio at the secondary level. There was one administrator for every 164 students.

Table 1

Demographic Information

	Caucasian	African American	Hispanic	Asian	Native Hawaiian/Pacific Islander	American Indian	2 or more races
School	5.7	93.1	0.0	0.0	0.0	0.0	1.2
District	10.0	88.4	0.9	0.0	0.0	0.0	0.6
State	51.4	18.3	23.0	4.1	0.1	0.3	2.8

Information taken from study district website

Table 2

School, District, and State Comparisons

	Percent low income*	Percent Limited English Proficient	Percent IEP	Chronic Truancy Rate	Mobility Rate	Attendance Rate	Total Enrollment
School	95.8	0	21.0	1.8	19.9	94.6	405
District	90.2	0.1	20.0	7.0	23.2	90.3	4237
State	48.1	8.8	14.0	3.2	12.8	94.0	2,074,806

*Low income refers to students who come from families receiving public aid

Information taken from study district website

Additional statistics pertaining to the community and the schools are provided in Table 2. The site for this research study was one of the district's seven kindergarten through eighth grade schools, which was classified as requiring federal improvement funding. The participants in this study included 44 seventh-grade students in two classes at the same elementary school. Demographically, the building's seventh-grade population consisted of 100% African American students divided into two classes, which were formed by random assignment of students during the registration process.

Experimental vs. Control Group

Students in the experimental group, which combined virtual manipulatives with concrete manipulatives, were scheduled for instruction in the computer laboratory three times per week. The teacher utilized eight websites when directing students with virtual manipulatives (Appendix A). These websites contained both dynamic and static manipulatives. *Math Playground* enabled students to explore mathematics concepts in a user-friendly manner. Students enjoyed this manipulative and were eager to explore the lesson objective. *Fraction Maker* was enlisted so students could learn fractions; depending on their skill level, they would start with an identifying fractions program and then continue on to programs that taught them how to rename , compare, add, subtract, and divide fractions. Students also experimented with different patterns by increasing and decreasing the size of fractions. *Waldo's Math* applet allowed students to try different procedures to learn how to solve simple equations. Each time the student clicked on "New Problem," a new equation appeared for them to solve. There were five levels of difficulty, and students chose which one they wanted to try. *Tangrams*, based on the ancient Chinese tangram blocks, is a virtual manipulative available through the National Library of Virtual Manipulatives website. The blocks can be dragged, rotated, and flipped to enable easier copying of models. *Pan Balance – Numbers* is one of a series of virtual manipulatives available through the Illuminations website that assists students in investigating the concept of equivalence. *Arcytech Educational Java Programs*, designed by Jacob Bulaevsky, includes interactive tools for several manipulatives used in the middle school grades; those used in this study included base 10 blocks and fraction bars. *Matti Math* helps students understand mathematics using visual objects that display mathematical relationships and applications. *Interactivate Manipulatives* allows students to visually examine, explore, and develop concepts. Students sorted colored shapes into bar graphs using this manipulative that helps students to practice sorting by shape or by color. *Study Island* engaged students in web-based instruction, practice, and assessment. This site covered all of the required content areas for mathematics instruction in alignment with the state standards.

Table 3

Demographics of Control and Experimental Groups

Group	**# of Girls**	**# of Boys**	**Race by %**	**# of Special Ed. Students**
Experimental	12	10	100% African American	0
Control	11	11	100% African American	0

Students in the experimental group were assigned a computer in the school's computer laboratory, which housed 30 computers. There were five rows with six computers in each row. The setup of the computer laboratory afforded ease of conversation between the students and enabled the teacher/researcher to easily maneuver around the room to watch and listen to student interactions.

There was no recruitment of participants. Mathematics is a core subject that all students are required to take. The experimental conditions were based on the ability of the mathematics teacher to bring only one of her classes to a computer lab to use virtual manipulatives.

Procedures

The same teacher taught mathematics to students in both the experimental and control groups. She employed only concrete manipulatives to teach mathematics in the control group, but used both concrete and virtual manipulatives to teach mathematics in the experimental group. An example of manipulatives that were used both in a virtual form and a concrete form were the two-sided chips. Red on one side and yellow on the other, these chips were used when introducing students to the concept of adding integers. The students in the control group used the concrete manipulative (two-sided chips) to explore ways to solve addition problems. The students and teacher explored with the chips to establish the rules for adding integers. The experimental group used their computers and the interactive whiteboard in their classroom. The teacher modeled the lesson on the interactive whiteboard, and then the students accessed the website on their individual computers. They used the interactive integer website to manipulate the two-sided chips just as the control group had, but in a virtual manner. The experimental group experienced the same steps as the control group, beginning with an introduction to the lesson and then establishing the rules for adding integers.

Both groups of students were taught for the same amount of time in the morning. The mathematics teacher administered the Iowa Test of Basic Skills (ITBS) in the fall and again in the spring after the independent variable (both concrete and virtual math manipulatives) had been employed. The fall test enabled the researcher to calculate an average mathematics score for both the control and the experimental group before experimentation. It represented a starting point and served as a pre-test displaying the mathematical abilities of each student (See Table 4).

The ITBS was administered again in the spring as a posttest to both the control and experimental groups. The researcher used a *t*-test to compare differences in the means of the dependent samples in the average mathematics score of the control group from the fall ITBS to the spring ITBS administrations (Bluman, 2008). Bluman (2008) stated that a *t*-test is, “a statistical test for the mean population and is used when the population is normally or approximately normally distributed” (p. 415). The researcher then used a *t*-test to compare the differences in mean mathematics scores on the spring ITBS between students in the control group and students in the experimental group.

Table 4

Average ITBS Scores by Grade Equivalencies

	Pre-ITBS	Post-ITBS
Experimental	7.7	9.3
Control	6.0	6.6

The teacher did not alter any part of the existing curriculum in order to implement virtual manipulatives. Each lesson had a virtual component that was adapted so that it could be taught using concrete manipulatives or no manipulatives at all (only paper and pencil).

The researcher observed students in both the experimental and control groups as they were learning the same lessons in their respective class periods each morning. The researcher observed each group approximately two hours per week for seven months (see observation form in Appendix B). The researcher's observations were focused on the engagement of the students and teacher. The researcher watched closely to see if there was greater engagement in the experimental group or the control group, as well as to assess the motivation level demonstrated by the mathematics teacher.

The experimental group utilized the computer laboratory on Mondays, Wednesdays, and Fridays during the morning and worked in groups of four that allowed them to engage in discussion while collaboratively using concrete manipulatives or working through problems with paper and pencil. Within their groups, students were assigned classroom locations and roles for each week and were presented with clear classroom procedures by the teacher using an interactive whiteboard situated in front of the room. One student retrieved the group members' reflective journals, another obtained the manipulatives they would be using, another gathered miscellaneous supplies (e.g., markers, pencils, paper, rulers), and the last group member secured the group's books. Students gathered these items quickly, so little time was spent on this process. Students listened to the teacher's explanation of procedures for the day's lesson and then began working. The teacher circulated around the classroom observing students and listening as they interacted with each other. Students began working independently and then were given time to collaborate with their group members. After a designated period of time, the teacher asked each group to present their solution to the problem of the day. If some groups experienced difficulty reaching a correct solution, she assigned a peer tutor and let them explore possible solutions to the problem with the tutor’s help. The entire class assembled toward the end of the session to discuss findings. The class always worked out the problem with the manipulative and wrote the steps taken to reach their solution. The last few minutes of class were dedicated to writing a journal reflection on the day's lesson.

Students in the experimental group completed written reflections at the end of the lessons, some of which were based on questions such as, "How did the virtual manipulatives help you to learn mathematics today?” and “How did you feel about using technology in class today?” Students in the control group wrote reflections on similar questions, such as “How well did you understand the objective of the lesson today?” “Did the tool you used today make it easier for you to complete the task” and “Is there anything that could have helped you to learn the lesson easier?" Reflective journal writing from both the experimental and control groups was utilized to gauge student motivation, progress, and attitudes toward the use of manipulatives, both concrete and virtual. Burns (1996) noted that journal writing can serve as an effective procedure to augment a student's mathematical thinking and communication skills. When students are encouraged to write reflectively in a mathematics journal, it allows them opportunities to self-assess their learning. When students engage in journal writing, they are reflecting on their problem-solving skills for each activity. They must think about the steps they took to problem solve and then accurately and clearly communicate those steps. This review of the mathematics problem and the process used to solve it encourages deeper thinking and allows the student to gain an important perspective about the manner in which the challenge was solved. Math journals also allow the teacher to determine if students are forming deep understandings of the presented concepts. The journal works as an assessment tool for the teacher. Students also can use the journals to communicate with the teacher about any specific concepts they do not understand in the lesson (Burns, 1996; Burns & Silby, 2001). In this study, the classroom teacher used the reflections, which were class assignments, to assist in improving her classroom instruction.

On days when the experimental group worked in the computer laboratory, the teacher took a cart on wheels containing the reflective journals, pencils, instructions, and papers needed to record findings; this enabled the students to maintain the same procedures as those used in the regular classroom. The three days spent in the computer laboratory each week began with an introduction to the virtual manipulative applet that the students would utilize for each particular instructional period. Students received their assignment for the computer laboratory, as well as a detailed objective sheet with instructions for using the virtual manipulative. Students also received paper on which to record their findings, which also helped them to stay focused on the specified learning objective. The teacher read the directions and assisted students in accessing the necessary website before beginning independent work.

The virtual and concrete manipulative treatment groups were designed to be treated equally with the exception of the environment. Uniformity in lesson design with the same teacher serving as the instructor for both groups was important to minimize extraneous variability between the two environments. The only difference between the two groups was the lesson sheet. The control group had a lesson sheet written on paper, while the experimental group accessed problems on both computer screen and paper. Both groups were required to write reflections in their journals.

Development of the Instruments

This study utilized a mixed methods design consisting of both quantitative and qualitative data collection. One aspect of the qualitative part of the design was the use of students' reflective journal writings. Journal writing afforded students an opportunity to expand their metacognitive skills, enabling them to interpret the factors influencing their thinking. Burns (1995) stated that students could examine their journals to evaluate themselves on items they understood and determine which items were still new and confusing to them. The students received a list of writing prompts that would require them to reflect critically upon their lessons (Appendix B). Students were not required to write on all prompts; rather, they were instructed to choose from the list an appropriate prompt for the day's lesson, without using the same one or two each time. Such reflective writing required students to show their understanding of the procedure they followed in order for others to replicate it. Reflective journals also served as a useful assessment tool for the teacher and offered further insight as to how the students viewed their own learning (Burns, 1995; Burns & Silby, 2001). By reading student journals, the teacher was able to determine if further practice was needed or if a true understanding of the material had been reached. Additional qualitative components consisted of the interview with the mathematics teacher and the observations of the two classrooms by the researcher. The observations enabled the researcher to focus on student engagement and motivation during the lesson. The researcher also observed the teacher and recorded her interaction with the students and the lesson. The researcher chose those specific observable characteristics to determine if the virtual manipulatives were more engaging and motivating than their concrete counterparts for the students and teacher. The questions presented to the teacher focused on the importance of the virtual manipulatives in her classroom. Her insight enabled the researcher to gain a deeper understanding of the teacher's eagerness to incorporate the virtual component into her lessons. The teacher answered the following questions:

1. How do you integrate virtual manipulatives into your mathematics lesson plans?
2. How does the addition of virtual manipulatives to your existing mathematics curriculum allow for differentiating instruction to meet the needs of individual students?
3. How has the use of virtual manipulatives affected student engagement/motivation in your mathematics lessons?
4. What kind of feedback does the use of virtual manipulatives provide for the student?
5. How does the use of virtual manipulatives in mathematics affect student understanding of the purpose/target(s) for each lesson you teach?
6. How does the use of a computer in mathematics allow students to use virtual manipulatives to authentically discover mathematical relationships?
7. How does the use of virtual manipulatives promote independent and autonomous student learning?
8. How does the use of virtual manipulatives encourage students to collaborate with their peers?
10. How has student understanding/learning in mathematics been affected by the use of virtual manipulatives?

11. Would you recommend to teachers the use of virtual manipulatives in their mathematics classrooms? Why?

The quantitative data in this research included scores on the Iowa Test of Basic Skills pretest and posttest, the Illinois Standards Achievement Test, which is an end-of-year standardized assessment, and the district's M-COMP pretest and posttest, which measures gains in student achievement.

Quantitative Measures

Pretest. At the beginning of the study, students took their fall ITBS. This test assessed their grade equivalency level prior to the start of manipulative use. The ITBS pretest also allowed the researcher to determine the amount of growth the two groups achieved between taking the pretest and posttest of the ITBS. The pretest and posttest items were identical to assist the researcher in assuring reliability between the two tests. The pretest score constituted an average score for each group prior to the experimental period.

A posttest was administered at the conclusion of the experimental period. This was a standard test administered to the entire study district. The researcher calculated the average score for each group and conducted a statistical analysis to compare the change in achievement between each group over the instructional period in which the experiment occurred. Students also took the district test, the M-COMP, a series of revised math computation assessments serving as the leading assessment and data management system for Response to Intervention (RTI) implementation. RTI is a method of academic intervention used in the United States to provide early, systematic assistance to children who are having difficulty learning. RTI seeks to prevent academic failure through early intervention, frequent progress measurement, and increasingly intensive research-based instructional interventions for children who continue to experience difficulty. The M-COMP, in terms of format, assesses a number of benchmarks and progress-monitoring probes at all grade levels. The M-COMP probes aid in identifying students who would benefit from the RTI method, thus allowing for early intervention and progress measurement. The M-COMP essentially is a math computation assessment that assesses students' mathematical levels, tracks their math understanding over time, and helps teachers differentiate instruction according to students' needs. The M-COMP is without bias because it is computer generated and contains enhanced content to provide a greater depth of information and increase alignment of its content closely with the district's curriculum standards (Pearson, 2011). This test was administered once during the first quarter of the school year and again during the middle of the fourth quarter.

Table 5

Averages for M-COMP

	Pre M-COMP	Post M-COMP
Experimental	30.1	38.2
Control	22.1	25.5

The Illinois Standards Achievement Test (ISAT) assesses the academic gains made by students in the school district. According to the Illinois State Board of Education (ISBE) (2011), “The ISAT measures individual student achievement relative to the Illinois Learning Standards. Results of this score are applied to the No Child Left Behind Act, for purposes of identifying failing schools” (ISBE, 2001, para. 1). The ISAT is a state-mandated test administered to all third- through eighth-grade students in the state during the same mandatory spring testing period. The ISAT enables teachers to identify where individual student achievement gains and problems exist. This test was administered at the end of the third quarter of the study district's school year. The researcher evaluated students' scores and formulated an average for both classes to determine if the data aligned with the ITBS and M-COMP outcomes.

Table 6

Spring 2010 ISAT Summary

	Warning	Below	Meets	Exceeds
State	2.2%	13.5%	53.9%	30.4%
District	2.7%	20.7%	68.7%	7.9%
Building	2.5%	22.8%	65.8%	8.9%
Experimental Group	0%	5%	73%	23%
Control Group	5%	32%	55%	9%

Information taken from the study district website

Qualitative Measures

Field notes, the researcher's observations and interactions with students, students' reflective journals, and the interview with the classroom teacher were used to evidence the importance of virtual manipulatives in the mathematics curriculum. The researcher used coding to analyze the qualitative data, having established a set of codes prior to observing the two classes. Inductive coding was used initially to keep track of behaviors, activities, conversations, and participation by the students and teacher. The researcher then found relationships that connected her codes and grouped them according to their common themes. The researcher spent two hours per week for seven months observing the students and teacher during classroom instruction. The researcher took notes and observed students as they interacted with one another and the teacher. In particular, the researcher was looking for positive engagement between the manipulatives, the students, and the teacher. Her observations also centered on verifying a deeper level of understanding of mathematical concepts because of student interactions with both peers and the teacher. She used an observation form that focused on the objectives of each lesson, which type of manipulatives were employed, and the interest level and attitudes displayed by students (Appendix C). The researcher observed teacher and student attitudes as they interacted during each lesson. In some instances, students communicated directly with the researcher during the observation.

The students also kept a reflective journal with the answers to questions posed by their teacher. The journals showed students' perceptions based on their attitudes and academic progress during the various lessons. Questions posed by the teacher were always open-ended and led to further discussion as students talked amongst themselves. The researcher was looking for ease of the mathematics lesson for students, engagement, and collaboration among students and the teacher. The researcher also was searching for evidence of the attitudes of the students and the teacher in both groups.

The researcher also interviewed the mathematics teacher to better understand her thoughts on the use of the manipulatives. Discussion evolving from this interview centered on the teacher's insights as to student engagement and motivation during the lesson, as well as student feedback regarding the use of virtual manipulatives and perceived enhancement of learning based on the manipulatives' authenticity to students.

Reliability and Validity

It has been the researcher's experience that student learning can be affected by the time of day in which it occurs. Therefore, in order to assure validity, both classes in this study were taught within one hour of each other, the first from 8:30 - 9:30 am and the second from 9:35 - 10:35 am.

The same math teacher instructed both the control and experimental groups to eliminate the effects that differences in teaching styles might contribute to the study. Curriculum materials were identical for both the control and experimental groups.

Reliability and Validity of the Instruments

Fraenkel and Wallen (2008) stated that, "Validity refers to the appropriateness, correctness, and usefulness of any inferences a researcher draws based on data obtained through the use of an instrument" while "Reliability refers to the consistency of scores or answers provided by an instrument" (p. 165).

Pearson (2011), the publisher of the M-COMP, has a web-based universal screening and progress-monitoring system, AIMSweb, which now features an updated assessment of math computation probes for grades one through eight. The content validity of the assessment was revised using feedback from the previous assessments, the M–CBM and the M–CBM2. The data indicated that applying the weighted scoring system to the M-COMP minimized the scoring time, maximized sensitivity to growth, and made it easier to control for the students who skipped to the easiest problems (Pearson, 2011). Pearson also noted that the reliability of the M-COMP assessment supported longitudinal data when scores were equated to the former assessments, the M-CBM and the M-CBM2. Pearson (2011) once again relied on user feedback from the former assessments to re-evaluate the current scoring process against a weighted scoring system on another assessment (M-CAP). Pearson also was able to improve upon the reliability of the psychometric soundness of the process. This is of importance according to Gardner because it deals with the design, administration, and interpretation of quantitative tests for the measurement of psychological variables, such as intelligence, aptitude, and personality traits (Gardner, 1991).

The Iowa Test of Basic Skills (ITBS), developed by the faculty and professional staff at The University of Iowa and published through Riverside Publishing, contends that researchers have assessed the validity and reliability of the ITBS. Time limits on the assessment were determined during empirical studies and observations to yield maximum information regarding student achievement. Time blocks were set to ensure concentration and limit student distraction (University of Iowa).

The ISAT includes a combination of items produced by Pearson-San Antonio and items written by Illinois teachers. Items from these two sources were combined into new forms that are scored and analyzed as a single test (ISBE):

> The Pearson items are part of the Stanford Achievement Test, Tenth Edition (SAT 10) and allow reporting of nationally norm-referenced results such as national percentile ranks, stanines, and the percent of students in national quarters. However, students' ISAT scale scores, which designate one of four performance levels (Exceeds Standards, Meets Standards, Below Standards, Academic Warning) are based on all items combined (i.e., SAT 10 and Illinois-developed

items). The resulting mix of items fully covers the Illinois Learning Standards. (ISBE, 2001, para. 4)

Dependent and Independent Variables and Internal Validity

The researcher controlled the use of the virtual manipulatives in this study in order to determine if these manipulatives were associated with a measureable increase in student achievement. In an experimental study, “The researcher manipulates one of the variables and tries to determine how the manipulation influences other variables” (Bluman, 2008, p. 14). According to Bluman (2008), “The independent variable in an experimental study is the one that is being manipulated by the researcher. The independent variable is also referred to as the explanatory variable. The resultant variable is called the dependent variable or the outcome variable” (p. 14). The experimental group in this mathematics manipulative study was instructed with both virtual and concrete manipulatives, while the control group was instructed with concrete manipulatives alone.

According to Fraenkel and Wallen (2006), “Internal validity means that observed differences on the dependent variable are directly related to the independent variable, and not due to some other unintended variable” (p. 169). The researcher identified possible threats to the internal validity of the study. One type of threat is participant selection, “Which are the biases that may result in the selection of comparison groups, a counter-attack against this is the randomization or random assignment of the group membership” (Yu & Ohlund, 2010, p. 1). Participant characteristics did not threaten the internal validity of this study because the sample was selected through the process of the school district’s registration. Grisham and McCauley (2011) noted that “Scores can change due to maturation occurring in subjects due to the passage of time, in order to validate the maturation of the study, it needs a control (comparison) group that does not receive the intervention/course" (p. 3). Participants will naturally change over time due to maturational growth; thus, the researcher's observations and the students' reflective journals were important to the validity of the study. The researcher's observations and student reflective journals were chosen as a method of data collection to acquire student understanding. The study was composed of an experimental and a control group. Loss of participants was a concern due to the district's high mobility, truancy, and absenteeism rates. Only students present for the ITBS pretest were allowed to participate in the data collection process. To reduce the possibility that one group would be treated advantageously all of the mathematics instruction was conducted in the study school's computer laboratory or the mathematics teacher's classroom. According to Fraenkel and Wallen (2008), implementation threat, "Raises the possibility that the experimental group may be treated in ways that are unintended and not necessarily part of the method, yet which give them an advantage of one sort or another" (p. 179). In this study, implementation threats were minimized by one teacher teaching both the control and experimental groups. The researcher observed and interviewed the teacher to gain insight into her feelings toward the use of the concrete and virtual manipulatives.

Summary

The research design allowed data to be collected pertaining to student achievement while learning with both concrete and virtual manipulatives in the mathematics classroom, as well as to their attitudes towards mathematics. Students need motivation to grasp and execute different mathematical concepts, as well as to create conceptual knowledge for future endeavors. Brown (2007) claimed that manipulatives are not just fillers for class time; they are the keys to making the connection from abstract to concrete understanding in everyday situations. As such, they are critical for enhancing student achievement.

The intent of the mixed methods study was to determine if adding virtual manipulatives to existing concrete manipulatives in the seventh-grade mathematics curriculum increased students' mathematics composite scores on standardized and teacher-created assessments. This chapter included a discussion of the overall design of the study. The instrumentation and alignment of the instruments were discussed, along with the validity and reliability of the instruments, in order to demonstrate that the instruments were suitable assessment tools. Finally, the quantitative and qualitative data analysis procedures were discussed and will be analyzed further in Chapter 4 in a detailed discussion of the data.

Chapter Four: Results

This study analyzed the effect of using concrete and virtual mathematics manipulatives in teaching mathematics in a seventh-grade classroom. The purpose of this study was to determine if adding virtual manipulatives to existing concrete manipulatives in the seventh-grade mathematics curriculum would result in a measureable change in students' mathematics composite scores on standardized and teacher-created assessments. The researcher also compared the two groups' written reflections on their own learning using manipulatives. This study utilized a mixed methods research design, which will be described in chapter 4 as the use and analysis of both quantitative and qualitative methods. The qualitative data consisted of the interview with the teacher, the researcher's observations, and the information garnered from the students' reflective journal writings. The quantitative data included the statistical and descriptive results of the ITBS, M-COMP, and ISAT assessments.

Research Question #1

The first research question was: How do students perceive the effectiveness of their learning/understanding when taught mathematics with both concrete and virtual manipulatives?

Students kept a reflective journal in which they recorded their thoughts to prompts given by the teacher. One student wrote, "The computer made math easy and fun for me, I was able to actually figure out how to multiply fractions. I think it was because the computer lets you know if you are right or wrong and then it helps you figure out how to do the problem. I have a computer at home and tonight I am going to go home and try this again." Another student wrote, "I enjoyed the lesson today, because I could really move the angles and that let me see what they looked like. Before this lesson I wasn't sure what some of the angles looked like, but I was able to move the arms on the angle and saw the degrees." Another student wrote, "The class went by super-fast today, we were working on fractions and we were able to compare our answers with the person next to us. This really let me see if I was on track or not. Plus the computer told us if we were right or had to try again. We tried to get every answer right kind of like a game." Another journal entry stated, "Today I had a really good time in math class. I don't really like math and can't wait for lunch. Today when we were working on conversions on the computer it was fun. If I didn't get an answer right away I could talk to the person next to me or the teacher. If that didn't work the computer helped me work through the conversions. They can be really tough but I learned the formulas for converting them and I feel that I understand what I am doing." Ninety-five percent of the students believed that the combination of the virtual and concrete manipulatives was effective in enhancing their learning. The students believed that when they not only saw the manipulatives but were able to see and touch them, they more quickly achieved a deeper, more authentic understanding of the objective. They enjoyed working with the manipulatives, and felt more motivated and engaged. Several students indicated that they went home and continued their lessons on their own computers.

Research Question #2

The second research question was: How does the teacher who has experience using both concrete and virtual manipulatives to teach mathematics perceive her effectiveness when using only concrete manipulatives? The teacher felt that she was effective, but also had some issues. Some of the issues that surfaced centered on the lack of enough available manipulatives and the tendency of students to discard or take the manipulatives with them. When she lacked manipulatives for a lesson she would either have two students team up or she would ask colleagues if she could borrow the needed manipulative from them. Some students would get off task and start building shapes and lose focus of the concept she was trying to incorporate. When the teacher noticed off-task behavior, she would attempt to make eye contact with the student(s). If this did not work, she would approach the student(s) and quietly start modeling what she expected them to work on. She would whisper to them (so that she did not distract the rest of class) that she believed they were not following directions, and with this redirect, students were soon working appropriately.

The questions presented to the study teacher were answered as follows:

1. *How do you integrate virtual manipulatives into your mathematics lesson plans?*

The textbook that I use has an "explore" section that allows integration of the virtual manipulative. I use the interactive whiteboard to demonstrate the proper manner for students to use the virtual manipulative. If some students need further assistance, they can work through their problems with me as I go through the steps on the whiteboard.

2. *How does the addition of virtual manipulatives to your existing mathematics curriculum allow for differentiating instruction to meet the needs of individual students?* The virtual manipulative affords me the opportunity to differentiate instruction to varying levels of mastery. The students can use the manipulative for a support as long as necessary until they master the concept. More advanced

students can work on more challenging problems, which alleviates boredom with the lesson.

3. *How has the use of virtual manipulatives affected student engagement/ motivation in your mathematics lessons?* The students are highly engaged because many are kinesthetic learners and become absorbed in the lesson. Many students view the virtual manipulative as a game and try to finish the problem as quickly as they can. Students who normally would become bored or fall asleep were awake and engaged.

4. *What kind of feedback does the use of virtual manipulatives provide for the student?* The virtual manipulatives provide immediate feedback to the student. They also provide visual feedback for students who might have difficulty in grasping abstract mathematical concepts and connecting these to their real-life learning. Virtual manipulatives also have an auditory component that enables the students to hear the question and the provided prompt(s), which helps push them towards the correct answer.

5. *How does the use of virtual manipulatives in mathematics affect student understanding of the purpose/target(s) for each lesson you teach?* The virtual manipulatives positively contribute to student understanding of the target for each lesson by providing the opportunity for students to use the manipulatives in ways that make the lesson understandable for all learners. Students are able to turn, flip, and rotate the dynamic virtual manipulative so that they can form a more concrete understanding of the once abstract idea.

6. *How does the use of a computer in mathematics allow students to use virtual manipulatives to authentically discover mathematical relationships?* Students use the manipulatives to authentically discover mathematical relationships to explore possible solutions to problems. Problem solving is a part of everyday life; the computer is a tool that most students own, so they can take what they learn and apply these concepts to their own lives.

7. *How does the use of virtual manipulatives promote independent and autonomous student learning?* The virtual manipulatives promote independent, autonomous student learning because they empower students to work through problems without seeking constant assistance from a teacher. Students come to realize that there is more than one solution to a problem, and they collaborate with peers to find a resolution rather than relying solely on the teacher.

8. *How does the use of virtual manipulatives encourage students to collaborate with their peers?* Virtual manipulatives encourage collaboration because students peer tutor each other as they work. They explain what they did and how they got there to peers so that their peers can then duplicate the successes as well.

9. *How has student understanding/learning in mathematics been affected by the use of virtual manipulatives?* Student understanding and learning have been greatly affected due to the use of virtual manipulatives. Students are excited to come to their mathematics class and are eager to use virtual manipulatives. They evidence their enjoyment of the immediate feedback and collaboration with their peers as they find solutions to mathematics problems. The use of virtual

manipulatives has increased student scores on computer-generated and teacher-designed assessments.

10. *Would you recommend to teachers the use of virtual manipulatives in their mathematics classrooms? Why?* Yes, I would recommend the use of virtual manipulatives to teachers in their mathematics classroom. The virtual manipulatives enable the teacher to reach all students at varying levels. Mathematics lessons can be easily differentiated to meet the needs of individual learners. Students are more actively engaged in the lesson and stay on task for longer periods of time. Students are more willing to rework areas that are giving them problems, and when assistance is needed, they collaborate with peers instead of wanting the teacher to do all the work for them. Students seem to take more pride in their mathematics lessons and share their results with each other. If problems arise, they work through them and show each other the steps they took to reach the solution, which mimics what we do in everyday life.

Research Question #3

The third research question was: How does the combination of virtual and concrete manipulatives affect the academic performance of students in the area of mathematics as opposed to the use of only concrete manipulatives? The teacher believed that the students using both concrete and virtual manipulatives were engaged in their learning and motivated to try even the more challenging problems. The learning seemed more authentic, thus enabling the students to make real-life connections. These personal connections formed a much more stable foundation for student learning. Students retained the information more quickly, and the teacher spent less time reteaching.

Students also collaborated with peers on possible solutions to their problems. The virtual manipulatives reinforced the learning from the concrete manipulatives to take it from the abstract to the tangible. From an instructional standpoint, virtual manipulatives provide students with instantaneous, corrective feedback (Clements & McMillen, 1996; Crawford & Brown, 2003; Durmus & Karakirik, 2006; Reimer & Moyer, 2005; Suh & Moyer, 2005). This immediate feedback benefitted the students immensely as they did not waste time waiting for the teacher to check their work. Many authors contend that this ability makes virtual manipulatives well suited to inquiry-based learning and problem solving (Clements & McMillen, 1996; Durmus & Karakirik, 2006). Another pedagogical benefit of virtual manipulatives demonstrated that they had the ability to provide multiple representations of a single concept at the same time (Clements & McMillen, 1996; Moyer et al., 2002; Suh & Moyer 2005). Reimer & Moyer (2005) argued that this ability provided an advantage over physical manipulatives, "Unlike physical manipulatives, electronic tools use graphics, numbers, and words on the computer screen to connect the iconic with the symbolic mode" (p. 7). It has also been proposed that this ability promoted transfer of knowledge from specific ideas to general knowledge (Clements & McMillen 1996; Durmus & Karakirik 2006; Moyer et al., 2002; Suh & Moyer 2005).

Null hypothesis (Ho^A) - Students taught mathematics with virtual manipulatives in addition to concrete manipulatives in a seventh-grade mathematics curriculum will not demonstrate a measureable change in mathematics composite scores when comparing Post-ITBS to Pre-ITBS.

According to Bluman (2007), an F-test or statistical test is used to compare two variances (p. 653). An F-test to check for similarities in the variances concluded that they were not similar and the study could continue with a t-test for difference in means with Unequal Variance. The F-test noted a p-value of 0.00 as compared to an alpha value of .05, which supports a decision to reject the null hypothesis (there is no difference in variance). The probability was below the .05 level, so the researcher chose to use a two-sample t-test assuming unequal variances.

Table 7

Two-Sample F-Test for Variances in Mathematics Scores

ITBS		
	Experimental	*Control*
Mean	7.495238095	6.014285714
Variance	3.07747619	1.015285714
Observations	21	21
df	20	20
F	3.031143005	
P(F<=f) one-tail	0.008403955	
F Critical one-tail	2.124153298	

Students who received instruction using virtual manipulatives obtained a mean score on the mathematics posttests of 9.16 (*SD*=2.89); students who received instruction using only physical manipulatives obtained a mean score on the mathematics posttests of 6.60 (*SD*=1.13). The data in Table 8 noted a p-value of 0.00 as compared to an alpha value of .05, which supports a decision to reject the null hypothesis. The probability was less than .05, so the researcher was able to reject the null hypothesis and conclude that there was a significant difference in the achievement scores between the experimental and control groups. Students receiving instruction with virtual manipulatives in addition to concrete manipulatives yielded a higher average than those using only concrete manipulatives.

Table 8

Two-Sample T-Test Assuming Unequal Variances

Post - ITBS		
	Experimental	*Control*
Mean	9.161904762	6.595238095
Variance	2.89147619	1.12547619
Observations	21	21
Hypothesized Mean Difference	0	
df	34	
t Stat	5.868549554	
P(T<=t) one-tail	6.37915E-07	
t Critical one-tail	1.690923455	
P(T<=t) two-tail	1.27583E-06	
t Critical two-tail	2.032243174	

Null hypothesis (Ho_B) - Students taught by a teacher using concrete manipulatives in the seventh grade mathematics curriculum will not evidence a measureable change in mathematics composite scores when comparing Post-M-COMP with students' use of concrete manipulatives to Post-M-COMP with students' use of virtual and concrete manipulatives.

An F-test to check for similarities in the variances concluded that they were not similar, so the study could continue with a t-test for difference in means for Unequal Variance. The F-test noted a p-value of 0.04 as compared to an alpha value of .05, which supports a decision to reject the null hypothesis. The probability was below the .05 level, so the researcher chose to use a two-sample t-test assuming unequal variances. Before choosing the appropriate t-test, the researcher tested the variances of the samples with an F-test for differences in variances.

Table 9

Two-Sample F-Test for Variances in Post-M-Comp

M-Comp		
	Experimental	*Control*
Mean	29.14285714	22.04761905
Variance	119.4285714	114.447619
Observations	21	21
df	20	20
P(T<=t) one-tail	0.017831638	
t Critical one-tail	1.720743512	
P(T<=t) two-tail	0.035663275	
t Critical two-tail	2.079614205	

Students who received instruction using virtual manipulatives obtained a mean score on the mathematics posttests of 38.18 (*SD*=121.4); students who received instruction using only concrete manipulatives obtained a mean score on the mathematics posttests of 25.45 (*SD*=142.6). The data noted a p-value of 0.00 as compared to an alpha value of .05, which supports a decision to reject the null hypothesis. The probability was less than .05, so the researcher was able to reject the null hypothesis and conclude that there was a significant difference in the achievement scores between the experimental and control groups. The experimental group indicated a significantly higher average on achievement than the control group.

Table 10

Two-Sample T-Test Assuming Unequal Variances

Post M-Comp		
	Experimental	*Control*
Mean	38.18181818	25.45454545
Variance	121.3939394	142.6406926
Observations	22	22
Pooled Variance	132.017316	
Hypothesized Mean Difference	0	
df	42	
t Stat	3.673806207	
P(T<=t) one-tail	0.000335425	
t Critical one-tail	1.681951289	
P(T<=t) two-tail	0.000670849	
t Critical two-tail	2.018082341	

After examining the results of the two groups' ISAT scores, it was verified that 95% of the students in the experimental group (concrete manipulatives coupled with virtual manipulatives) met or exceeded the state standards in mathematics on the test. The state requires that 87.3% of the students meet or exceed the state standards in the area of mathematics in order to make adequate yearly progress. Therefore, the experimental group made adequate yearly progress as mandated by the state. The control group (only concrete manipulatives) did not make adequate yearly progress as mandated by the state, with only 55% meeting and 9% exceeding the state standards.

Table 11

Achievement Based on State Standards

	Warning	Below	Meets	Exceeds
State	2.2	13.5	53.9	30.4
District	2.7	20.7	68.7	7.9
Building	2.5	22.8	65.8	8.9
Experimental Group	0	5	73	23
Control Group	5	32	55	9

Summary of the Results

This study analyzed the effect of using a combination of concrete and virtual mathematics manipulatives versus using concrete manipulatives alone in teaching mathematics in a seventh-grade classroom. The results indicated that adding virtual manipulatives to existing concrete manipulatives in the seventh-grade classroom was associated with a measurable change in student learning. The interview with the teacher indicated that she believed adding the virtual manipulatives enabled the students to garner a deeper understanding of the mathematical objective for each lesson. The students’ reflective journals indicated that they developed a deeper connection to the material. Students were able to connect the objectives to real life, which enabled them to make authentic discoveries in their learning. The researcher's observations in the two mathematics classrooms led her to believe that the students who were given the virtual manipulatives were eager to discover and share their findings with each other. They accepted the challenging problems and looked for results that could then be applied to real life. The students made real-life connections to the lessons that demonstrated ownership of and pride in their findings. When students received immediate feedback via the virtual manipulatives, it encouraged them to keep prodding for different ways to achieve their results.

An analysis of the quantitative data revealed that students who were instructed with both concrete and virtual manipulatives yielded a higher average than those using only concrete manipulatives on both the ITBS and M-COMP tests. An analysis of the ISAT scores showed that 95% of the students in the experimental group met or exceeded according to the state standards in mathematics on the state test. Further discussion of the findings and future research directions will be presented in Chapter 5.

Chapter Five: Discussion

This mixed methods study analyzed the impact of using computer-simulated (virtual) manipulatives and hands-on (concrete) manipulatives, as opposed to hands-on manipulatives alone, on seventh-grade student learning in mathematics. Students' composite mathematics scores on both standardized and teacher-created assessments were compared. The researcher also compared each of the two student groups' written reflections of their learning as it pertained to both concrete and virtual manipulatives.

The researcher analyzed the correlation between mathematics achievement and the use of both concrete and virtual manipulatives. The researcher believes that technology, in the form of virtual mathematics manipulatives, in conjunction with the concrete manipulatives already present in many classrooms, is essential for enhancing mathematics instruction by ensuring student understanding of mathematics concepts. Using the results of this study to restructure instructional practices could help to increase student achievement in the mathematics classroom.

This study found that students who used both the virtual and concrete mathematics manipulatives demonstrated a measureable change in mathematics scores. The results also indicated that students enjoyed using virtual manipulatives, which encouraged them to work on more challenging problems. The teacher also exhibited a positive attitude towards virtual manipulatives.

Interpretation

The objective of this study was to determine the effect of using concrete mathematics manipulatives coupled with virtual mathematics manipulatives (independent variable) on student achievement (dependent variable), as opposed to using concrete manipulatives alone. To this end, a statistical analysis of the experimental group's (concrete and virtual manipulatives) and the control group's (only concrete manipulatives) ITBS and M-COMP assessment scores was conducted using F-tests and t-tests to determine if the tests evidenced a measurable change. The hypothesis stated that students taught mathematics with virtual manipulatives in addition to concrete manipulatives in a seventh-grade mathematics curriculum would demonstrate a measureable change in mathematics composite scores on standardized and teacher-made assessments compared to students taught mathematics with only concrete manipulatives. The data analyses using *F*-tests for differences in variance and *t*-tests for difference in means enabled the researcher to render a decision regarding whether or not to reject the null hypothesis.

The results of the ITBS scores between the two groups evidenced a measurable change in the mathematics abilities of students instructed with concrete manipulatives coupled with virtual manipulatives. The *p* values of each group fell into the critical regions on a bell-shaped curve (using a 95% confidence interval), thus verifying that the students using both types of manipulatives demonstrated a significant measurable change in mathematics composite scores.

The results of the M-COMP scores between the two groups evidenced a measurable change in the mathematics abilities of the students instructed with concrete manipulatives coupled with virtual manipulatives. The *p* values of each group fell into the critical regions on a bell-shaped curve (using a 95% confidence interval), thus verifying that the students using types of manipulatives demonstrated a significant measurable change in mathematics composite scores.

The results of the ISAT scores between the two groups followed the same trend as those of the ITBS and the M-COMP assessments. Ninety-five percent of the students who used concrete manipulatives coupled with virtual manipulatives met or exceeded expectations on the state test. Furthermore, no students in the experimental group fell into the *academic warning* category, and only five percent of the students were in the *below meeting standards* category, whereas five percent of the students in the control group fell into the *academic warning* category, and 32% of them fell into the *below meeting standards* category. Also in the control group, only 63% of the students met or exceeded expectations on the state test, thus not reaching the 83.7% needed to make adequate yearly progress as mandated by the state of Illinois. The experimental group fared better than the district in all areas of the ISAT. The only area of the ISAT on which the experimental group did not achieve the state-required percentage was in the *exceeds expectation* category.

One of the guiding research objectives was to determine how students perceive the effectiveness of their learning/understanding when taught mathematics with both concrete and virtual manipulatives. Based on students' reflective journaling, the researcher concluded that the students believed that the virtual mathematics manipulatives enhanced their learning. Students indicated in their journals that the virtual manipulatives made it easier for them to learn the mathematic concept they were studying that day. They also wrote that they received immediate feedback and enjoyed collaborating with their teacher and peers. In addition, the students enjoyed working on the more challenging problems because the interaction occurred just between them and the computer, thus raising their confidence level.

Another guiding research question was posed to determine how the teacher familiar with using both concrete and virtual manipulatives to teach mathematics perceived her effectiveness when using only concrete manipulatives. According to the interview, she believed she was effective in her teaching when using the virtual manipulatives. She indicated that she was able to differentiate her lessons and that students stayed on task for longer periods. The teacher was comfortable using the concrete manipulatives because she had extensive training and classroom use with the manipulatives. She believed the students enjoyed the concrete manipulatives and that they did offer another level of understanding. The teacher also indicated that she noted higher test scores from the students that used both the concrete and virtual manipulative. She believes this is because the students feel a connection to the virtual lessons and a deeper level of understanding was achieved. A problem that occurred during instruction was there were not enough manipulatives for all students, some students used the manipulatives inappropriately, and there were times a few students kept the manipulatives.

The final question that guided this research concerned the manner in which the use of virtual manipulatives coupled with concrete manipulatives affects the academic performance of students in the area of mathematics as opposed to the use only of concrete manipulatives. Students taught with a combination of concrete and virtual manipulatives showed a measureable change in mathematics composite scores on standardized and teacher-made assessments.

Recommendations and Implications

Based on the results of this study, one recommendation is that mathematics educators incorporate both concrete manipulatives and virtual manipulatives in their mathematics curriculum. As demonstrated in the study, the combination of these two types of manipulatives enabled the students in this group to accomplish a measureable change in tested mathematical ability. Educators need to offer their students lessons that are authentic and interesting in order to hold their attention as they attempt to grasp the concepts. The different options also provide students with the needed differentiated instruction to suit their varied learning styles.

This study supports the belief that educators need to be discerning in selecting appropriate virtual manipulatives to use when teaching mathematical concepts. Students indicated in their journals that they believed the virtual manipulatives that provided immediate feedback were more helpful and encouraged them to pursue more challenging problems. Therefore, another recommendation is for schools to provide opportunities for their teachers to pursue ongoing professional development in the area of both concrete and virtual mathematics manipulatives. Collaboration and follow-up sessions are essential for educators to achieve success in their quest to provide varied instruction for their students.

According to the National Staff Development Council (NSDC) (2008), the term *professional development* is defined as, “A comprehensive, sustained, and intensive approach to improving teachers’ and principals’ effectiveness in raising student achievement” (p. 1). Staff development also, “fosters collective responsibility for improved student performance” (p. 1). The NSDC has asserted that professional development must be ongoing and should contain a component that provides immediate feedback for teachers so that they will be able to obtain the maximum effectiveness needed to increase their knowledge, as well as that of their students. Burch (2006) and Hiebert et al. (1997) asserted that well-established professional development fosters collaboration between teachers, and when teachers collaborate effectively, they can share their skills with those who need assistance with their classroom instruction. Also critical is application, which describes the ability of an individual to transfer his or her understanding to another situation; this ability allows teachers to routinely check the effectiveness of new learning in enhancing their performance and to make informed adjustments as needed. The NSDC also has advocated that teachers, whenever possible, be provided with common planning times during the school day so they can communicate with one another on all aspects associated with the professional development initiative. During this time, teachers can share ideas collaboratively about ways to implement their learning in their classrooms. Burch (2006) stated that teachers, as part of their professional development to improve their own teaching, should have ample opportunity to visit other classrooms in order to view what colleagues are implementing to improve student achievement. Burch (2006) continued that teachers often fear innovative teaching initiatives, but when provided with thorough professional development, they are more apt to overcome their fears and attempt to implement the strategies and techniques of the professional development program into their curriculum. The recommendation stemming from this study, then, is to approach professional development regarding manipulatives from the perspective of the NSDC guidelines.

Future Studies

The participants in this study consisted of a small sample of the seventh-grade population within one particular school district. Using a larger sample of students across different grade levels could fortify this study. Another interesting avenue of research would be to survey all of the seventh-grade teachers in the study district to investigate their beliefs about using concrete and virtual manipulatives. The gender of the students could also be considered in order to determine if gender differences play a role in the effect of using concrete and virtual mathematics manipulatives.

Summary

The significance of this study lies in its emphasis on the importance of incorporating virtual manipulatives into existing mathematics curriculums. The study strongly indicated that virtual manipulatives enhanced student learning in the mathematics classroom. The study revealed that students felt confident and challenged when provided with alternative methods for learning mathematics. The study teacher revealed that lessons could be differentiated to build upon student cognition. The results from this study support the use of virtual manipulatives in the mathematics classroom.

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Appendix A

Websites Used for Instruction with Virtual Manipulatives

http://www.homeschoolmath.net/worksheets/fraction_calculator.php
http://zirkel.sourceforge.net/doc_en/index.html
http://www.imaginationcubed.com/index.php
http://www.hbmeyer.de/eratosiv.htm
http://www.explorelearning.com/index.cfm
http://www.studyisland.com
http://www.classzone.com
http://www.brainpop.com
http://www.symphonymath.com
http://www.learner.org/interactives/geometry/
http://www.saltire.com/gallery.html
http://www.harveyshomepage.com/Harveys_Homepage/Welcome.html
http://illuminations.nctm.org/
http://www.shodor.org/interactivate/
http://www.brocku.ca/mathematics/resources/learningtools/learningobjects/index.php
http://www.ies.co.jp/math/java/index.html
http://mathforum.org/mathtools/
http://nlvm.usu.edu/en/nav/vlibrary.html
http://www.pbs.org/teacherline/resources/interactives.cfm?cc=tlredir
http://standards.nctm.org/document/eexamples/
http://phet.colorado.edu/
http://www.tangoes.com/indexH.htm
http://www.teacherled.com/
http://www.georgehart.com/virtual-polyhedra/vp.html
http://www.visualfractions.com/
http://www.cet.ac.il/math/function/english/
http://www.visualmathlearning.com/index.php
http://www2.dsu.nodak.edu/users/edkluk/public_html/ViElMath/ViElMath.html
http://www.waldomaths.com/
http://www.fi.uu.nl/wisweb/en/
http://zonalandeducation.com/ezGraph/ezGraph.html

Appendix B

Observation Form

Observation form for: *Success to Increased Achievement Scores in the Middle School Mathematics Class*

observer/researcher: ____

Date: ____________________ Time: start _________ end ____________

Objective of lesson:

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Which type of manipulative was used?

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Students interest level:

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Student attitude toward the manipulative/lesson:

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Student academic progress/understanding (noted during the lesson):

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Teacher attitude toward the manipulative/lesson:

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Notes:

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Vitae

Elaine Diane Doias attended Southern Illinois University – Edwardsville, where she obtained a Bachelor of Science in Elementary Education. She then furthered her education by earning a Masters of Arts in Educational Administration at Lindenwood University.

Elaine currently teaches seventh grade in the Cahokia 187 School District in Illinois. She is highly qualified in the areas of Language Arts and Social Science. Elaine also holds an administrative certificate. She has served as a mentor for teachers in her building and has worked with several student teachers in her classroom. Elaine works closely with her special education teacher to ensure that the goals of her students are being met. She is a member of the Gateway to Excellence in Math and Science organization, which works to enhance educational experiences for teachers so that they can relay these experiences to their students.