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Singapore Math: A Longitudinal Study of Singapore Math in One School District from 2007 to 2012

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by

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Declaration of Originality

I do hereby declare and attest to the fact that this is an original study based solely upon my own scholarly work here at Lindenwood University and that I have not submitted it for any other college or university course or degree here or elsewhere.

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Abstract

For the last several years, Americans have fallen behind in the area of mathematics when compared to their peers in industrialized countries around the world. Singapore, on the other hand, was at the top of the world rankings in mathematics in the last four Trends in International Math and Science Study (TIMSS) assessments taken by fourth and eighth graders every four years. This project focused on the impact of the Singapore Math program on two cohorts of students by utilizing their Missouri Assessment Program (MAP) scores from the mathematics subtest. The first cohort, A, was comprised of students who were in third, fourth, and fifth grade during the first years of the implementation of the Singapore Math program in 2007, 2008, 2009, and compared with students in Cohort B who were exposed to the math program since first grade, as intended by the publisher. The students of Cohort B were in third, fourth, and fifth grade in 2010, 2011, and 2012, respectively. Data were also analyzed to see if the program had a correlation with a decrease in gender, ethnic, or socioeconomic (SES) achievement gaps when compared to Cohort B. Three tests were given in order to triangulate the results of the MAP test: difference in means by way of a z-test for a difference in means, a comparison of students scoring proficient and advanced through the utilization of a z-test for difference in proportions, and an F-test for difference in variance in MAP scores.

Results of the study yielded mixed results. While there was not a significant statistical difference in achievement between Cohort A and B in third, fourth, or fifth grade, there was evidence to support that the subgroups that were included in the study

(female students, Black students, and students with Free and Reduced Lunch status) performed commensurately with their peers in Cohort B.

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Chapter One: Introduction

Background

At the time of this writing, as America endeavors to retain its competitiveness on a global scale, it is not surprising that the American education system is once again on the forefront of the discussion. America was striving to improve performance in mathematics for years, and educators have sought new ways to help students understand math in order to meet the growing need for professionals in Science, Technology, Engineering, and Mathematics (STEM) and STEM-related careers (Huetinck & Munshin, 2008). In the 1950s and 1960s, there was “new math” where students were taught from textbooks which contained algorithms and abstract theories without application problems in order to jump start their mathematics skills to catch up with countries who were considered to be ahead (Huetinck & Munshin, 2008). During the 1970s, educators attempted to remedy math education by focusing on the basic skills, and by the 1980s and 1990s Clark (2009) suggested that problem solving began to emerge as the prominent focus in classrooms.

Results from the Trends in International Mathematics and Science Study (TIMSS) in 1995, 1999, 2003, 2007, and 2011 proved that American students were performing mediocrely on the world-wide stage (Ball, Hill, & Bass, 2005) on international math tests. The TIMSS was intended to measure achievement in math and science skills at the fourth and eighth grade levels. American fourth graders increased little in world rankings from 12th in 2003 to 11th in 2007, while eighth graders ranked 16th in 2003 and rose to ninth in the latest TIMSS assessment, in 2011. According to the TIMSS Average Scale, an average score on the test was 500. American fourth and eighth graders were ranked just above average at 529 and 508, respectively, while countries in the top three scored close to or above 600 (U.S. Department of Education, 2009a, pp. 6-7).

Due to the concerns with regard to international performance, there was renewed interest in revamping mathematics (Prystay, 2004) in order to effectively teach to all students. It was the hope of educators that the nationwide focus on improvement in the field of mathematics will improve test scores and make programs more competitive on the global stage (Ferrini-Mundy, 2001). Many district officials, however, were reluctant to try new strategies, since federal funding was linked to performance on state exams set in place by the No Child Left Behind Act (NCLB) (Prystay, 2004). Reform proved difficult because the U.S. did not use a national curriculum (National Council of Teachers of Mathematics [NCTM], 2011; Reys et al., 2006), and each state was responsible for setting its curriculum standards and districts within the state decided the best way to teach those standards (Prystay, 2004).

Too few U.S. students left schools with sufficient knowledge, skill, or confidence in math to be competitive in their current market (Stacey, 2002). There were a plethora of possible reasons why students were ill-prepared in mathematics. One suggested reason was the heightened importance on state exams in 21st century classrooms, which caused teachers to focus on covering as much content as possible in order to expose students to as many topics before the test in spring. This mile-wide and inch-deep curriculum resulted in re-teaching material in subsequent years, so little new knowledge was learned or retained (Garelick, 2006).

Another issue was with the growing demand of jobs in the field of science and technology. Graduates needed to be able to solve complex problems using higher-level math skills in order to compete in the global market. The mathematics that was taught in the schools of yesteryear, which had a practical base, was no longer relevant. Repetitive memorization of algorithms was not as valuable as it once was and did not guarantee meaningful understanding of the concept (Yang, Reys, & Wu, 2010).

Finally, with the implementation of the Common Core State Standards (CCSS) and their accompanying assessments, students would be accountable for "making sense of problems, reasoning abstractly and quantitatively, constructing viable arguments, and critiquing the reasoning of others, modeling with mathematics, using appropriate tools strategically, communicating precisely, and looking for and making meaning of structure" (Dessoff, 2012, p. 54).

Kool (2003) found that the field of mathematics was changing constantly, and student mastery of computation strategies and math concepts at elementary, middle, and high school could keep up with the demand by employers. Historically speaking, mathematics at the early grades focused on computation and arithmetic, and middle and high school courses followed with a generally procedural approach to algebra. The Mathematical Association of America found that this approach was largely unsuccessful with respect to achievement tests. It was for this reason that educators were advocating for change, beginning at the elementary level (Landel & Nelson, 2007).

Professional organizations, such as the National Council of Teachers of Mathematics (NCTM), policy makers, and researchers emphasized the importance of building algebra skills at earlier ages and using a different approach than what was used in the past (Katz, 2007). The NCTM also suggested that mathematics at the elementary level needed to strengthen student understanding of math by emphasizing conceptual understanding through problem solving (Kajander, 2010), By the time students leave eighth grade, they should be able to understand mathematical concepts, compute problems fluently, solve real-world problems, and utilize logical reasoning skills (Stacey, 2002).

The CCSS embraced these approaches and contended that by building a strong foundation, students in kindergarten through fifth grade could engage in hands-on learning of algebra (Common Core State Standards, 2012b). The foundation of algebra included the cultivation of mathematical habits that addressed the deeper structure of mathematics (Katz, 2007), as well as computation and problem-solving skills, which should be developed between kindergarten and eighth grade (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007). The new approach would embed algebra throughout the students' school experience (Katz, 2007).

As educators searched for strategies to climb the global ladder, some turned their focus toward the small island of Singapore (Garelick, 2006) to strengthen the American math program. Singapore began to garner world-wide attention when its fourth and eighth grade students were ranked first in the world for three consecutive TIMSS assessments 1995, 1999, and 2003 (Leinwand & Ginsburg, 2007) and remained in the top three in the assessments in 2007 and 2011. According to the U.S. Department of Education (2009a), in each of the three tested years, Singapore beat out as many as 49 international competitors.

The repeated success of Singapore was not an accident. There were many factors that contributed to Singapore's global reign in math (Leinwand & Ginsburg, 2007). Singapore's math curriculum was reformed in the early 1990s to better emphasize problem solving. Its curriculum was also a coherent and national curriculum that was followed in all of its schools. There was also less of a focus on content coverage, in terms of quantity, in a year. Students gained a deeper understanding and mastered topics in greater depth, and topics built on each other as the student progressed through school (Abdul-Alim, 2006; Curriculum Review, 2010; Garelick, 2006; Leinwand & Ginsburg, 2007).

In U.S. schools that adopted the Singapore program, math typically lasted 60 to 90 minutes daily in elementary classrooms, first through sixth grade, and followed a prescriptive path to lead the students to mathematical success. Classes began with mental math problems that focused on strengthening the foundation upon which the program was built. The students used a technique known as model drawing to solve word problems. This method helped students to visualize problems, as well as introduced them to algebraic and geometric concepts (Institute of Education Sciences, 2009).

Another factor of Singapore's success lay within the richness of the problems its text provided. Leinwand and Ginsberg (2007) found that students were often expected to complete multistep problems, find intermediary values before the solution, and apply various skills. Too often, American curriculum and texts did not push students to learn, thus depriving them of important opportunities to learn and succeed at math.

Statement of the Problem

As the world became more competitive in STEM-related areas, American educators were looking for ways to bolster their curriculum. Ball et al. (2005) contended that the United States was unlikely to succeed on the global stage unless factors of other countries' success, such as Singapore, were examined. As more schools were striving to make improvements, putting forth more effort than effect (Ball et al, 2005), students still were enrolling in universities unprepared (ACT, 2011b; Dervarics & O'Brien, 2012; National Science Foundation, 2013). In Maryland, for example, it was reported that 49% of students had to enroll in a remedial math class for no credit before being able to enroll in math courses for credit (Ganem, 2011, p. 1), and the Center for Public Education found that almost two-fifths of graduates country-wide were not prepared for entry-level jobs or college-level courses (Dervarics & O’Brien, 2012). The achievement gaps that continued to plague school districts were also alarming. Disparities on achievement tests existed between male and female students (Georgiou, Stavrinides, & Kalavana, 2007; Perry, 2012), Black and White students (Burchinal et al., 2011; Riegle-Crumb & Grodsky, 2010), and students from impoverished and wealthy families (Goldberger & Bayerl, 2008).

It is unreasonable to assume that students in Singapore were inherently better at math than American students, so there must be something in the Singapore system that was better than the system used in the United States (American Institutes for Research, 2011). The focus on higher-level mathematics in U.S. high schools was essential to stay competitive in the global market. Students who passed Algebra II in their high school courses were more than four times more likely to graduate college than their peers (Dervarics & O'Brien, 2012). To address the gap between elementary, high school, and college, the NCTM proposed embedding algebraic topics in the elementary level (Ketterlin-Geller et al., 2007). In order to make these effective, new elementary teachers needed to receive more intense training in math. Ball et al. (2005) explained that a strong mathematics curriculum and framework were essential to the growth of students, and in order for teachers to effectively instruct and assess student understanding, teachers also required a deep understanding of mathematics.

The Singapore Math program took the idea of embedding algebra and higher level concepts at the elementary level one step further by providing teachers and students the tools to make the math relevant. Leinwand and Ginsburg (2007) found that students in early elementary classrooms may use concrete objects such as counters to introduce variables, while older students used visual aids. All students in the program utilized the model drawing technique which provided a reliable process to solve a myriad of problems (Hoven & Garelick, 2007). The Singapore Math program utilized a combination of curriculum, high quality teachers, and textbooks to develop students' problem-solving skills; which was a necessity to thrive in the 21st century (Clark, 2009).

Employers, too, were expecting workers to transfer work-related problems from the classroom. They needed to be able to calculate discounts and solve complex problems. Math could also be helpful in everyday life. Using math, one could apply formulas to determine potential profits in a business venture or calculate the average miles per gallon of gas. As such, it was important that students were able to make a connection with the math, rather than see concepts as disconnected facts (Burns, 2007).

Purpose of the Study

The purpose of this study was to determine if student achievement and performance in one school district in the area of mathematics had improved since the implementation of Singapore Math at the elementary level. Data were collected and analyzed from the Missouri Assessment Program (MAP) test. Data were examined comparing two cohorts of randomly sampled students in the district who had different exposure to Singapore Math in grades 1 and 2. Cohort A consisted of a random sample enrolled in the district and in the third, fourth, and fifth grade in the spring of 2007, 2008, and 2009, respectively. These students did not participate in the Singapore Math program in grades 1 and 2. Cohort B consisted of randomly sampled students enrolled in the district and in the third, fourth, and fifth grade in 2010, 2011, and 2012, respectively. These students participated in the Singapore Math program beginning in first grade. The researcher also analyzed three randomly sampled student subgroups of Cohort B: female students, Black students, and students with free and/reduced lunch status to determine whether or not scores among the subgroups were commensurate with Cohort B's sample.

The implementation of the Singapore Math program in the researched school district began in 2007. The district addressed concerns, questions, and professional development from administrators, teachers, and parents to ensure the success of the program. Prior to implementation, the teachers' professional development included a weeklong training in the summer. Additionally, the district provided four training sessions during the school year. Training included the use of manipulatives, instructional strategies, utilization of the text books, and the scope and sequence of the program in first through fifth grade. Since its launch in 2007, every elementary teacher in first through fifth grade in the researched school district received ongoing training on Singapore Mathematics methods.

In an effort to help parents with the new and different techniques that Singapore Mathematics utilized, each school in the district held parent education nights during the first two years of implementation. In addition, videos were created and posted to the district's website on specific topics in various grade levels. In many classrooms, upcoming topics and examples were provided through weekly newsletters.

Research Questions and Hypotheses

Does the use of Singapore Mathematics improve the understanding of mathematics in elementary students in the studied district, as measured by the MAP test?

Hypotheses:

Hypothesis 1: The MAP test index scores of students in Cohort A will increase from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

Hypothesis 2: The MAP test index scores of students in Cohort A will increase from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

Hypothesis 3: The MAP test index scores of students in Cohort B will increase from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

Hypothesis 4: The MAP test index scores of students in Cohort B will increase from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

Hypothesis 5: Cohort B's MAP scores will increase in grades three, four, and five when compared to Cohort A's MAP scores in grades three, four, and five.

Hypothesis 6: There is a difference between average math MAP test scores of female students and students in Cohort B.

Hypothesis 7: There is a difference between average math MAP test scores of Black and non-Black students.

Hypothesis 8: There is a difference in average math MAP test scores of students on Free and Reduced Lunch and students who are in Cohort B.

Limitations

Area of Study. This cohort study was limited to two groups of students from one school district from 2007 to 2012. Results may have been different if more areas and districts were included in the study.

Sample Size. Fifty students were randomly chosen to make up each cohort. Additional random samples were pulled to compare data for Black students, students with free or reduced lunch status, and female students; 30 students, 50 students, and 50 students respectively. Only 30 Black students were randomly pulled due to the lower number of third, fourth, and fifth grade Black students enrolled in the district. Population sizes from which the samples were taken were limited because of the number of students in each cohort that attended all three grade levels (third, fourth, and fifth) in the studied school district. Since the sample size was relatively small, the data may not necessarily be reflective of a general pattern if the study is replicated elsewhere. Also, students who took the alternative MAP assessment (known as MAP-A), were not included in the study, because their curriculum did not necessarily follow the student's grade level expectations, and the Singapore Mathematics program was not utilized in those settings.

Researcher Bias. At the time of this writing, the researcher was an Assistant Principal at one of the schools in the studied district, and was part of the implementation initiative of the Singapore Mathematics program. The data given to the researcher was scrubbed of identifying information, such as name and school in an effort to ensure anonymity of students.

Data. This study utilized data collected from the math section of the MAP test. It is acknowledged that one piece of data does not gain a full comprehensive look at mathematical progress. Data from district common assessments was not available for Cohort A, and was limited from Cohort B. In an effort to strengthen the reliability of the data, three different analyses were utilized in the study.

Definition of Terms

Common Core State Standards (CCSS). The set of curricular standards created and adopted by a majority of states in the U.S. to align curriculum with college and career expectations. The CCSS created a clear and consistent framework and addressed curriculum specifically in Mathematics and English Language Arts (CCSS, 2012a). Students will be assessed on these standards beginning in the 2014-2015 school year (Smarter Balanced Assessment Consortium, 2012).

The Department of Elementary and Secondary Education (MODESE). The Administrative arm of the State Board of Education. MODESE regulated and monitored public school districts to ensure that students were able to receive a high quality education (MODESE, 2012a).

International Association for Evaluation of Educational Achievement (IEA). An independent, cooperative research institution comprised of international government and research institutions. The group was based out of Boston and was responsible for conducting the TIMSS study every four years to participating countries (International Association for the Evaluation of Educational Achievement, 2012).

Missouri Assessment Program (MAP). Missouri's annual state-wide augmented norm-referenced assessment that measured the proficiency of students in the areas of Mathematics and Communication Arts in third through eighth grade (MODESE, 2012b).

National Assessment of Educational Progress (NAEP). A national assessment given periodically in mathematics and other subjects and used as a common metric to gauge the achievement of students across all states (National Center for Education Statistics [NCES], 2014).

National Council of Teachers of Mathematics (NCTM). A public organization for mathematics teachers that provided support through professional development opportunities, research, and publications (NCTM, 2012).

Singapore Math. The mathematics series was initially developed in Singapore by the Ministry of Education in 1981 to specifically address the curricular goals in Singapore. In 1998, it was adapted and distributed in the U.S. for American students (Singapore Math, Inc., 2006). The study was conducted utilizing the American adaptation of Singapore's mathematics program. The books used at the elementary level were titled Primary Mathematics Series (Singapore Math, Inc., 2006).

Traditional Math Program. A program which followed the belief that students needed to learn basic math facts and algorithms with automaticity in order to understand more difficult and abstract concepts (What is traditional math?, 2012). There were a wide variety of publishers that wrote books that addressed many states' curricular goals, as well as the Common Core State Standards.

Trends in International Math and Science Study (TIMSS). An international test given to fourth grade and eighth grade students in order to measure proficiency in math and science. TIMSS data was collected every four years since 1995 (NCES, 2013a).

Summary

As American educators improved their mathematics programs and prepared students to become more marketable and competitive on a global scale by addressing gaps and overlaps in curriculum, as well as building a strong mathematical foundation, it was necessary for educators to examine researched successful mathematics programs. The focus of this study was on a country's program that experienced ongoing success in international mathematics testing since 1995. American students must experience success in higher-level math courses in order to compete globally. To achieve success in higher-level math courses, students at the elementary level required a solid foundational understanding of mathematical concepts and skills to support the abstract thinking required in advanced mathematics courses. Singapore's repeated top-world ranking in mathematics warranted a closer look at the strategies it employed to create successful students, and educators utilized those same strategies in order to keep American students mathematically competitive.

The intent of this study was to gauge the effectiveness of the Singapore Mathematics program in the elementary school setting. Since the world was becoming more competitive and jobs requiring higher-level math skills were in demand, educators in the United States recognized the need to strengthen practices in mathematics to help students become more proficient at math, thus helping them to be better prepared for challenges in the 21st century.

This study focused on determining whether there was an increase in students' performance on the MAP test since the implementation of Singapore Mathematics in the studied school district. The study utilized data from 2007 to 2012 in a district where Singapore Mathematics was employed in the first through fifth grades. Two groups of students were investigated to examine the effectiveness of the program based on the number of years of student participation in the Singapore program. Cohort A (2007-2009) was introduced to Singapore Math in the third grade. Cohort B (2010-2012) consisted of students who participated in the program since the first grade. Prior to district implementation and throughout the year, elementary teachers in this district received training on Singapore Mathematics. New teachers then received training at the beginning of the school year and intermittently throughout the year. The questions answered in the study spoke directly to student achievement in mathematics, which was determined by the comparison of MAP scores between the two cohorts.

Chapter Two focuses on background information of both the U.S. and Singapore in the area of mathematics performance, educator preparation, population, and classroom environment in order to orient the reader to the educational contexts of each country. Then, the three subgroups of students examined in the studied school district were explored: gender, minority, and poverty. These groups of students have been identified as being part of the achievement gap in American education. Next, components, strategies, and structures of the Singapore Mathematics system that have drawn the attention of Americans were explored. Finally, the chapter addresses how the Singapore

Mathematics program supported the Common Core State Standards (CCSS), Algebra, and how an understanding of mathematics helps students well beyond school.

Chapter Two: The Literature Review

Chapter two includes a review of literature that juxtaposes the features of the American education system and the features of Singapore's education system. Specifically, a historical background in mathematics, the importance of educator preparation, population and diversity, and teacher and classroom statistics of each of the two countries are discussed. There are three specific factors that have an effect on student performance in America: .gender, ethnicity, and poverty. These factors are considered part of the achievement gap in America and are discussed next as American schools work to eliminate the achievement gap. The components, framework, organization, and details of the Singapore Math program are explored. Finally, discussions on how Singapore Math addresses the CCSS, upper level mathematics courses, and the importance of mathematics in today's economy are explored.

Historical Background in Mathematics

There were many factors that contributed to a country's success in mathematics, such as teacher preparation, length of school year, quality of instruction, and programs used (Cavanagh, 2012). It was acknowledged that a mathematics program alone could not be implemented and improve or sustain a country's performance. Therefore, the background, progress, and status of the United States and Singapore's educational systems are addressed.

U.S. historical background in mathematics.

At the time of this writing, competing in a global market was the focus of American education, and there was an awareness that American students in elementary and secondary schools were falling behind their international peers (Cavanagh, 2012;

Garelick, 2006; Sami, 2012). A key element in the discussion was the quality of schools, as measured by the testing of mathematics skills (Lee & Fish, 2010). Every four years since 1995, American students participated in the TIMSS. The results consistently demonstrated that eighth graders in the United States ranked below many other industrialized countries (Garelick, 2006), but demonstrated more recent improvement by entering the top 10 in 2007. Table 1 shows how the United States' fourth and eighth graders ranked in the mathematics portion of TIMSS.

Table 1.

United States' Results on the Mathematics Section of the TIMSS

	Fourth Grade		**Eighth Grade**	
Year	Average Score	World Rank	Average Score	World Rank
1995	518	12	492	21
1999	Not Available	Not Available	502	19
2003	518	12	504	15
2007	529	11	508	9
2011	541	11	509	9

Note. The study in 1999 only collected data from eighth graders, so no data were available for fourth graders in that study. From the NCES (2013a).

Examining the scores by subtests, and focusing in on the algebra subtest, American students scored below many global competitors (Ketterlin-Geller et al., 2007) and students in underdeveloped countries (Sami, 2012). Dixon (2005) found that only about 9% of American students reached the 90th percentile on the math portion of the test on the 2003 administration of the assessment. In 2011, the TIMSS data showed 4% increase for fourth grade students, but decreased 2% for American eighth graders (NCES, 2013a).

Singaporean historical background in mathematics.

Much of the attention drawn to the Singapore Math program stemmed from the success of Singapore's students on the international testing stage (Garelick, 2006; Leinwand & Ginsburg, 2007). However, prior to its success in the early 1990's, Singapore's math and science achievement was mediocre (Leinwand & Ginsburg, 2007). The country's poor performance served as the catalyst that drove national efforts to boost Singapore's success in mathematics (Leinwand & Ginsburg, 2007). In three consecutive TIMSS administrations in 1995, 1999, and 2003, Singapore rose to the top of the ranks beating out math powerhouses such as Japan and Taiwan (Garelick, 2006). Singapore continued to maintain its high performance with scores that placed their students among the top three ranking countries in 2007, and in 2011. The fourth grade students regained the top position while the eighth grade students' scores came in second just behind the Republic of Korea (Mullis, Martin, Foy, & Arora, 2013; U.S. Department of Education, 2009a).

The country of Singapore not only out-performed many countries, but Bhattacharjee (2004) found that Singapore students rose to the top ranks out of as many as 49 countries. According to the TIMSS Report, 46% of Singaporean students reached the top 10% in terms of test scores on the math test in 1999 (Dixon, 2005, p. 626), and in 2011, 48% of Singaporean students scored in the top 10% of all TIMSS test takers (NCES, 2013a), while only 1% of students were designated as low performers on the test (Mullis et al., 2013). Ng (2001) found that some of this improvement occurred because Singapore was intentional in aligning its curriculum with the TIMSS assessment.

Mathematics Curricula

The U.S. and Singapore had very different curricula. Curricula in the U.S. was not nationally prescribed, while the curricula in Singapore was nationalized and the basis of the Singapore Math program. Therefore, an examination of curricula follows from both countries.

Mathematics curricula in the U.S.

The U.S. did not have a national curriculum. The U.S. Constitution protected the rights of each state to oversee its own education (Dennis, 2000). As such, there was not a rigorous central core of standards, which would provide a focus for states to follow (American Institutes for Research, 2011; Wang-Iverson, Myers, & Lim, 2009; Yang et al., 2010). The 2001 NCLB Act required that states adopt more challenging academic standards in math that specified what the learner should be able to know and able to do. Kauerz and McMaken (2004) found that at the heart of NCLB was the requirement of Adequate Yearly Progress (AYP). AYP required all states in the U.S. to ensure that all students progressed each year and were considered to be at least ‘proficient’, as measured by each state’s standardized test by 2014. Curricula were to contain coherent and rigorous concepts and support the teaching of advanced skills. There was no commonality among states regarding what content to teach at a particular grade level (Reys et al., 2006) which meant that mathematical proficiency was measured differently from state to state (Kauerz & McMaken, 2004). This inconsistency led to a lack of college or career preparedness (Barth, 2003; Sloan, 2010).

In 2000, the NCTM, during the Standards 2000 Project, developed a set of standards, which contained two components for the instruction of mathematics: Content and Process (Leinwand & Ginsburg, 2007). The NCTM Standards were used at a national level for many states to develop coherent and rigorous curricula. Missouri was one of many states that employed the NCTM Standards as the basis of Grade Level Expectations for mathematics (MODESE, 2014a).

As student achievement improvements moved at a snail's pace, Bandeira de Mellow (2011) and Reys et al. (2006) reported that learning expectations for mathematics varied greatly from state to state. Inconsistencies in curricula continued, as out of the 52 entities in the United States, including the 50 states, the District of Columbia, and the Department of Defense Education Activity, only 37 identified specific Grade Level Expectations (GLEs) for each grade level. Six states established GLEs in some grade levels, and eight states established 'grade band' documents, which were expectations for groups of grades such as Kindergarten through fourth grade or fifth through eighth grade (Reys et al., 2006).

In 1990, an assessment called the National Assessment of Educational Progress (NAEP) was given nationwide to a sampling of fourth, eighth, and 12th grade students (Hughes, Daro, Holtzman, & Middleton, 2013). It was the first assessment of its kind to take a national sampling of students and accurately compare the results between states (Grissmer, Flanagan, Kawata, & Williamson, 2000). Schmidt (2004) contended that discrepancies between state curriculums could have an effect on achievement between states. Table 2 illustrates the variety of proficiency on the mathematics section of the NAEP test that was given in 2013.

Table 2.

Average Scores and Achievement: NAEP Mathematics for Fourth Grade Students; 2013

	State/Jurisdiction	Average Score	% Below Basic	% Basic	% Proficient	% Advanced
Percentage of students achieving proficient or advanced is higher than national public 4th graders	Minnesota	253	10	31	44	16
	New Hampshire	253	7	34	46	12
	Massachusetts	253	10	32	43	16
	Indiana	249	10	38	42	10
	Vermont	248	13	36	41	11
	Colorado	247	13	37	39	11
	New Jersey	247	13	38	39	10
	Washington	246	14	38	38	10
	North Dakota	246	11	41	41	7
	Ohio	246	14	37	38	10
	Wyoming	247	10	42	41	7
	Iowa	246	13	39	38	9
	Kansas	246	11	41	40	8
	Maine	246	12	40	39	9
	Virginia	246	12	40	38	9
	Wisconsin	245	15	38	38	9
	Maryland	245	18	36	33	13
	Hawaii	243	17	37	37	9
	North Carolina	245	13	42	37	8
	Connecticut	243	17	38	36	9
	Montana	244	14	41	38	7
	Nebraska	243	16	40	37	8
Percentage of students achieving proficient or advanced is not significantly different than national public 4th graders	Pennsylvania	244	15	41	36	8
	Utah	243	17	39	36	8
	Rhode Island	241	17	40	35	7
	Delaware	243	14	44	35	7
	Kentucky	241	16	42	35	6
	Texas	242	16	43	34	7
	Florida	242	16	43	34	7
	South Dakota	241	16	44	35	5
	Tennessee	240	20	40	33	7
	Oregon	240	19	41	33	8
	Idaho	241	17	43	33	6
	New York	240	18	43	33	7
	Arizona	240	18	42	32	7
	Georgia	240	19	42	32	7
	Arkansas	240	17	44	34	5
	Illinois	239	21	39	31	8
	Missouri	240	17	44	33	5
Percentage of students achieving proficient or advanced is lower than national public 4th graders	Michigan	237	23	40	30	7
	Alaska	236	23	40	30	6
	Oklahoma	239	17	46	32	5
	West Virginia	237	19	46	31	4
	Nevada	236	20	46	30	4
	California	234	26	41	27	5
	New Mexico	233	26	43	27	4
	Alabama	233	25	45	26	3
	DC	229	34	39	22	6
	Louisiana	231	25	48	24	3
	Mississippi	231	26	48	24	2

Note. DoDEA = Department of Defense Education Activity; DC = District of Columbia. Adapted from DOE, 2014

The states and jurisdictions were ranked in descending order by the percentage of students attaining proficient or advanced and are grouped as being significantly higher than, not significantly different, or significantly lower than the national average.

In 2009, a draft form of a common set of standards was introduced to the states. They were internationally benchmarked and were developed to assist students in becoming better prepared for postsecondary education or the workforce (Sloan, 2010) and addressed the curriculum-expectation gap that had grown amongst states (Achieve, 2010; Kober & Stark-Rentner, 2011). In June 2010, Missouri, along with more than 40 states, adopted these common set of standards, which became known as the CCSS (ASCD: Common Core State, 2013; MODESE, 2010; MODESE, 2013b).

Proponents of a common curriculum among states believed it should be the priority of the education system, so that rigorous expectations could be the same for American students from coast to coast, rather than vary depending on where they lived (Schmidt, 2004). And, if well executed, such standards could produce improvement in mathematics achievement (Schmidt 2004) to be competitive with the best in the world (NCTM, 2014). Secretary of Education Duncan corroborated their findings when he stated at the 2009 National Press Club:

> When children are told they are ‘meeting a standard,’ the logical assumption for that child or for that parent is to think they are on-track to be successful. But because these standards have been dummied down and lowered so much in so many places, when a child is ‘meeting the state standard’ they are in fact barely able to graduate from high school (U.S. Department of Education, 2009b, p. 1).

Opponents to a nationalized mathematics curriculum, such as Stotsky (2013) were concerned that a one-size-fits-all curriculum would not enhance the performance of states and their students, but rather diminish it in states that had high performance on such tests as the NAEP and lead to lower outcomes for all. McCluskey (2010) and Usiskin (2007) found that some of the highest performing countries on the 2007 TIMSS had a national mathematics curriculum, but so did 11 of the 12 lowest performing countries. Usiskin (2007) went on to state that the variance in state standards was evidence that even the most intelligent people could not agree on what math concepts were most important to teach or when to teach them. Schmidt (2004) found that there were many ways to sequence topics in mathematics, but topics could not be placed arbitrarily together. If the sequence did not follow a logical path set by the cumulative nature of mathematics, then concepts would become a list of items that students memorized for tests, but soon forgot.

Mathematics curricula in Singapore.

A national framework was the first structure put in place to improve mathematical achievement in Singapore. Singapore's curriculum was based upon this framework that contained and described necessary "concepts, skills, processes, attitudes, and metacognition" in order for students and teachers to be successful in math (Leinwand & Ginsburg, 2007, p. 32). The Singapore mathematics curriculum was well-defined (Gross, 2009), and the elements of the math curriculum were intentionally designed to support one another (Leinwand & Ginsburg, 2007). In the Singapore system, concepts were learned to mastery (American Institutes for Research, 2011). Singapore equally weighted content and process, placed problem solving at the heart of the framework, and included five necessary components to create connections and integrate the goals of the curriculum (Leinwand & Ginsburg, 2007; Ng, 2001). Singapore's framework curriculum becomes increasingly more advanced in detail in each successive grade (Institute of Education Sciences, 2009). Figure 1 illustrates Singapore's nation-wide math curriculum (Clark, 2009).

How educators were prepared to instruct students was unique to each country. In this section, the importance of a strong educator preparation program was reviewed. Additionally, U.S. and Singaporean teacher preparation strategies were contrasted by way of coursework, student teaching requirements needed to acquire an elementary education degree, and differences in professional development for in-service teachers.

Educator preparation in the U.S.

In undergraduate and graduate math preparation, teachers should have developed confidence in an intelligible mathematical knowledge base and been able to articulate it to students effectively (Langfield, 2000). Mathematical knowledge was essential to the success of reform-based classes and education programs needed to address changes to content and pedagogy effectively to prepare teacher candidates for the classroom (Kajander, 2010). Tsui (2007) contended that in United States' classrooms, many elementary teachers did not have sound enough academics to teach mathematics. Elementary teachers often did not have specialized instruction with mathematics and lacked the understanding and skill base to teach mathematics effectively (Ketterlin-Geller et al., 2007).

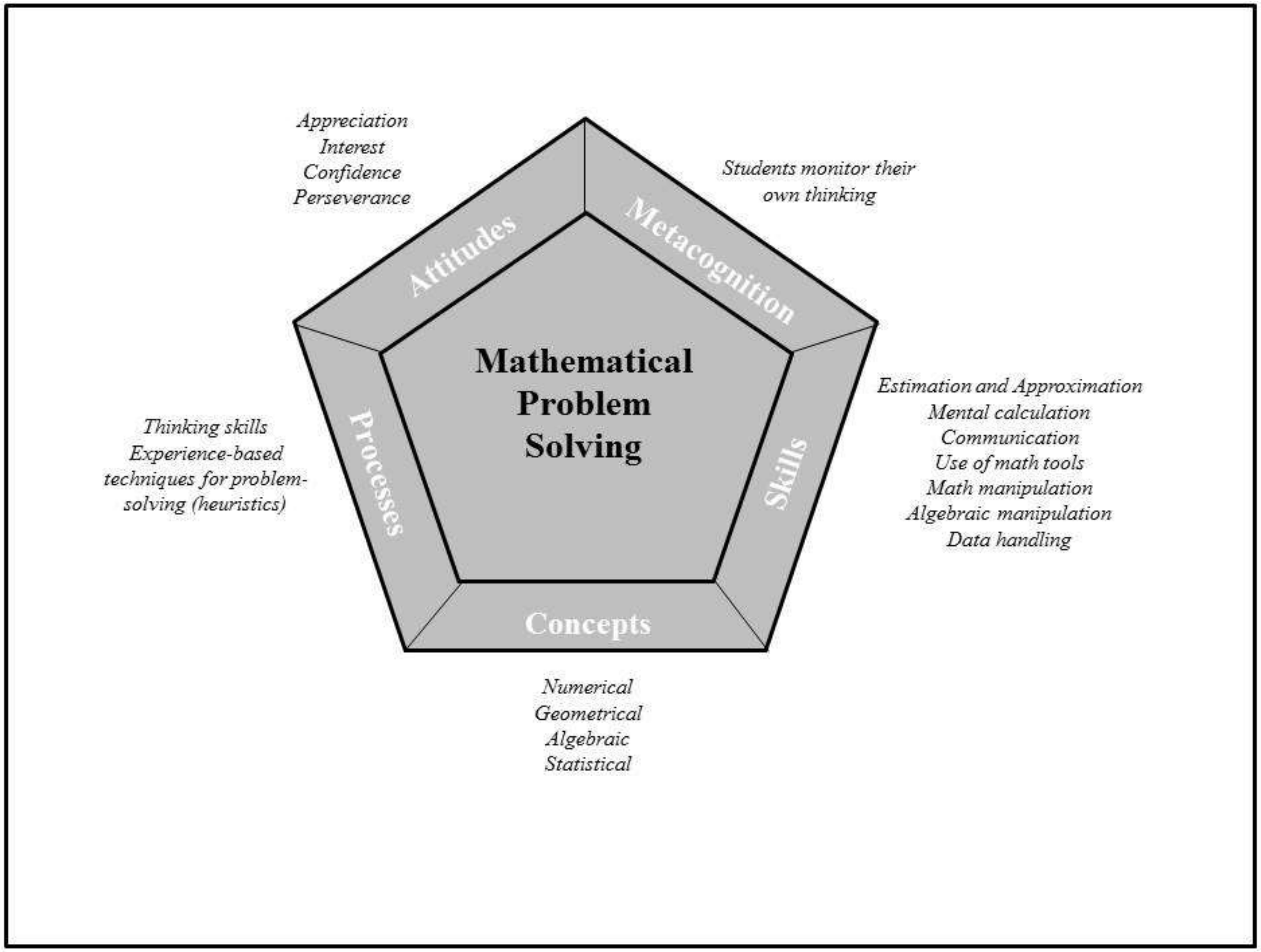


Figure 1. The pentagonal representation of Singapore's math curriculum. Adapted from Clark, 2009, p. 1.

Elementary teachers were being outperformed by teachers internationally in the understanding of mathematical concepts as well as their instruction of math (Tsui, 2007). At the elementary level, it was especially evident that mathematics training was urgently needed for teachers (Ketterlin-Geller et al., 2007).

To earn a teaching degree in America the requirements for elementary, middle school, and high school varied. At the elementary level, teachers were required to teach all core content areas and there was not a specific elementary math undergraduate degree available to be earned (Education Portal, 2011). The mathematics requirement for an elementary teacher in Missouri, according to MODESE (2014b), was a minimum of two math classes and a mathematics pedagogy course. An elementary teacher could, if they chose, have an area of concentration if they took at least 21 semester hours in a particular content (MODESE, 2014b). Pre-service teachers typically ended their coursework with a minimum of eight semester hours of student teaching (MODESE, 2014b).

Finally, an advanced specialist degree in mathematics was available for teachers who wished to specialize in the instruction of mathematics. The state of Missouri required the program to have 24 hours of coursework, which included four components: mathematical content knowledge, pedagogy content of mathematics, foundations of leadership, and clinical experience (MODESE, 2014c).

Educator preparation in Singapore.

The Singaporean educator program had high expectations for their in-service teachers (Prystay, 2004), and in Singapore, teaching was considered a very honorable and desired profession ("Strong performers and successful reformers in education", 2013). The Pearson Foundation found that the top third of each cohort of students was selected by panels to become teacher candidates (“Strong performers and successful reformers in education”, 2013). These candidates needed to possess the character, ability, and skill in the field to teach and develop students. If selected, prospective teachers attended Singapore’s National Institute of Education (NIE), which ensured that all teachers received the same rigorous education in pedagogy and mathematics (Jackson, 2010a). It was essential for Singaporean teachers to have a solid base in numbers, algebra, geometry, functions, statistics, probability, and trigonometry in order to be prepared to teach courses (Langfield, 2000). In exchange for their time for training, prospective teachers received fully-paid tuition at NIE and a monthly stipend commensurate with graduates from other fields (“Strong performers and successful reformers in education”, 2013). After they completed their teacher training, they were required to commit to the teaching field for at least three years (National Center on Education and the Economy [NCEE], 2013).

Curriculum was supported by ongoing professional development that kept all educators informed of the content and best practices that led to student achievement (Gross, 2009). There was an emphasis on a higher level of training for teachers in Singapore (Ezarik, 2005). Teachers logged 100 hours of professional development each year at no cost to the teacher (“Strong performers and successful reformers in education”, 2013). The Ministry of Education recognized that teachers could not successfully implement the program by flipping to the back of the book to find the answers. They understood that educators must take the time to learn how to effectively implement the program and have an understanding of math, as well (Prystay, 2004). Teachers in Singapore received help from mentors during the first two years of teaching and were given many opportunities to collaborate and grow professionally (“Strong performers and successful reformers in education”, 2013).

Population and Diversity

Population and the diversity of the population in each country varied. These differences are discussed due to their possible effects on implementation of programs, student performance, and decision-making as it related to education. This section will explain aspects of population and diversity in the U.S. first, followed by an explanation of population and diversity in Singapore.

Population and diversity in the U.S.

The United States’ population was approximately 317 million people, and increased by one person roughly every 14 seconds. The U.S. was currently the third most populous country in the world behind China and India (United States Census Bureau, 2013). The minority population of the United States was estimated at 35% of the total population in the 2010 census (Santa Cruz, 2010, p. 1) and a 2013 press release by the U.S. Census Bureau stated that while English was the national language, nearly 11% of people spoke Spanish, 0.75% spoke Chinese, and about .50% of the people spoke French. According to the Census Bureau (2013) in 2011, in more than 20% of homes, a language other than English was spoken and, the diversity of languages was expected to rise over the next 50 years in the U.S.

Population and diversity in Singapore.

The country of Singapore was an island located south of Malaysia, roughly three and a half times the size of Washington, DC (Central Intelligence Agency, 2013), with a population of about 5.4 million people (*Singapore Demographics Profile*, 2013, p. 1).

Singapore's population was 24% Indian or Malay while the majority of the population, about 76%, was Chinese (Central Intelligence Agency, 2013; Leinwand & Ginsburg, 2007). While there were some fluctuations in the population, the diversity proportions was roughly the same since around 1900 (Dixon, 2005). Index Mundi estimated that the population growth rate was at 1.993% ("Singapore Demographics Profile", 2013).

Singapore was considered a bilingual country (Dixon, 2005), with 35% of the population speaking Chinese (Mandarin) and 23% speaking English. Singapore also had two other official languages amongst the eight languages spoken: Malay and Tamil (Central Intelligence Agency, 2013). The government mandated in 1966 that math and science classes be instructed in English starting in first grade (Dixon, 2005). Therefore, English was considered to be the working language in Singapore (Central Intelligence Agency, 2013). The policy mandating bilingualism in Singapore proved successful on international tests in which the bilingual students performed as well as, or better than, monolingual students in monolingual countries. (Dixon, 2005). As the country had no natural resources to speak of, politicians reminded the people that the development of its citizens through education was the most important resource in the country (Ng, 2001).

Teacher and Classroom Statistics

The public education classroom environment was another difference between the U.S. and Singapore. For the purpose of the literature review, the classroom environment included the number of students in the classroom, average salary, percentage of males and females in the field of education, and percentage of teachers that earned more than a bachelor's degree. This section will first explain teacher and classroom statistics from schools in the U.S., then provide an explanation for schools in Singapore.

Teacher and classroom statistics in the U.S.

Elementary classrooms in the United States varied in size, but according to the Institute of Education Sciences (2013), the student-to-staff ratio was roughly 15:1 in 2009. The average elementary public school teacher in America had taught for 14 years and made $53,100 per year (NCES, 2013b, p. 3). *The Economist* found that bonuses for U.S. public education teachers were not widespread, but were being experimented with in large cities, such as Boston, Baltimore, Cincinnati, Cleveland, and Newark (Bonus Time, 2012). The study also showed that just over three-fourths of the more than 3 million American teachers were female and that about 52% of teachers had attained a master's degree or higher. Newark teachers could receive a bonus up to $12,000 per: $5,000 for student achievement, $5000 for working in low-performing schools, and up to $2,500 for teaching a subject that was a difficult position to fill (Bonus Time, 2012, para. 1).

Teacher and classroom statistics in Singapore.

Elementary, or primary classes as they were called in Singapore, were often comprised of 30 to 40 students, and the schools maintained an overall teacher-to-student ratio of 1:18 in (Ministry of Education: Singapore, 2013b, p. 1). There was no mandate for maximum class sizes, but Singapore's Ministry of Education (2013b, p. 1) stated that they were trying to decrease the teacher-to-student ratio to about 1:16 by 2015, though the Ministry had not found conclusive evidence that smaller class sizes were beneficial. They stated that smaller class sizes should ensure that schools had necessary resources and that learning could be better organized to meet the needs of their students (Ministry of Education: Singapore, 2013a).

According to the National Center on Education and the Economy (NCEE, 2013), the average annual salary for 25 to 29 year-old teachers in Singapore was $43,563 and successful teachers could earn retention and/or performance bonuses of up to 30% of their salary through high performance on their teacher evaluations. Data from the Encyclopedia of the Nations (2013) showed that about 81% of the teachers in Singapore were female in a population of more than 29,300 educators (Ministry of Education: Singapore, 2013a, p. 1) and the Ministry of Education was looking to increase the teacher population by more than 4,000 in the two to three years following the 2013 report (Ministry of Education: Singapore, 2013b, p. 1).

Factors Affecting Student Performance in the U.S.

There were considerable differences in performance on national and state mathematics assessments between groups of students. This was often defined as the achievement gap. This gap was visible in grades, standardized tests, dropout rates, and course selection, among other areas (Education Week, 2011), and was battled in U.S. classrooms for decades (Education Commission of the States, 2014). Though the origins of achievement gaps among subgroups of students remained hotly debated, they indisputably existed (Burchinal et al., 2011).

The Education Commission of the States (2014) cited some of the factors that they found contributed to the achievement gap, including racial background, economic background, education level of the student's parents, access to high-quality preschool, and curricular and instructional quality. Since these subgroups had not performed at a level commensurate with their peers, NCLB, passed in 2001, required accountability in closing the gap. Since that time, data was disaggregated to identify the subgroups and create awareness and interventions to close the gap nationwide (Education Week, 2011).

According to MODESE’s (2013c) description of a subgroup in Missouri, a student could be a member of a subgroup if they received free or reduced lunch, had a racial or ethnic background, were an English Language Learner, or was a student with a disability. In the studied school district, factors such as gender, ethnicity, and poverty were considered significant subgroups in the testing data and were included in the research study data.

Gender gap.

As the need for graduates pursuing careers in STEM increased, so too did the focus on gender roles and sexism, or the gender gap, in academic achievement between males and females (O'Shea, Heilbronner, & Reis, 2010). The term ‘gender gap’ purported the favoring of males in mathematics and was evidenced by scholars as early as kindergarten and widened as students grew older (Li-Grining, Votruba-Drzal, Maldonado-Carreno, & Haas, 2010; Penner & Paret, 2008; Robinson, Lubienski, & Copur, 2011). While there was a vast number of factors that could be attributed to the gender gap, it was important to investigate them, because could lead to interventions which could boost female performance and attitudes towards math (Gunderson, Ramirez, Levine, & Beilock, 2012), as the large and consistent gap in mathematics achievement between males and females continued to exist (Georgiou, et al., 2007; O'Shea et al., 2010). A study in 2011 by Robinson et al. showed that by the end of third grade, there was a male advantage of about 0.25 standard deviations, and differences in Scholastic Aptitude Test (SAT) scores increased to 0.3 standard deviations (O'Shea et al., 2010; Perry, 2012, p. 2).

Higher performing females, those scoring 700 or above, only comprised 36.5% of the population on the SAT in 2010 (O'Shea et al., 2010, p. 237). These trends were uninterrupted since 1972 (Perry, 2012, p. 2; Tsui, 2007, p. 2). On the fourth grade version of the TIMSS 2011, American girls scored an average of nine points lower than their male counterparts, a designation worthy of ‘statistically significance’ by the International Association for the Evaluation of Educational Achievement (IEA) (Mullis et al., 2013).

The achievement gap had deeper implications than scores on high stakes assessments. It was affecting women’s admissions into competitive colleges and universities (O'Shea et al., 2010) because lower self-confidence and lack of interest in math tended to lead female students to take fewer math classes than male counterparts, which resulted in lower performance on math tests and entrance exams (Tsui, 2007). Furthermore, disparities led to disproportions and the under-representation of women in mathematics and engineering fields where women held only 18% of jobs (O'Shea et al., 2010; Robinson et al., 2011) and less than one third of doctoral degrees in chemistry, math, physics, computer sciences, and engineering (National Science Foundation, 2013; O'Shea et al., 2010).

One factor of the gender achievement gap was anxiety towards math. When females had anxiety towards math, their performance in math tended to be lower (Gunderson et al., 2012). Teachers may inadvertently have reinforced anxiety amongst females by giving males more opportunities to respond to higher-order questions and learning opportunities, while giving direct help or even completing tasks for female students (O'Shea et al., 2010). When student anxiety was coupled with a female teacher who had math anxiety, their achievement was lower still (Gunderson et al., 2012). Gunderson et al. (2012) went on to state that, on the other hand, a female student may also be intimidated by a female teacher who was good at math because the teacher's confidence was threatening and the students were worried about letting their teacher down.

Another factor was the long-standing stereotype that males were inherently better at math than females and that students were susceptible to that stereotype (Gunderson et al., 2012; Tsui, 2007). While teachers were not credited for creating such a stereotype, their non-verbal and verbal behaviors such as stating, 'I was never really good at math' served to reinforce stereotypes that were acquired from sources outside of the classroom (Gunderson et al., 2012). Gunderson et al. (2012) and Tsui (2007) concurred that females did not necessarily believe the mathematics stereotype, but were threatened because of their awareness that the stereotype existed and their scores were negatively impacted. While a few scholars suggested that boys were innately better at math than girls, most U.S. researchers cited the disparity in mathematics performance to social and environmental factors, such as social conditioning and expectations from society regarding how boys and girls should perform (Tsui, 2007). Female students were more vulnerable to the gender stereotype if they believed that math was something in which they were inherently good or bad, because when struggles came along, girls tended to question their abilities and think, 'I must not be good at math' (Dweck, 2012). Studies showed that teachers tended to rate math skills of girls lower than their male counterparts (Robinson et al., 2011) and they attributed boys' success in mathematics to ability while girls' success stemmed from their efforts (O'Shea et al., 2010). In Tsui's

(2007) study of eighth grade students, mathematics was perceived as a subject taken for males, and they felt less confident in their math ability and tended to like math less than boys. Female students who believed that understanding in mathematics was possible and its concepts were worthwhile may develop confidence, which led to academic achievement (O'Shea et al., 2010) and sustained interest in learning mathematics (Dweck, 2012). Teachers who challenged females to defend their answers, showed passion for mathematics, and employed varied pedagogical strategies that engaged both sexes tended to have more mathematical success among both the male and female students (O'Shea et al., 2010). Though there were many outside factors that contributed to the gender gap in mathematics, Robinson et al. (2011) found that when teachers showed consistent expectations to males and females, girls would lose 40% to 75% less ground in mathematics achievement. Tsui (2007) contended that the gender difference in mathematics performance was eliminated altogether when the test giver lowered the stereotype threat by stating that the test did not produce results with a difference in gender.

If female students could be convinced to think that abilities in math could be cultivated rather than a gift bestowed to them, then the gender stereotype became less credible (Dweck, 2012). A female's belief about her abilities, her confidence in math, and the perceived relevance of mathematics in her life may be essential in promoting female's talents in STEM-related careers (O'Shea et al., 2010) and may make her more likely to work hard to achieve (Dweck, 2012). A challenging curriculum accompanied by test-taking strategies was needed in order to increase female achievement in the mathematics achievement gap (O'Shea et al., 2010).

Ethnic minorities.

One of the most pervasive and persistent gaps was one between Black and White students (Flores, 2007; Goldberger & Bayerl, 2008). This substantial gap in achievement on standardized tests was one of the most maleficent problems facing America at the time of Burchinal et al.'s (2011) writings and was the most evident and predictive variable in achievement in schools (Hines III, 2008). Though racial subgroups made some gains in closing the achievement gap in mathematics tests since the 1990s and there was increasing evidence that the gap was capable of closing (Goldberger & Bayerl, 2008), White students continually outperformed ethnic minorities at every grade level, with the highest-performing Black students often performing at the same level as the lowest quartile of White students (Aud, Fox, & Kewal-Ramani, 2010; Education Commission of the States, 2014; Holloway, 2004; Riegle-Crumb & Grodsky, 2010). Black students were also less likely to be in higher-level math courses (Riegle-Crumb & Grodsky, 2010), and the lack of preparation in math in early grades affected Black students as they grew older (Holloway, 2004) resulting in 91% of Black students unable to reach proficiency in mathematics (Flores, 2007). Even as some strides were taken to enroll more Black students in higher-level courses nationwide, inequality in achievement still remained (Flores, 2007; Riegle-Crumb & Grodsky, 2010). Riegle-Crumb and Grodsky (2010) contended that the gap was even wider between Black and White students in the most rigorous high school math classes.

The American College Test (ACT) composite scores from 2011 demonstrated that White students averaged a score of 22.1 on the math portion of the test, while their Black counterparts scored 17.2 (ACT, Inc., 2011a, p. 4). This gap, according to ACT results, existed since 2007, and increased by three tenths of a point (ACT, Inc., 2011b). The ACT's (2011b) report on college and career readiness stated that only 14% of Black students met college readiness benchmarks while 54% of Whites did. The SAT, formerly known as the Scholastic Aptitude Test, revealed similar results. In the 2012 subtest of mathematics, the mean score of a college-bound Black student was 428, compared to a White college-bound student's mean score of 536 (College Board, 2012, p.1). Without equitable levels of achievement comparable to their White counterparts, Black students remained in a position of disadvantage in higher-level math courses (Riegle-Crumb & Grodsky, 2010).

While there were many barriers in and out of school that faced ethnic minorities, Holloway (2004) and Flores (2007) stated that minorities, as a group, tended to have a less rigorous curriculum, and these lower expectations prohibited the opportunity for them to take higher level math courses in high school and college because of the inadequate mathematical foundation. Flores (2007) found that in classrooms where more than half of the students are Black, teachers spent their time teaching and assessing lower-level objectives.

With the minority population continuing to rise in the United States and the growing number of jobs requiring higher education and skills, it became imperative that educational outcomes for minority students were more drastically advanced (Riegle-Crumb & Grodsky, 2010). The NCTM (2005) stated that the achievement gap could be closed if all students had access to an equitable and challenging mathematics curriculum and high expectations (Pascopella, 2006) from well-qualified teachers who made connections to the needs, backgrounds, and cultures of all students.

Poverty.

The achievement gap did not stop with race or gender, as research concluded that SES also created an achievement gap (Hines III, 2008). Flores (2007) found that school funding was largely derived from local property taxes. As a result, districts in impoverished areas did not receive as many funds per pupil as other higher-income areas. This resulted in poor and uneven instruction and intensified an already existent gap, due to the inability to attract and retain highly qualified teachers (Goldberger & Bayerl, 2008).

According to Jordan, Kaplan, Locuniak, and Ramineni (2007), a mathematics screening of low-income kindergarteners indicated an achievement gap, and their trajectories tended to flatten over time. These students were unable to gain significant amounts of mathematics knowledge throughout kindergarten and first grade, which widened the gap with middle and upper-class peers (Jordan et al., 2007). Between fourth and eighth grade, it was found that students in poverty fell further behind academically (Balfanz & Byrnes, 2006). According to NAEP, since 2000 fourth graders who took the math assessment and qualified for reduced lunch scored roughly 14 points lower than students who did not. Students who qualified for free lunch scored lower still; achieving about 24 points lower than ineligible students (Which Student Groups are Making Gains?, 2014, p. 1). In high schools, it was found that schools in which there were large numbers of underprivileged students, there were fewer students who enrolled in college preparatory classes (Balfanz & Byrnes, 2006; Flores, 2007) than in nonimpoverished areas.

There were many factors that contributed to the widening gap. Riegle-Crumb and Grodsky (2010) stated that children from more advantaged families could be successful because their parents were more familiar with how the school system worked and they taught their students at very young ages to apply themselves and communicate in ways that were rewarded in schools. These parents were also more likely to be able to help their child with more complex mathematics homework (Riegle-Crumb & Grodsky, 2010).

Factors Affecting Student Performance in Singapore

Factors affecting Singaporean students were different from the factors that affected U.S. students. Despite the success of the Singaporean students, Chen (2014) found that important issues, such as the achievement gap existed, but were rarely explored. The 2011 TIMSS report showed a trend from 1995 to 2011 that female students scored higher than males, with tests in 2003 and 2007 being significantly higher. By 2011, fourth grade scores between the genders ranged by only four points (Mullis et al., 2013, p. 68). Chen (2014) found that roughly 60% of the low-achieving Singaporean fourth graders tested on the TIMSS in 2007 were male and 40% were female (p. 14).

Socio-economic status (SES) may have also played a role in creating an achievement gap in Singapore (Baird, 2012). Baird (2012) collected information on how many educational books were in the home to determine students who were considered high and low SES. Roughly 18% of the gap could be linked directly to socioeconomic status (Baird, 2012, p. 484). Baird (2012) and Chen (2014) attributed the achievement gap more to the difference in student characteristics such as work ethic, attitude towards math, age, self-concept, and ethnicity.

Singapore Math Program Structure

In this section, the aspects of the Singapore Math program were explored and juxtaposed with the traditional approaches and methods of the instruction of mathematics in America. The Singapore Math program was adapted for use in America, and the features of the Singapore Math program included scope and pacing, components of the lesson, the employment of the Model Drawing method, textbooks, and the integration of algebra.

Scope and pacing.

The Singapore Math program was taught at a deliberately slower pace because of the intentional focus to master essential skills before moving on to the next unit (Hoven & Garelick, 2007; Singapore Math Inc., 2014). In traditional American mathematics, many concepts were covered throughout the year, but with little depth (Singapore Math Inc., 2014; Wang-Iverson et al., 2009). The Singapore Math program slowed down the process of learning and permitted students the time to form a foundation upon which more complex skills could be built (Curriculum Review, 2010). By slowing the pace, students could learn various concepts by using blocks, graphs, charts, and other visualization tools. This resulted in students retaining their current skills and decreased the amount of instructional time to revisit the same skills year after year (Curriculum Review, 2010). The result of slowing down the pace and going deeper in the concepts resulted in meaningful understanding and more rapid progress for all students (Hoven & Garelick, 2007).

Components of a Singapore math lesson.

The intent of the Singapore Math program was to focus on fewer topics per grade level and teach those topics to mastery, so those concepts did not formally have to be reviewed each year (Chen, 2008; Garelick, 2006). According to Chen (2008), a math class should be taught for 60 to 90 minutes each school day, and include the distinct segments outlined in Table 3 in order to appropriately cover the material in the lesson.

Table 3.

Segments of a Typical Singapore Math Lesson

Segment	**Length of Time**
Mental math	10 minutes
Teacher-directed lesson	20 minutes
Activity	20 minutes
Problem-solving	15 minutes
Independent practice	15 minutes

Note. Adapted from Chen, 2008, p. 36.

Model drawing.

The improvement of student achievement using Singapore Math could be in part attributed to the bar model or model drawing technique that the Singapore program employed. This straightforward tool was introduced to third graders solving uncomplicated word problems, and was versatile enough to be used with fifth and sixth graders on more complex problems (Hoven & Garelick, 2007). Students typically had difficulty with word problems because they were weak in mathematical language, had limited understanding of mathematical reasoning, and/or were unable to understand what the problem was asking them to do (Teach kids math with model method, 2011). Students used these model drawings, which guided them to determine what mathematical operations were useful in solving problems (FAQ-Primary Math, 2011). The model drawing process, outlined in Figure 2, has eight steps.

Systematic Model Drawing Approach

1. Read the whole problem.

2. Decide and write who is involved in the problem.

3. Decide and write what is involved in the problem.

4. Draw unit bar (or bars) of equal length.

5. Chunk the problem, placing information on model as indicated.

6. Put the question mark in place to identify what needs to be found.

7. Work computations to the side or underneath the problem.

8. Answer the question in a complete sentence

Figure 2. The eight step model drawing approach. Adapted from Hogan & Char Forster, 2007.

The bar model is a representation that allowed students to bridge the gap between concrete and abstract concepts and reinforced visualization and understanding of mathematical concepts (Hogan & Forster, 2007). Using the bar model approach, students were able to calculate simple one-step problems, as well as ratios, proportions, rates of change, and part-whole computations (Hoven & Garelick, 2007). Although American textbooks asked students to draw a picture to help them solve a problem, with this specific variant of the ‘draw a picture’ strategy, students were able to read the problem, know what type of bar model to draw, could see the information graphically, and know how to find the necessary information (Hoven & Garelick, 2007).

With the foundation of representation firmly built at an early age, students who were taught with Singapore Math’s bar model were able to understand algebraic concepts more easily (Hoven & Garelick, 2007). Consider the problem in Figure 3 from a third grade Singapore textbook:

Margo and Bob have $20 between them. Margo has $4 more than Bob.

How much money does each person have?

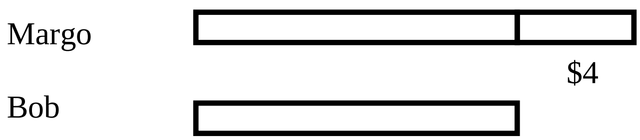


Figure 3. Third grade math problem. Adapted from Singapore Math, Inc. (2006, p. 21)

In the first step of the problem, the students would recognize that two bars were needed because they needed to find out how much each person had. Margo's line is longer because she has more money. The extra amount is shown in the extended bar. Margo and Bob's bar portions are equal in length if one removed Margo's four extra dollars. Now the sum of the two bars must equal sixteen dollars, since four dollars was taken away from the total of twenty dollars, leaving the students to find that two times Bob's bar equals sixteen dollars, and the difference between their amounts is four dollars (Garelick, 2006).

With practice using the bar model method, students learned a technique that worked for a variety of different problems. Garelick (2006) stated that by introducing and reinforcing the bar model method, teachers could eliminate less effective American traditional methods such as 'guess and check.' Additionally, using bar modeling, students were given the opportunity to identify the known and unknown in a given problem, which was essentially algebraic depiction. When problems became too complex to be represented by pictures, students could begin to use the intentionally infused foundational algebraic skills to solve the problems (Hoven & Garelick, 2007).

To illustrate the effectiveness of visualization, a problem from the Massachusetts state test was published in the *Boston Globe*; a problem which Abdul-Alim (2006) cited more than half of the 72,000 10h graders incorrectly answered. Using the bar model and no calculators, a class of sixth graders in Massachusetts successfully answered the problem (Abdul-Alim, 2006). The consistent use of a single powerful pedagogical tool, like the bar model used in Singapore Math, was a component that was nonexistent in American classrooms.

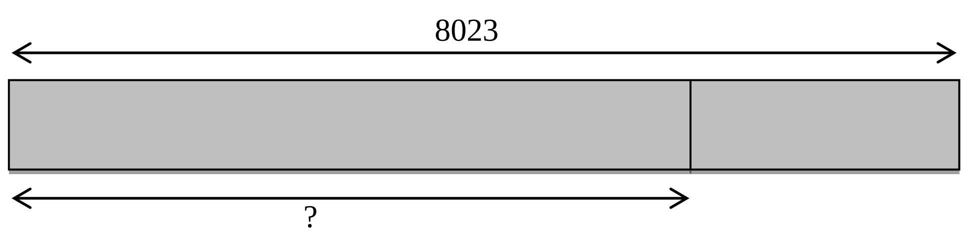
Textbooks.

One major difference in the Singapore program when compared to the United States was the textbooks that were used. American publishers, such as Houghton Mifflin, Harcourt, Scott Foresman - Addison Wesley, and McGraw-Hill included colorful pictures and graphics; however, the pictures and illustrations had little or no connection to the problem or concept on which the students were working (Leinwand & Ginsburg, 2007; Yang et al., 2010). American texts were full of games and activities, but seldom challenged students (Garelick, 2006).

Singapore's texts were small paperback books that contained few pictures and illustrations unless they directly pertained to the problem (Garelick, 2006). They were written to specifically address their mathematical framework and supported all five of the Singaporean elements in math (Clark, 2009). A report by the American Institutes for Research (AIR) found that problems in the sixth grade Singapore Math texts were more challenging than the eighth grade NAEP assessment (American Institutes for Research, 2011). They also found that traditional American texts focused on definitions and formulas, whereas Singapore texts intentionally developed students in a problem-based context.

Singapore Math books contained multiple representations and small cartoons, which suggested strategies for problem solving, provided information about the problem, or asked specific questions (Leinwand & Ginsburg, 2007). Students could, for example, learn how to solve problems with fractions by the text's concrete representations (five out of nine clowns are smiling), pictorial representations (six out of eight congruent regions of the square are shaded), and abstract representations (2/3 = 8/12 = 6/9). One or more of these representations were followed by a think bubble to give the teacher and student several ways to develop skills and conceptual understanding (Leinwand & Ginsburg, 2007).

(a)What is the number 1408 less than 8023?



(b) The difference between the largest and smallest of the three numbers 1277, 687, and 5768 is __________.

Figure 4. Samples of problems typical in the Singapore Math program. Adapted from Singapore Math, Inc., 2004, p. 16.

Figure 4 shows two subtraction problems found in a Singapore text book for third grade. Problem A uses a pictorial representation while students conceptualize the topic in question B (Singapore Math, Inc., 2004).

Garelick (2006) contended that the Singapore Math program was full of rich problems that gave students many different opportunities to view problems and apply concepts that were taught. Students also worked on problems in multiple ways. They were taught to use mental math and concrete examples in addition to model drawing. Kelli-Palka (2010) stated that the authors of the Singapore Math program realized that students did not learn the same way and offered a number of methods to help them. Problems were much more complex, usually consisting of two or three steps each (Clark, 2009). Topics typically had questions, such as the one shown in Figure 4, over five to ten pages and were covered in several days (Garelick, 2006).

Another difference was the amount of content in textbooks that was utilized in each country. Textbooks had a profound influence on what and how a concept was taught to students (Organization for Economic Co-operation and Development, 2003; Yang et al., 2010). Scott-Foresman, an American publisher included 729 pages, 32 chapters, and 164 lessons, which averaged about four pages per lesson (Leinwand & Ginsburg, 2007). The Singapore program books covered 42 lessons and averaged about 12 pages for each lesson (Leinwand & Ginsburg, 2007). The discrepancy between sizes of text books raised questions about when and how math topics were covered in the United States (Yang et al., 2010).

Math textbooks in Singapore and the United States were different with respect to the creation process and their final product (Schmidt, 2004). The number of texts available for purchase by school districts also differed between the U.S. and Singapore. While there were varieties of textbooks in Singapore, they were required to conform to national regulations. In order for a textbook to be marketed and sold to schools in

Singapore, the texts were reviewed to be sure that they met the Ministry of Education's standards (Yang et al., 2010).

American publishers were often in competition with one another and contained conflicting material, which made focus and coherence almost impossible to achieve (Leinwand & Ginsburg, 2007). Since there was no national curriculum in the United States, textbook series were created based upon state standards, which varied from state to state, resulting in more than ten different textbook publishers marketing and selling a variety of textbooks to American schools (Yang et al., 2010). Professional organizations such as the NCTM provided recommendations regarding texts, but their influence varied from state to state (Yang et al., 2010). The decisions of what to put in the textbooks was not made by educators, but by the authors and publishers of the textbooks with the influence of state standards (Yang et al., 2010). In order to keep up with the fluctuating standards in each state, textbooks underwent continuous development (Reys & Reys, 2006). Then, they were recommended to schools by a state adoption committee or, as was the case with about half of the states, the local school district (Yang et al., 2010).

Garelick (2006) discussed another notable difference between each country's textbooks. In American textbooks, a wide range of topics was covered throughout the school year. In the Singapore program, particularly in elementary grades, the program covered a relatively small number of topics in more depth, when compared to the typical math book utilized in the United States (Institute of Education Sciences, 2009). Singapore Math covered an average of 15 topics per year in grades one through six (Leinwand & Ginsburg, 2007). Each year, American teachers spent a great deal of instructional time revisiting and covering previously taught material. Singapore Math required students to master the material they were learning before progressing to the next level (Garelick, 2006). Students learned traditional math facts, but they took more time and used pictorial and abstract methods to help students understand (Kelli-Palka, 2010; Singapore Math, Inc., 2004).

The number of math topics covered per year in the United States varied per state. In Florida and New Jersey, at least 50 topics were covered per year in elementary schools (Leinwand & Ginsburg, 2007). The thicker the text and greater the number of concepts taught in a year, the less likely it would be that American students would achieve greater results on international tests (Abdul-Alim, 2006). In programs used in America, students might learn various and disjointed topics, such as time and money in the same week, and then not cover them again until a week later (Curriculum Review, 2010). Such disjunction and arbitrariness made learning math more difficult, particularly for students who were disadvantaged or struggling academically (Schmidt, 2004). According to NCTM, when teachers focused on a smaller number of key areas, students had a deeper experience with mathematical concepts and skills. Once students could sustain a deeper understanding, they could begin to apply and generalize their knowledge (NCTM, 2011). According to Leinwand and Ginsburg (2007), Singapore's success indicated that fewer topics should be covered in depth with less repetition of topics each year. States that adopted a similar model, North Carolina and Texas, covered 18 and 19 topics per year, respectively. Those two states have been recognized for their performance on the NAEP (Leinwand & Ginsburg, 2007).

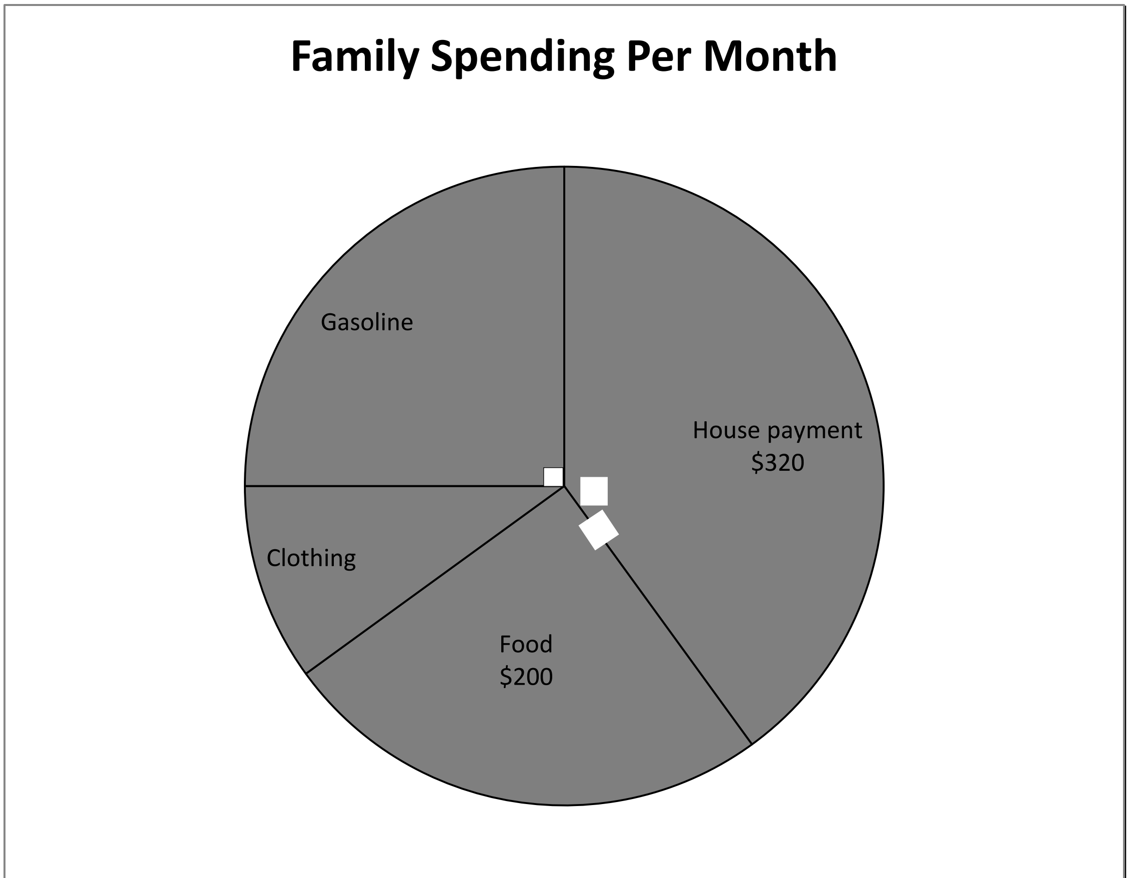
Though texts in the United States covered many different topics, the questions and problems that appeared in textbooks were often one-step tasks that required students to recall or apply the mathematical process in routine exercises (Leinwand & Ginsburg, 2007). The Singapore method was a paradigm shift from the traditional math strategy of memorization of algorithms and strategies. Students needed to think critically, as well as visualize math problems through various representations (PR Newswire, 2011).

American textbooks often included word problems at the bottom of a page after students practiced procedural sequences on basic problems (Jackson, 2011), while the Singapore texts embedded problem solving as a central part of the lesson every time a new concept was introduced (Clark, 2009). Multi-step and higher order questions were necessary to become more proficient in math (Leinwand & Ginsburg, 2007). Figure 5 is a typical problem from a third grade Singapore textbook:

> *120 children took part in a mathematics competition. 53 of them were girls. How many more boys than girls were there in the competition?*

Figure 5. Third grade math problem. Adapted from Singapore Math, Inc. (2006, p. 37).

Students must complete multi-step problems while applying a range of previously learned skills and concepts. These multi-step problems required students to find intermediate answers before arriving at a final solution. Multi-step problems of this caliber enhanced the breadth and depth of mathematical understanding (Leinwand & Ginsburg, 2007). A practice sheet from the Houghton Mifflin Math (2011) series contained 16 problems. The first 15 were number problems, saving the word problem for last (Houghton Mifflin Math, 2011). Figures 6 and 7 show two pie chart problems; One typical of a Singapore Math book compared to a problem that could be found in a Scott-Foresman-Addison Wesley math book.



Monthly Family Budget:

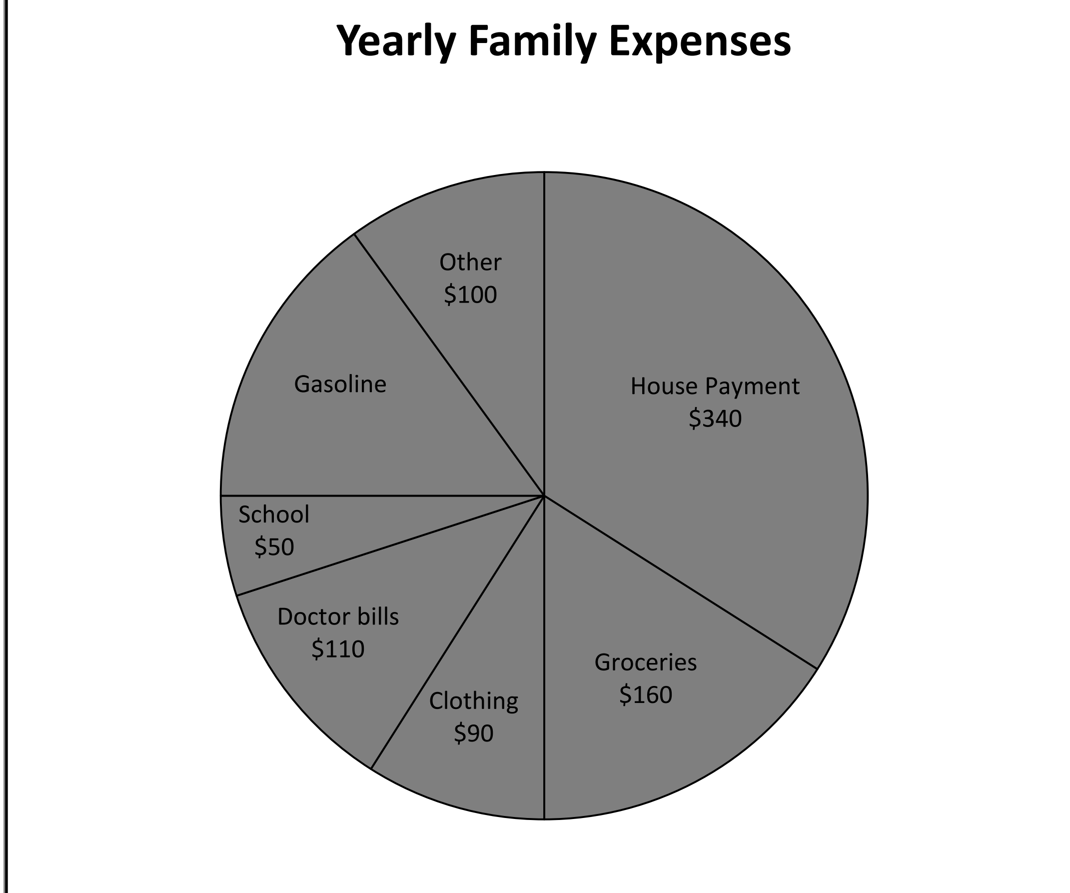
a. What fraction of the total budget went to Gasoline?

b. What was the total amount of money the family budgeted for the month?

c. How much money did the family budget for clothing?

d. What was the ratio of the amount the family budgeted for the house payment to the amount budgeted for food?

Figure 6. A word problem that is expected of students to solve in a Singapore textbook. Adapted from Leinwand & Ginsburg, 2007, p. 36.



Annual Family Expenses (for every $1,000)

a. How much does the family spend on gasoline for each $1,000?

b. For each $1,000, how much more does the family spend on the house payment than on groceries and doctor bills?

c. TEST PREP: If the family spends $4,000, about how much is spent on clothing?

a. $100 b. $200 c. $400 d. $1000

d. For what category does the family spend about three times what it spends on doctor bills?

Figure 7. A word problem that is expected of students to solve in a typical textbook in America. Adapted from Leinwand & Ginsburg, 2007, p. 36.

The Singapore Math problem in Figure 6 required six steps to solve. Students must use previous understandings of right angles and what those angles mean in a pie chart before attempting to solve the problem. Leinwand and Ginsburg (2007) stated that using such a wide range of skills to solve problems and interpret pie charts helped students develop a deeper understanding of the concepts. The complexity of the Singapore problem highlighted the low-level expectations that the Scott-Foresman text required (Leinwand & Ginsburg, 2007).

With Singapore's textbooks available in the United States, it was tempting for districts to adopt the textbooks and expect achievement, but Wang-Iverson et al. (2009) and a former president of the NCTM argued that there were many different aspects of their approach to mathematics that contributed to the repeated success of math achievement in Singapore and we must look beyond a textbook or program to find sustained success (Garelick, 2006).

Singapore Math and Algebra.

In years recent to these writings, algebra became a major focus of schools and districts in the United States (Ketterlin-Geller et al., 2007, Vogel, 2008; Wu, 2001). There were several factors that contributed to the push of integrating Algebra early into the lives of students, but for many educators, the biggest concern was the performance of U.S. students compared to their international peers (Ketterlin-Geller et al., 2007). In American mathematics, algebra was often the first math class that required the understanding of abstract concepts and problem solving; skills that were invaluable, even if the student did not go on to utilize algebraic concepts in the post-high school career (Vogel, 2008). National legislators, professional organizations, such as the NCTM, and researchers emphasized the need for American students to develop algebra skills earlier in the curriculum (Ketterlin-Geller et al., 2007; Wu, 2001), because, as Vogel (2008) and Wu (2001) pointed out, success in algebra became the gate keeper for college and career success in the 21st century global economy.

In the algebra strand, there were few topics that students were expected to master in the elementary and middle school level. The NCTM created a curriculum focus for grades K-8 and recommended in 2006 that teachers lay the groundwork for algebra as early as Kindergarten (Ketterlin-Geller et al., 2007). As it stands at the time of this writing, many American math curriculums utilize algebra as a separate strand of math. These facets of mathematics were not exclusive from each other, but rather, should be taught together in such subtopics as patterns, functions, coordinate systems, and data analysis (Ketterlin-Geller et al., 2007). Vogel (2008) contended that algebraic thinking should begin at an earlier age and that more manipulatives and problem-solving activities should be incorporated to bolster mathematical skills.

Many student misconceptions about algebra stemmed from early elementary grades when students learned mathematical ‘truths’ in adding, subtracting, multiplying, and dividing, without a firm conceptual foundation (Ketterlin-Geller et al., 2007). Wu contended in his 2001 article that the proper study of fractions at the elementary level would greatly impact algebraic success. As students progressed through math classes, the rules they learned in earlier grades no longer generalized and students relied on memorization, instead of a foundation, for success in math, such as when adding a negative number or dividing fractions (Ketterlin-Geller et al., 2007). Algebraic reasoning built deeper meaning in numbers and relationships (Ketterlin-Geller et al.,

2007) and buildt problem-solving skills (Vogel, 2008). Algebra and geometry were integrated progressively and intentionally as students reached middle and high school age (Institute of Education Sciences, 2009). Garelick (2006) found the Singapore Math program allowed for students at the elementary level to begin building a foundation in algebra through its problem solving techniques and concrete illustrations. When students performed model drawing, they were symbolically representing a problem, which was a mainstay in algebraic concepts (Garelick, 2006). A rigorous math curriculum and quality instruction were vital at the elementary level in order for proficiency in algebra to be achieved (Tsui, 2007).

Singapore Math and its Relevance to the Common Core State Standards

The CCSS were developed by groups of states in the U.S. with the input of teachers, administrators, and educational experts (Dessoff, 2012; Sloan, 2010) and were adopted by many states, including Missouri, for implementation and assessment starting in the 2014-2015 academic year (Dessoff, 2012). The CCSS were designed to provide a more united curricular front in states that chose to adopt the initiative in the areas of Mathematics and English Language Arts (Achieve, 2010), close nationwide achievement gaps (Sloan, 2010), and outline the skills and knowledge necessary for students to be successful after graduating high school in both college and career-ready fields (CCSS, 2012a). The states that created the CCSS used evidence of success from other countries around the globe in order to help students be more globally competitive (CCSS, 2012a). In fact, the CCSS were somewhat based on Singapore’s national math curriculum (iSingaporemath.com, n.d.).

At the elementary level, Achieve (2010) found that by the end of fourth grade, the rigor between the CCSS and Singapore Math programs was comparable. For example, each document required students to have fluency in addition, subtraction, multiplication, and division of whole numbers, a deep understanding and application of place value, and classification of two-dimensional shapes (Achieve, 2010). When comparing Singapore's syllabus to the CCSS at the middle school level, there were also some substantial parallels in rigor. Algebra was a concept utilized frequently in both documents with emphasis on proportions, linear expressions, inequalities, and applying those concepts to real-world situations. The expectations found in the secondary Singapore Math books reflected what was expected of eighth graders according to the Common Core (Achieve, 2010). At the high school level, standards of rigor continued to be seen in both the Common Core and Singapore documents. While some concepts in the CCSS were not covered in the Singapore curriculum, the Singapore syllabus covered math commensurate with a calculus course, whereas the CCSS provided a pre-calculus base for high school students (Achieve, 2010).

Overall, there were key components that were comparable when examining both documents throughout the K-12 curriculum. A solid foundation for number sense and skill building reinforced the rigors of mathematics at the high school level. It also emphasized learning concepts more deeply, eliminating the need for unnecessarily re-teaching concepts year-to-year (Achieve, 2010).

CCSS assessments required students to think deeply and conceptually, reason mathematically, and demonstrate their knowledge differently than they had in the past (Dessoff, 2012). Rote memorization or other similar methods did not embody the spirit of the Common Core (iSingaporemath.com, n.d.) and students were no longer held accountable for such items in the same way. When asked about the online format of the assessments, an advisor for one of the assessments stated, “It won’t just be a paper-and-pencil test put on a screen” (Dessoff, 2012, p. 57). Students may see questions applied to real-world settings and have to explain their answers, rather than filling in blanks. If teachers and administrators know what is expected from the CCSS, then they will know what will be assessed and how to prepare them (Dessoff, 2012). Singapore Math textbooks were better aligned with the CCSS than many American textbooks and went through a sequence of learning concepts more deeply by mastering concepts concretely, pictorially, and abstractly (iSingaporemath.com, n.d.).

Importance of Math in America’s Future

The decline in mathematics and algebra skills did not stop at the high school level; it already had an effect at the university and job market (Prystay, 2004). Politicians, professors, and business members agreed that Americans were not as globally competitive as they once were, and American high schools were not producing graduates with math and technical skills that helped to succeed in higher education or in a globally competitive market (Public Education Network, 2006), despite the push to bolster the mathematics curriculum in America (Ganem, 2011). Eventually, according to Sami (2012), the deficiencies in math and STEM-related subjects would lead to a shortage of American engineers and scientists. Prystay (2004) corroborated this by finding that from 1994 to 2001, the number of U.S. students majoring in engineering and science dropped 10%, while foreign enrollments increased by 35% (p. 1).

U.S. students were leaving school with inadequate depth and breadth in math knowledge skill and confidence (Stacey, 2002) and were unable to meet the demands of mathematics beyond high school. The widespread availability of technology diminished the need for basic computation skills that were necessary 20 years previous to the reporting, but at the same time, companies and employees were tasked with more complex and quantitative ideas that required a deeper understanding of math (Stacey, 2002). About 40% of students reported to Achieve (2010) that they wished they had taken more advanced math courses in high school. Ganem (2011) noted that disconnects between math at the high school and college level made it difficult for students to transfer knowledge to higher level courses or work experience.

In 2006, the Public Education Network found that less than half of American graduates were prepared for math courses at the college level and that number was on the decline (Vogel, 2008). According to a 2013 report, one out of five freshmen needed to take a remedial math course (Sparks & Malkus, 2013). In Maryland, about 49% of high school graduates took a remedial math course for no credit before they can take other math courses for credit (Ganem, 2011) and the deficiency in mathematics is evident in community colleges across America as well (Sami, 2012). Conversely, Donovan (2008) found that students who are able to finish a high school math course beyond Algebra II were twice more likely to obtain their Bachelor’s degree. Ten years out of high school, Achieve (2013) found that students who took high level math courses in high school had higher incomes, regardless of their grades, backgrounds, or college degrees.

Because of this gap between high school math, college math, and the work force, American workers risked losing more jobs to their foreign cohorts (Prystay, 2004).

Meanwhile, jobs requiring skills in math, engineering, and technical skills were projected to increase 17% to 24%, or roughly 6.3 million jobs, between 2004 and 2018 (Achieve, Inc., 2014; Public Education Network, 2006, p. 1). Achieve (2014) found that 88% of the most notable and successful engineers and entrepreneurs all had a degree in a STEM-related field, linking a sound foundation of mathematics to innovation. The shortage of qualified workers would have a negative impact both economically and socially in the short and long-term (Achieve, Inc., 2014; Norton, 2010). If U.S. achievement in the area of mathematics could be commensurate with high achieving nations around the world, economists said that the U.S.'s gross domestic product could gain as much as 36% (Achieve, Inc., 2014).

Nations like Singapore, on the other hand, recognized the need to create a high quality workforce. The ministry believed this focus enabled them to take advantage of the modern, technology-driven society through effective schooling. Their emphasis on math, science, and technology allowed for the collective work between government agencies to anticipate needs in these areas 10 to 15 years in the future (Jackson, 2010b).

Conclusion

In America, many teachers and districts looked for quick fixes and reached out for the latest fad (Leinwand & Ginsburg, 2007). The Singapore Math program should not be regarded as such, but rather, it should be utilized to help identify essential features that the United States adopts to become more mathematically competitive in the world (Leinwand &Ginsburg, 2007). Jacobs (2011) found that it generally took one or two years to learn how the Singapore Math series worked. Teachers would be better served to look at the learning of mathematics as a marathon instead of a sprint and put long-term structures in place to ensure meaningful learning over a long period of time. This was especially true for schools that were in need of drastic improvements (Jackson, 2010a).

It was acknowledged that moving all students forward and closing the achievement gap would require more than a change in a mathematics program. Goldberger and Bayerl (2008) contended that it would require much more than conventional solutions, such as teacher development and engaging lessons. Still, a program which contained higher standards, expectations and rigor was critical to address the variations in mathematics assessments in classrooms across the United States (Lee & Fish, 2010; Burchinal et al., 2011) and ensured that U.S. schools and their students were meeting the needs of tomorrow's work force.

The research in this chapter outlined the current status of the United States and Singapore regarding facets of mathematics education. There were differences between the education systems of the U.S. and Singapore, and it was important to establish background knowledge when drawing comparisons between two such entities. Secondly, education was a complex system and there were many factors outside of the school that played a part in the education for all students; gender, ethnicity, and poverty were only three factors. These three were chosen because they were prevalent subgroups, identified by the state of Missouri, for which the studied school district was accountable on the MAP. Aspects of the Singapore Math program were also explored and compared to American math texts. Finally, the connection between Singapore Math and the CCSS and algebra was examined and the importance of math after completing high school and college was reiterated.

This study, searching for an effective way of teaching mathematics, was well-suited for a longitudinal study using MAP scores as a comparison between two cohorts and their subgroups. The study utilized two groups: one group of students that was not exposed to the Singapore Math program until the fifth grade year, and the second group, which received Singapore Math instruction beginning in first grade. The methodology for this study is outlined in the Chapter Three.

Chapter Three: Methodology

This study analyzed the relationship between the implementation of the Singapore Math program and elementary scores on the mathematics MAP. The purpose was to determine the effectiveness of the Singapore Math program, as indicated through the mathematics section of the MAP assessment for the years from 2007 to 2012.

The MAP was a criterion-referenced assessment given to students who attended public school districts state-wide annually in spring. These assessments were based on the state's grade level expectations (MODESE, 2013a) and should provide an accurate assessment of students' cumulative knowledge in mathematics. The researcher analyzed the data from two cohorts, A and B, in order to determine if scores were statistically different. Cohort A was not introduced to Singapore Math until fifth grade, and Cohort B was introduced to Singapore in first grade in the studied school district.

This study was designed for researchers to determine if the Singapore Math program was an effective option for elementary schools in the United States to utilize in order to contribute to development of a solid mathematical foundation for upper level mathematics courses. In this study, the Singapore Math program functioned as the independent variable that may or may not have led to a significant change in students' performance on the math section of the MAP test, which functioned as the dependent variable.

Demographics

The study was conducted in a rapidly growing suburban school district in Missouri. MAP data was analyzed for two groups of third, fourth, and fifth grade students. At the time of the study, kindergarten through fifth grade enrollment in the district was 6,939 students across 10 elementary schools (Tyler Systems, 2012, p. 1). District-wide demographics were as follows: 2.1% Asian, 7.9% Black, 2.4% Hispanic, .4% Indian, and 87.1% White (p. 1). Students enrolled with Free and Reduced Lunch status totaled 22.3%, and females made up 48.87% of the district's population. The Average Daily Attendance for the studied district was 95.75% (p. 1).

Research Question and Hypotheses

Research question.

Does the use of Singapore Math improve the understanding of mathematics in elementary students in the studied district as measured by the MAP test?

Hypotheses:

Null Hypothesis 1: The MAP test index scores of students in Cohort A will not increase from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

An increase would be evidenced by:

- Difference in means on the math subtest of the MAP through the utilization of a *z*-test for a difference in means
- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a *z*-test for difference in proportions
- Difference in variance on the math subtest of the MAP through the utilization of an *F*-test for difference in variance

Null Hypothesis 2: The MAP test index scores of students in Cohort A will not increase from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

An increase would be evidenced by:

- Difference in means on the math subtest of the MAP through the utilization of a z-test for a difference in means
- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a z-test for difference in proportions
- Difference in variance on the math subtest of the MAP through the utilization of an F-test for difference in variance

Null Hypothesis 3: The MAP test index scores of students in Cohort B will not increase from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

An increase would be evidenced by:

- Difference in means on the math subtest of the MAP through the utilization of a z-test for a difference in means
- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a z-test for difference in proportions
- Difference in variance on the math subtest of the MAP through the utilization of an F-test for difference in variance

Null Hypothesis 4: The MAP test index scores of students in Cohort B will not increase from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

An increase would be evidenced by:

- Difference in means on the math subtest of the MAP through the utilization of a z-test for a difference in means

- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a *z*-test for difference in proportions
- Difference in variance on the math subtest of the MAP through the utilization of an *F*-test for difference in variance

Null Hypothesis 5: There will be no increase in scores on the MAP between Cohort A and Cohort B in third, fourth, and fifth grade.

A difference would be measured by:

- Difference in means between the two cohorts on the math subtest of the MAP through the utilization of a *z*-test for a difference in means
- Difference between the two cohorts in percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a *z*-test for difference in proportions
- Difference in variance between the two cohorts through the utilization of an *F*-test for difference in variance

**Null Hypothesis 6:* There is no difference in average math MAP test scores of female students and students in Cohort B.

A difference would be measured by:

- A difference in means between the sample and female students on the math subtest of the MAP through the utilization of a *z*-test for a difference in means
- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a *z*-test for a difference in proportions
- A difference in variance between the sample and female students through the utilization of an *F*-test for difference in variance

**Null Hypothesis 7:* There is no difference in average math MAP test scores of Black and students in Cohort B.

A difference would be measured by:

- A difference in means between Black students and the sample on the math subtest of the MAP through the utilization of a *z*-test for a difference in means
- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a *z*-test for a difference in proportions
- A difference in variance between Black students and the sample through the utilization of an *F*-test for difference in variance

**Null Hypothesis 8:* There is no difference in average math MAP test scores of students on Free and Reduced Lunch and students who are in Cohort B.

A difference would be measured by:

- A difference in means between students with Free and Reduced Lunch and the sample on the math subtest of the MAP through the utilization of a *z*-test for a difference in means
- Percentage scoring proficient and advanced on the math subtest of the MAP through the utilization of a *z*-test for a difference in proportions
- A difference in variance between students with Free and Reduced Lunch and the sample through the utilization of an *F*-test for difference in variance

**The null hypothesis and the researcher's claim are the same.*

Data Collection and Analysis Procedures

The MAP test score data were acquired through the researched district's database at Central Office. Permission was requested and granted from the Assistant

Superintendent of Student Services and data collected were scrubbed of identifying information except for gender, ethnicity, Free and Reduced Lunch status, year the assessment was taken, and student ID and exported into an Excel document. In Cohorts A and B, the student ID number was necessary in order to ensure that the data collected were gathered from the same student for the three years of data collected. In order to select data for each group, a random sampling method was utilized for each. Each group of students was separated and put in its own spreadsheet. The researcher then input the total number of lines of student data into software provided by a website called random.org. Fifty numbers were chosen randomly, and the numbers corresponded with the rows in the Excel spreadsheet. This process was repeated for Cohort A, Cohort B, female students, Black students, and students with Free and Reduced Lunch status. Students who were in Cohort B were also eligible to be randomly sampled in each of the three subgroups (female, Black, Free and Reduced Lunch). Once the random samples were pulled, the student ID number was scrubbed from the data.

The state of Missouri developed a rubric and scoring criteria for the MAP assessment, which assigned students to one of four categories: Below Basic, Basic, Proficient, and Advanced, based on student performance. In order to meet state requirements for AYP, it was the goal of the state's educators to have 35.8% of students score proficient or advanced in 2007, 45% in 2008, 54.1% in 2009, 63.3% in 2010, 72.5% in 2011, and 81.7% in 2012 (MODESE, 2013c). Under NCLB, states were required to have 100% of their student populations score Proficient or Advanced by 2014, according to the state's AYP report (MODESE, 2013c).

Participants

This study included 50 students in each of the two cohorts, 50 students who made up the female subgroup, 30 students who made up the Black subgroup, and 50 students who made up the Free and Reduced Lunch status subgroup. A student's could be present in one of the cohorts and duplicated in subgroup data. The elementary students in each of the two cohorts were chosen through use of a cluster sampling method. In order to be eligible for the study, the student must have attended the school district from first through fifth grade, in order to ensure that they were exposed to a commensurate elementary math curriculum. Cohort A reflected a group of students who were not exposed to the Singapore Math program until their fifth grade year. They attended the fifth grade at the school in the 2008-2009 school year. The second group of students, Cohort B, began the Singapore Math program in first grade and continued throughout their elementary careers. These students were in fifth grade during the 2011-2012 school year. Fifty female students, 30 Black students, and 50 students with Free or Reduced Lunch status were also chosen as subgroups from the original two cohorts, in order to analyze progress in comparison with their cohort. The population of Black students that was in the district at the time of the study did not provide a count high enough to yield a sample of 50 students; 30 students was more commensurate when compared to the population size. Their MAP scores were taken in third grade (2009-2010), fourth grade (2010-2011), and fifth grade (2011-2012).

MAP data collected from the samples were compared throughout the students' elementary careers in order to check for a statistical difference in mathematical performance on the MAP test by applying three analyses: a *z*-test for a difference in means, a *z*-test for difference in proportions, and an *F*-test for difference in variance. These three tests were applied to all of the groups analyzed. The tests compared Cohort A's third graders to fourth grade students, Cohort A's fourth graders to fifth grade students, Cohort B's third grade students to fourth grade students, Cohort B's fourth grade to fifth grade students, Cohort A's third grade students to Cohort B's third grade students, Cohort A's fourth grade students to Cohort B's fourth grade students, Cohort A's fifth grade students to Cohort B's fifth grade students, Cohort B's female students in third, fourth, and fifth grade to Cohort B's female third, fourth, and fifth graders, respectively, Cohort B's Black students in third, fourth, and fifth grade to Cohort B's Black third, fourth, and fifth graders, respectively, and Cohort B's third, fourth, and fifth grade students with Free and Reduced Lunch status to Cohort B's third, fourth, and fifth grade students with Free and Reduced Lunch status, respectively.

Conclusion

This study utilized quantitative data collected from MAP assessments from 2007 to 2012 in order to determine whether or not there was a nexus between the implementation of the Singapore Math program and success on the math portion of the MAP test. Students were split into two groups: students who completed fifth grade in 2009, Cohort A, and students who completed fifth grade in 2012, Cohort B. Students were then randomly sampled in order to apply three statistical tests to the data: a *z*-test for a difference in means, a *z*-test for a difference in proportions, and an *F*-test for a difference in variance. The tests were utilized to triangulate MAP test data for potential statistical differences.

Chapter Four: Results

At the time of Garelick's (2006) writing, when compared to their international peers on assessments, American students were a little better than average; a trend over the past two decades. There were many factors that contributed to the mathematical success of students, including strong standards, a strong curriculum, and a teacher's knowledge of mathematics (Ball et al., 2005), but the mathematics program which supported the standards was also a critical component. Hoven and Garelick (2007) stated that the Singapore Math program was well-liked by mathematicians because of its logical and coherent sequence of topics and its focus on building an algebraic foundation.Table 4 shows the percentage of elementary students who scored Proficient or Advanced on the MAP mathematics portion in the studied district from 2007 to 2012, as well as Missouri's Annual Proficiency Target. This was intended to provide a context for overall mathematical achievement in the elementary schools during the time that each of the cohorts were enrolled in the district.

Table 4.

Percentage of Students Scoring Proficient or Advanced on the Math Section of the MAP

Year	Target	3rd Grade	4th Grade	5th Grade	District
2007	35.8	46.1	50.2	51.2	49.1
2008	45.0	47.5	46.4	54.9	49.5
2009	54.1	51.8	51.4	57.1	53.3
2010	63.3	55.1	56	64.2	58.3
2011	72.5	60.8	60.4	66.2	62.4
2012	81.7	60.9	64.2	69.8	67.3

Note. Target = Missouri Annual Proficiency Target; District = third, fourth, and fifth grade students in the district.

Research Question: Singapore Math and MAP Data

The primary purpose of the study was to analyze the impact of the Singapore Math program in one district over a period of time in the elementary setting. The effectiveness of the program was measured based on the results of the following research question: Does the use of Singapore Math improve the understanding of mathematics in elementary students in the studied district as measured by the MAP test?

The researcher utilized three tests in order to triangulate the results of the MAP assessment scores: a *z*-test for a difference in means, a *z*-test for a difference in proportions, and an *F*-test for a difference in variance. The *z*-tests utilized a confidence level of 95% and critical values of -1.960 and +1.960. The *F*-test was a one-tailed test at a confidence level of 95%.

Null Hypothesis 1

The MAP test index scores of students in Cohort A will not increase from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

A *z*-test for difference in means was applied to the data. The *z*-test value of -3.874 was in the critical region, therefore the researcher rejected the null hypothesis that there was not an increase of mean scores on the MAP. The data supported the alternate hypothesis that students made progress between the third and fourth grade assessments. Results are shown in Table 5.

Table 5.

Cohort A Mathematics MAP Index Scores: Third vs. Fourth Grade

	3rd	*4th*
Mean	619.340	645.900
Known Variance	1126.964	1223.031
Observations	50	50
z	-3.874	
P(Z<=z) one-tail	5.350	
z Critical one-tail	1.644854	

Note. Confidence Level = 95%

Null Hypothesis 1a

There will be no difference in proportions of students scoring proficient and above in Cohort A from the third grade administration of the MAP test to the fourth grade administration of the MAP test

The second test applied to third and fourth grade data from Cohort A was a *z*-test for a difference in proportions of students scoring proficient and above. The critical values were -1.96 and +1.96. The *z*-test value was 0.203321. The researcher did not reject the null hypothesis that there was no difference in the proportion of students who scored proficient or advanced on the MAP test. Therefore, the alternate hypothesis was not supported, and there was no difference between the third and fourth percentages of students proficient and above on the mathematics MAP.

Null Hypothesis 1b

There will be no difference in variance in MAP test index scores of students in Cohort A from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

The third test applied to Cohort A data during third and fourth grade was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.085. The researcher did not reject the null hypothesis that there was not a difference of variance in scores on the MAP test. Therefore, there was no difference in the variance of mathematics MAP scores between third and fourth grade. Table 6 shows the results of the *F*-test.

Table 6.

Cohort A Difference in Variance: Third vs. Fourth Grade

	4th	*3rd*
Mean	645.900	619.340
Variance	1223.031	1126.964
Observations	50	50
df	49	49
F	1.085	
P(F<=f) one-tail	0.387	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overview of hypothesis 1.

Overall, in the first hypothesis, the researcher did not reject the null hypothesis on two of the three tests, which meant that two of the three tests showed no significant change in MAP scores in Cohort A between third and fourth grade.

Null Hypothesis 2

The MAP test index scores of students in Cohort A will not increase from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

A *z*-test for difference in means was applied to the data. The *z*-test value of -3.417 was in the critical region; therefore the researcher rejected the null hypothesis that there was not an increase in MAP scores. The data supported the alternate hypothesis that students made progress between the fourth and fifth grade assessments. Results are shown in Table 7.

Table 7.

Cohort A Mathematics MAP Index Scores: Fourth vs. Fifth Grade

	4th	*5th*
Mean	645.900	670.080
Known Variance	1223.031	1281.381
Observations	50	50
z	-3.417	
P(Z<=z) one-tail	0.0003	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 2a

There will be no difference in proportions of students scoring proficient and above in Cohort A from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

The second test applied to fourth and fifth grade data from Cohort A was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 1.80. The researcher did not reject the null hypothesis that there was no difference in the proportion of students who scored proficient or advanced on the MAP. This signified that there was no difference between fourth and fifth grade scores.

Null Hypothesis 2b

There will be no difference in variance in MAP test index scores of students in Cohort A from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

The third test applied to Cohort A during third and fourth grade was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.048. The researcher did not reject the null hypothesis that there was no difference in variance in scores on the MAP. Therefore, there was no difference in the variance of MAP scores between fourth and fifth grade. Table 8 shows the results of the *F*-test.

Table 8.

Cohort A Difference in Variance: Fourth vs. Fifth Grade

	5th	*4th*
Mean	670.080	645.900
Variance	1281.381	1223.031
Observations	50	50
df	49	49
F	1.048	
P(F<=f) one-tail	0.436	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overview of hypothesis 2.

Overall, in the second hypothesis, the researcher did not reject the null hypothesis on two of the three tests, which meant that two of the three tests showed no significant change in MAP scores in Cohort A between fourth and fifth grade.

Null Hypothesis 3

The MAP test index scores of students in Cohort B will not increase from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

A *z*-test for difference in means was applied to the data. The *z*-test value of -3.982 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was an increase of mean scores on the MAP. The data supported the null hypothesis that students of Cohort B did not make statistically significant progress between the third and fourth grade assessments. Results are shown in Table 9.

Table 9.

Cohort B Mathematics MAP Index Scores: Third vs. Fourth Grade

	3rd	*4th*
Mean	632.940	658.740
Known Variance	1128.058	970.400
Observations	50	50
z	-3.982	
P(Z<=z) one-tail	3.410	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 3a

There will be no difference in proportions of students scoring proficient and above in Cohort B from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

The second test applied to third and fourth grade data from Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 0.629. The researcher did not reject the null that there was no difference in the proportion of students who scored proficient or advanced on the MAP test. This signified that there was no difference between third and fourth grade scores.

Null Hypothesis 3b

There will be no difference in variance in MAP test index scores of students in Cohort B from the third grade administration of the MAP test to the fourth grade administration of the MAP test.

The third test applied to Cohort B during third and fourth grade was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.162. The researcher did not reject the null hypothesis that there was no difference in variance in scores on the MAP. Therefore, there was no difference in the variance of MAP scores between third and fourth grade. Table 10 shows the results of the *F*-test.

Table 10.

Cohort B Difference in Variance: Third vs. Fourth Grade

	3rd	4th
Mean	632.94	658.74
Variance	1128.05755	970.4004
Observations	50	50
df	49	49
F	1.16246607	
P(F<=f) one-tail	0.30014773	
F Critical one-tail	1.60728946	

Note. Confidence Level = 95%

Overview of hypothesis 3.

Overall, in the third hypothesis, the researcher did not reject the null hypothesis on all three of the tests, which meant that the tests showed no significant change in scores in Cohort B between third and fourth grade.

Null Hypothesis 4

The MAP test index scores of students in Cohort B will not increase from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

A *z*-test for difference in means was applied to the data. The *z*-test value of -3.952 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not an increase of mean scores on the MAP test. Results are shown in Table 11.

Table 11.

Cohort B MAP Index Scores: Fourth vs. Fifth Grade

	4th	5th
Mean	658.740	684.340
Known Variance	1128.058	970.400
Observations	50	50
z	-3.952	
P(Z<=z) one-tail	3.880	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 4a

There will be no difference in proportion of students scoring proficient and above in Cohort B from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

The second test applied to fourth and fifth grade students from Cohort B was a *z*-test for difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 0.891. The researcher did not reject the null that there was no difference in the proportions of students who scored proficient or advanced on the MAP test. This signified that there was no difference between fourth and fifth grade MAP scores.

Null Hypothesis 4b

There will be no difference in variance in MAP test index scores of students in Cohort B from the fourth grade administration of the MAP test to the fifth grade administration of the MAP test.

The third test applied to Cohort B during fourth and fifth grade was an *F*-test for difference in variances. The critical values for the test were 1.607. The *F*-test value was found to be 1.162. The researcher did not reject the null hypothesis that there was not a difference in variances in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between fourth and fifth grade. Table 12 shows the results of the *F*-test.

Table 12.

Cohort B Difference in Variance: Fourth vs. Fifth Grade

	5th	*4th*
Mean	684.340	658.740
Variance	1473.331	970.400
Observations	50	50
df	49	49
F	1.518	
P(F<=f) one-tail	0.074	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overview of hypothesis 4.

Overall, in the fourth hypothesis, the researcher did not reject the null hypothesis on all three tests, which meant that the tests showed no significant change in scores in Cohort B between fourth and fifth grade.

Null Hypothesis 5 [third grade]

There will be no difference in scores on the MAP between Cohort A and Cohort B in third, fourth, and fifth grade.

Third Grade Students

A *z*-test for difference in means was applied to the data for third grade students in each cohort. The *z*-test value of -2.025 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference in mean scores on the MAP test. The data supported that students of Cohort B had no difference in scores when compared to third grade students in Cohort A. Results are shown in Table 13.

Table 13.

Cohort A and Cohort B MAP Index Scores: Third Grade

	3rd A	3rd B
Mean	619.340	632.940
Known Variance	1126.964	1128.058
Observations	50	50
z	-2.025	
P(Z<=z) one-tail	0.02140	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 5a

There will be no difference in proportions of students scoring proficient and above in Cohort A and Cohort B in third, fourth, and fifth grade.

The second test applied to third grade students from Cohorts A and B was a *z*-test for difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 2.200. The researcher rejected the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to Cohort A. This signified that third graders in Cohort B had a higher proportion of students who earned proficient or advanced on the MAP test than Cohort A.

Null Hypothesis 5b

There will be no difference in variances in MAP test index scores of students in Cohort A and Cohort B in third, fourth, and fifth grade.

The third test applied to third graders in Cohorts A and B was an *F*-test for difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.001. The researcher did not reject the null hypothesis that there was not a difference variances in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between Cohort A and Cohort B. Table 14 shows the results of the *F*-test.

Table 14.

Cohort A and Cohort B Difference in Variance: Third Grade

	3rd B	3rd A
Mean	632.940	619.340
Variance	1128.058	1126.964
Observations	50	50
df	49	49
F	1.001	
P(F<=f) one-tail	0.499	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overall, in the analysis of third graders from Cohorts A and B, the researcher failed to reject the null hypothesis on two of the three tests, which meant that two of the three tests showed third graders in Cohort B scores were not significantly different when compared to third graders in Cohort A.

Null Hypothesis 5 [fourth grade]

There will be no difference in scores on the MAP between Cohort A and Cohort B in third, fourth, and fifth grade.

Fourth Grade Students

A *z*-test for difference in means was applied to the data for fourth grade students in each cohort. The *z*-test value of -1.939 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference of mean scores on the MAP test between the cohorts. The data showed that there was no difference in means between the cohorts of fourth graders. Results are shown in Table 15.

Table 15.

Cohort A and Cohort B MAP Index Scores: Fourth Grade

	4th A	4th B
Mean	645.900	658.740
Known Variance	1223.031	970.400
Observations	50	50
z	-1.939	
P(Z<=z) one-tail	0.026	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 5a

There will be no difference in proportions of students scoring proficient and above in Cohort A and Cohort B in third, fourth, and fifth grade.

The second test applied to fourth grade students from Cohorts A and B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 2.613. The researcher rejected the null that there was no difference in the proportions of students in Cohort B who scored proficient or advanced on the MAP test when compared to Cohort A. This signified that fourth graders in Cohort B had a higher proportion of students who earned proficient or advanced on the MAP test than Cohort A.

Null Hypothesis 5b

There will be no difference in variances in MAP test index scores of students in Cohort A and Cohort B in third, fourth, and fifth grade.

The third test applied to fourth graders in Cohorts A and B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.260. The researcher did not reject the null hypothesis that there was not a difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between the fourth graders of Cohort A and Cohort B. Table 16 shows the results of the *F*-test.

Table 16.

Cohort A and Cohort B Difference in Variance: Fourth Grade

	4th A	4th B
Mean	645.900	658.740
Variance	1223.031	970.400
Observations	50	50
df	49	49
F	1.260	
P(F<=f) one-tail	0.210	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overall, in the analysis of fourth graders from Cohorts A and B, the researcher failed to reject the null hypothesis on two of the three tests, which meant that two of the three tests showed fourth graders in Cohort B had scores that were not significantly different when compared to fourth graders in Cohort A.

Null Hypothesis 5 [fifth grade]

There will be no difference in scores on the MAP between Cohort A and Cohort B in third, fourth, and fifth grade.

Fifth Grade Students

A *z*-test for difference in means was applied to the data for fifth grade students in each cohort. The *z*-test value of -1.921 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference of mean scores on the MAP test between the cohorts. The data showed that there was no difference in means between the cohorts of fourth graders. Results are shown in Table 17.

Table 17.

Cohort A and Cohort B MAP Index Scores: Fifth Grade

	5th A	5th B
Mean	670.080	684.340
Known Variance	1281.381	1473.331
Observations	50	50
z	-1.921	
P(Z<=z) one-tail	0.027	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 5a

There will be no difference in proportions of students scoring proficient and above in Cohort A and Cohort B in third, fourth, and fifth grade.

The second test applied to fifth grade students from Cohorts A and B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 1.715. The researcher failed to reject the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to Cohort A. This signified that fifth graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as Cohort A.

Null Hypothesis 5b

There will be no difference in variances in MAP test index scores of students in Cohort A and Cohort B in third, fourth, and fifth grade.

The third test applied to fifth graders in Cohorts A and B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.260. The researcher did not reject the null hypothesis that there was not a difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between the fifth graders of Cohort A and Cohort B. Table 18 shows the results of the *F*-test.

Table 18.

Cohort A and Cohort B Difference in Variance: Fifth Grade

	5th B	5th A
Mean	684.340	670.080
Variance	1473.331	1281.381
Observations	50	50
df	49	49
F	1.150	
P(F<=f) one-tail	0.314	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overall, in the analysis of fifth graders from Cohorts A and B, the researcher failed to reject the null hypothesis on all three tests, which meant that fifth graders in Cohort B had scores that were not significantly different when compared to fifth graders in Cohort A.

Overview of hypothesis 5.

In the comparison of third, fourth, and fifth grade students of Cohorts A and B, the researcher failed to reject the null hypothesis in seven of the nine tests, which showed that the Cohorts' scores on the MAP tests were not significantly different from each other.

Null Hypothesis 6 [third grade]

There is no difference in average math MAP test scores of female students and students in Cohort B.

Third Grade Students

A *z*-test for difference in means was applied to the data for female third grade students and students in Cohort B. The *z*-test value of 0.270 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference of mean scores on the MAP test. The data supported that third grade female students had no difference in scores when compared to third grade students in Cohort B. Results are shown in Table 19.

Table 19.

Cohort B MAP Index Scores: Third Grade Female Students

	3rd Female	3rd Cohort B
Mean	634.720	632.940
Known Variance	1043.879	1128.058
Observations	50	50
z	0.270	
P(Z<=z) one-tail	0.394	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 6a

There will be no difference in proportions of students scoring proficient and above of female students and students in Cohort B.

The second test applied to third grade female students and Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 0.205. The researcher failed to reject the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to female students. This signified that third graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as female students.

Null Hypothesis 6b

There will be no difference in variances in MAP test index scores of female students and students in Cohort B.

The third test applied to female third graders and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.081. The researcher did not reject the null hypothesis that there was not a difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between female students and students in Cohort B. Table 20 shows the results of the *F*-test.

Table 20.

Cohort B Difference in Variance: Third Grade Female Students

	3rd Cohort B	3rd Female
Mean	632.940	634.720
Variance	1128.058	1043.879
Observations	50	50
df	49	49
F	1.081	
P(F<=f) one-tail	0.394	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overall, in the analysis of female third graders and Cohort B, the researcher failed to reject the null hypothesis on all three tests, which meant that third graders in Cohort B had scores that were not significantly different when compared to female third graders.

Null Hypothesis 6 [fourth grade]

There is no difference in average math MAP test scores of female students and students in Cohort B.

Fourth Grade Students

A *z*-test for difference in means was applied to the data for female fourth grade and students in Cohort B. The *z*-test value of 0.168 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference of mean scores on the MAP test. The data supported that fourth grade female students had no difference in scores when compared to fourth grade students in Cohort B. Results are shown in Table 21.

Table 21.

Cohort B MAP Index Scores: Fourth Grade Female Students

	4th Female	4th Cohort B
Mean	659.840	658.740
Known Variance	1179.566	970.4004
Observations	50	50
z	0.168	
P(Z<=z) one-tail	0.433	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 6a

There will be no difference in proportions of students scoring proficient and above of female students and students in Cohort B.

The second test applied to fourth grade female students and Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 1.236. The researcher failed to reject the null that there was no increase in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to female students. This signified that fourth graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as female students.

Null Hypothesis 6b

There will be no difference in variances in MAP test index scores of female students and students in Cohort B.

The third test applied to fourth grade female students and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.216. The researcher did not reject the null hypothesis that there was not a difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between female students and students in Cohort B. Table 22 shows the results of the *F*-test.

Table 22.

Cohort B Difference in Variance: Fourth Grade Female Students

	4th Female	4th Cohort B
Mean	659.840	658.740
Variance	1179.566	970.400
Observations	50	50
df	49	49
F	1.216	
P(F<=f) one-tail	0.249	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overall, in the analysis of female fourth graders and Cohort B, the researcher failed to reject the null hypothesis on all three tests, which meant that fourth graders in Cohort B had scores that were not significantly different when compared to female fourth graders.

Null Hypothesis 6 [fifth grade]

There is no difference in average math MAP test scores of female students and students in Cohort B.

Fifth Grade Students

A *z*-test for difference in means was applied to the data for female fifth grade and students in Cohort B. The *z*-test value of 0.795 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference of mean scores on the MAP test. The data supported that fifth grade female students had no difference in scores when compared to fifth grade students in Cohort B. Results are shown in Table 23.

Table 23.

Cohort B MAP Index Scores: Fifth Grade Female Students

	5th Female	5th Cohort B
Mean	690.340	684.340
Known Variance	1372.107	1473.331
Observations	50	50
z	0.795	
P(Z<=z) one-tail	0.213	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 6a

There will be no difference in proportions of students scoring proficient and above of female students and students in Cohort B.

The second test applied to fifth grade female students and Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 0.456. The researcher failed to reject the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to female students. This signified that fifth graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as female students.

Null Hypothesis 6b

There will be no difference in variances in MAP test index scores of female students and students in Cohort B.

The third test applied to fifth grade female students and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.074. The researcher did not reject the null hypothesis that there was no difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between female students and students in Cohort B. Table 24 shows the results of the *F*-test.

Table 24.

Cohort B Difference in Variance: Fifth Grade Female Students

	5th Cohort B	5th Female
Mean	684.340	690.340
Variance	1473.331	1372.107
Observations	50	50
df	49	49
F	1.074	
P(F<=f) one-tail	0.402	
F Critical one-tail	1.607	

Note. Confidence Level = 95%

Overall, in the analysis of female fifth graders and Cohort B, the researcher failed to reject the null hypothesis on all three tests, which meant that fifth graders in Cohort B had scores that were not significantly different when compared to female fifth graders.

Overview of hypothesis 6.

In the comparison of third, fourth, and fifth grade female students and students in Cohort B, the researcher failed to reject the null hypothesis in all nine tests, which indicated that the female's scores on the MAP tests were not significantly different from the students of Cohort B.

Null Hypothesis 7 [third grade]

There is no difference in average math MAP test scores of Black students and students in Cohort B.

Third grade students.

A *z*-test for difference in means was applied to the data for third grade Black students and students in Cohort B. The *z*-test value of -1.619 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was no difference of mean scores on the MAP test. The data supported that third grade Black students had no difference in scores when compared to third grade students in Cohort B. Results are shown in Table 25.

Table 25.

Cohort B MAP Index Scores: Third Grade Black Students

	3rd Black	3rd Cohort B
Mean	610.900	632.940
Known Variance	4884.093	1128.058
Observations	30	50
z	-1.619	
P(Z<=z) one-tail	0.053	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 7a

There will be no difference in proportions of students scoring proficient and above of Black students and students in Cohort B.

The second test applied to Black third grade students and students in Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 1.344. The researcher failed to reject the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to Black students. This signified that third graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as Black students.

Null Hypothesis 7b

There will be no difference in variances in MAP test index scores of Black students and students in Cohort B.

The third test applied to Black third graders and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.699. The *F*-test value was found to be 4.330. The researcher rejected the null hypothesis that there was no difference in variance in scores on the MAP test. Therefore, there was a statistical difference in the variance of MAP scores between Black students and students in Cohort B. Table 26 shows the results of the *F*-test.

Table 26.

Cohort B Difference in Variance: Third Grade Black Students

	3rd Black	3rd Cohort B
Mean	610.900	632.940
Variance	4884.093	1128.058
Observations	30	50
df	29	49
F	4.330	
P(F<=f) one-tail	3.26E-06	
F Critical one-tail	1.699	

Note. Confidence Level = 95%

Overall, in the analysis of Black third graders and Cohort B, the researcher failed to reject the null hypothesis on two of the three tests, which signified that third graders in Cohort B had scores that were not significantly different when compared to Black third graders on two of the three tests.

Null Hypothesis 7 [fourth grade]

There is no difference in average math MAP test scores of Black students and students in Cohort B.

Fourth grade students.

A *z*-test for difference in means was applied to the data for Black fourth grade and students in Cohort B. The *z*-test value of -2.249 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was no difference in mean scores on the MAP test. The data supported that fourth grade Black students had no difference in scores when compared to fourth grade students in Cohort B. Results are shown in Table 27.

Table 27.

Cohort B MAP Index Scores: Fourth Grade Black Students

	4th Black	4th Cohort B
Mean	642.000	658.740
Known Variance	1079.790	970.400
Observations	30	50
z	-2.249	
P(Z<=z) one-tail	0.012	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 7a

There will be no difference in proportions of students scoring proficient and above of Black students and students in Cohort B.

The second test applied to fourth grade Black students and Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 0. The researcher failed to reject the null that there was no increase in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to Black students. This signified that fourth graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as Black students.

Null Hypothesis 7b

There will be no difference in variances in MAP test index scores of Black students and students in Cohort B.

The third test applied to fourth grade Black students and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.699. The *F*-test value was found to be 1.113. The researcher did not reject the null hypothesis that there was no difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between Black students and students in Cohort B. Table 28 shows the results of the *F*-test.

Table 28.

Cohort B Difference in Variance: Fourth Grade Black Students

	4th Black	4th Cohort B
Mean	642.000	658.740
Variance	1079.790	970.400
Observations	30	50
df	29	49
F	1.113	
P(F<=f) one-tail	0.363	
F Critical one-tail	1.699	

Note. Confidence Level = 95%

Overall, in the analysis of Black fourth graders and Cohort B, the researcher failed to reject the null hypothesis on all three tests, which meant that fourth graders in Cohort B had scores that were not significantly different when compared to Black fourth graders.

Null Hypothesis 7 [fifth grade]

There is no difference in average math MAP test scores of Black students and students in Cohort B.

Fifth grade students.

A *z*-test for difference in means was applied to the data for Black fifth grade and students in Cohort B. The *z*-test value of -1.358 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was no difference of mean scores on the MAP test. The data supported that fifth grade Black students had no difference in scores when compared to fifth grade students in Cohort B. Results are shown in Table 29.

Table 29.

Cohort B MAP Index Scores: Fifth Grade Black Students

	5th Black	5th Cohort B
Mean	670.900	684.340
Known Variance	2054.510	1473.330
Observations	30	50
z	-1.358	
P(Z<=z) one-tail	0.087	
z Critical one-tail	1.645	

Note. Confidence Level = 95%

Null Hypothesis 7a

There will be no difference in proportions of students scoring proficient and above of Black students and students in Cohort B.

The second test applied to fifth grade Black students and Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 2.038. The researcher rejected the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to Black students. This signified that fifth graders in Cohort B had a statistically higher proportion of students who earned proficient or advanced on the MAP test than Black students.

Null Hypothesis 7b

There will be no difference in variances in MAP test index scores of Black students and students in Cohort B.

The third test applied to fourth grade Black students and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.699. The *F*-test value was found to be 1.394. The researcher did not reject the null hypothesis that there was no difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between Black students and students in Cohort B. Table 30 shows the results of the *F*-test.

Table 30.

Cohort B Difference in Variance: Fifth Grade Black Students

	5th Black	5th Cohort B
Mean	670.900	684.340
Variance	2054.510	1473.330
Observations	30	50
df	29	49
F	1.394	
P(F<=f) one-tail	0.150	
F Critical one-tail	1.699	

Note. Confidence Level = 95%

Overall, in the analysis of Black fifth graders and Cohort B, the researcher failed to reject the null hypothesis on two of the three tests, which meant that fourth graders in Cohort B had scores that were not significantly different when compared to Black fourth graders on two of the three tests.

Overview of hypothesis 7.

In the comparison of third, fourth, and fifth grade Black students and students in Cohort B, the researcher failed to reject the null hypothesis in seven of the nine tests, which indicated that Black student's scores on the MAP tests were not significantly different from the students of Cohort B on seven of the nine tests.

Null Hypothesis 8 [third grade]

There is no difference in average math MAP test scores of students on Free and Reduced Lunch and students who are in Cohort B.

Third grade students.

A *z*-test for difference in means was applied to the data for third grade students who had Free and Reduced Lunch status and students in Cohort B. The *z*-test value of -0.836 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was no difference in mean scores on the MAP test. The data supported that third grade students who had Free and Reduced Lunch status had no difference in scores when compared to third grade students in Cohort B. Results are shown in Table 31.

Table 31

Cohort B MAP Index Scores: Third Grade Free and Reduced Lunch

	3rd F/R	3rd Cohort B
Mean	626.720	632.940
Known Variance	1637.800	1128.060
Observations	50	50
z	-0.836	
P(Z<=z) one-tail	0.201	
z Critical one-tail	1.645	

Note. Confidence Level = 95%; F/R = Students with Free and Reduced Lunch status

Null Hypothesis 8a

There will be no difference in proportions of students scoring proficient and above of Free and Reduced Lunch students and students in Cohort B.

The second test applied to third grade students with Free and Reduced Lunch status and students in Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 1.010. The researcher failed to reject the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to students with Free and Reduced Lunch status. This signified that third graders in Cohort B had a similar proportion of students who earned proficient or advanced on the MAP test as students with Free and Reduced Lunch status.

Null Hypothesis 8b

There will be no difference in variances in MAP test index scores of Free and Reduced Lunch students and students in Cohort B.

The third test applied to third graders with Free and Reduced Lunch status and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.452. The researcher failed to reject the null hypothesis that there was no increase of variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between students with Free and Reduced Lunch status and students in Cohort B. Table 32 shows the results of the *F*-test.

Table 32.

Cohort B Difference in Variance: Third Grade Free and Reduced Lunch

	3rd F/R	3rd Cohort B
Mean	626.720	632.940
Variance	1637.800	1128.060
Observations	50	50
df	49	49
F	1.452	
P(F<=f) one-tail	0.098	
F Critical one-tail	1.607	

Note. Confidence Level = 95%; F/R = Students who had Free and Reduced Lunch status

Overall, in the analysis of third graders that had Free and Reduced Lunch status and Cohort B, the researcher failed to reject the null hypothesis on all three tests, which signified that third graders in Cohort B had scores that were not significantly different when compared to third graders who had Free and Reduced Lunch status.

Null Hypothesis 8 [fourth grade]

There is no difference in average math MAP test scores of students on Free and Reduced Lunch and students who are in Cohort B.

Fourth grade students.

A *z*-test for difference in means was applied to the data for fourth grade students who had Free and Reduced Lunch status and students in Cohort B. The *z*-test value of -1.865 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was not a difference of mean scores on the MAP test. The data supported that fourth grade students who had Free and Reduced Lunch status had no difference in scores when compared to fourth grade students in Cohort B. Results are shown in Table 33.

Table 33.

Cohort B MAP Index Scores: Third Grade Free and Reduced Lunch

	4th F/R	4th Cohort B
Mean	646.880	658.740
Known Variance	1052.110	970.400
Observations	50	50
z	-1.865	
P(Z<=z) one-tail	0.031	
z Critical one-tail	1.645	

Note. Confidence Level = 95%; F/R = Students with Free and Reduced Lunch status

Null Hypothesis 8a

There will be no difference in proportions of students scoring proficient and above of Free and Reduced Lunch students and students in Cohort B.

The second test applied to fourth grade students with Free and Reduced Lunch status and students in Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 2.417. The researcher rejected the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to students with Free and Reduced Lunch status. This signified that fourth graders in Cohort B had a higher proportion of students who earned proficient or advanced on the MAP test than students with Free and Reduced Lunch status.

Null Hypothesis 8b

There will be no difference in variances in MAP test index scores of Free and Reduced Lunch students and students in Cohort B.

The third test applied to fourth graders with Free and Reduced Lunch status and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.084. The researcher failed to reject the null hypothesis that there was no increase of variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between students with Free and Reduced Lunch status and students in Cohort B. Table 34 shows the results of the *F*-test.

Table 34.

Cohort B Difference in Variance: Fourth Grade Free and Reduced Lunch

	4th F/R	4th Cohort B
Mean	646.880	658.740
Variance	1052.110	970.400
Observations	50	50
df	49	49
F	1.084	
P(F<=f) one-tail	0.389	
F Critical one-tail	1.607	

Note. Confidence Level = 95%; F/R = Students who had Free and Reduced Lunch status

Overall, in the analysis of fourth graders that had Free and Reduced Lunch status and Cohort B, the researcher failed to reject the null hypothesis on two of the three tests, which signified that third graders in Cohort B had scores that were not significantly different when compared to fourth graders who had Free and Reduced Lunch status on two of the three tests.

Null Hypothesis 8 [fifth grade]

There is no difference in average math MAP test scores of students on Free and Reduced Lunch and students who are in Cohort B.

Fifth grade students.

A *z*-test for difference in means was applied to the data for fifth grade students who had Free and Reduced Lunch status and students in Cohort B. The *z*-test value of -1.607 was not in the critical region, therefore the researcher failed to reject the null hypothesis that there was no difference of mean scores on the MAP test. The data supported that fifth grade students with Free and Reduced Lunch status had no difference in scores when compared to fifth grade students in Cohort B. Results are shown in Table 35.

Table 35.

Cohort B MAP Index Scores: Fifth Grade Free and Reduced Lunch

	5th F/R	5th Cohort B
Mean	672.160	684.340
Known Variance	1397.400	1473.330
Observations	50	50
z	-1.607	
P(Z<=z) one-tail	0.05398	
z Critical one-tail	1.645	

Note. Confidence Level = 95%; F/R = Students with Free and Reduced Lunch status

Null Hypothesis 8a

There will be no difference in proportions of students scoring proficient and above of Free and Reduced Lunch students and students in Cohort B.

The second test applied to fifth grade students with Free and Reduced Lunch status and students in Cohort B was a *z*-test for a difference in proportions. The critical values were -1.960 and +1.960 on the two-tailed test. The *z*-test value was 2.306. The researcher rejected the null that there was no difference in the proportion of students in Cohort B who scored proficient or advanced on the MAP test when compared to students with Free and Reduced Lunch status. This signified that fifth graders in Cohort B had a higher proportion of students who earned proficient or advanced on the MAP test than students with Free and Reduced Lunch status.

Null Hypothesis 8b

There will be no difference in variances in MAP test index scores of Free and Reduced Lunch students and students in Cohort B.

The third test applied to fifth graders with Free and Reduced Lunch status and students in Cohort B was an *F*-test for a difference in variances. The critical value for the test was 1.607. The *F*-test value was found to be 1.054. The researcher failed to reject the null hypothesis that there was no difference in variance in scores on the MAP test. Therefore, there was no difference in the variance of MAP scores between students with Free and Reduced Lunch status and students in Cohort B. Table 36 shows the results of the *F*-test.

Table 36.

Cohort B Difference in Variance: Fifth Grade Free and Reduced Lunch

	5th Cohort B	5th F/R
Mean	684.340	672.160
Variance	1473.330	1397.400
Observations	50	50
df	49	49
F	1.054	
P(F<=f) one-tail	0.427	
F Critical one-tail	1.607	

Note. Confidence Level = 95%; F/R = Students with Free and Reduced Lunch status

Overall, in the analysis of fifth graders with Free and Reduced Lunch status and Cohort B, the researcher failed to reject the null hypothesis on two of the three tests, which signified that third graders in Cohort B had scores that were not significantly different when compared to fourth graders who had Free and Reduced Lunch status on two of the three tests.

Overview of hypothesis 8.

In the comparison of third, fourth, and fifth grade students with Free and Reduced Lunch status and students in Cohort B, the researcher failed to reject the null hypothesis in seven of the nine tests, which indicated that Black student's scores on the MAP tests were not significantly different from the students of Cohort B on seven of the nine tests.

Conclusion

This was a quantitative study conducted to determine whether a relationship between the Singapore Math program and student achievement on the MAP existed. Analysis of the data resulted in the failure to reject the eight null hypotheses stating there was no difference in student achievement on the MAP tests. The implications of the failure to reject, however, differ between each of the hypotheses, warranting further study of the Singapore Math program.

Data from this study did not support the establishment of a correlation between students exposed to the Singapore Math program since first grade (Cohort B) and a higher understanding of math as measured by the MAP test. The data did, however, indicate on hypotheses six, seven, and eight that subgroups were able to perform commensurately with their peers on the MAP when the Singapore Math program was utilized. Chapter Five discusses the results and implications of the data analysis. Further analysis and recommendations are also discussed.

Chapter Five: Discussion and Reflection

Mathematics achievement in American classrooms were under scrutiny for many years, especially when it was compared to other industrialized countries on international assessments, such as the TIMSS. Educators and legislatures focused on finding solutions to make U.S. students more competitive in math, and therefore more competitive in the job market. One such effort was studying the implementation of the Singapore Math program, in hopes that students would leave elementary school with a solid foundation of mathematics in place, the motivation to take more advanced math courses, and the persistence to lead to a higher proportion of students receiving college degrees.

This study analyzed MAP data to determine whether a correlation could be found between the implementation of the Singapore Math program and increased achievement in mathematics in the studied school district. MAP scores from the mathematics portion of the test served as the independent variable, because it was an assessment given consistently to the two cohorts of students whose data were analyzed. The MAP also assessed the content knowledge students in Missouri were expected to acquire throughout a given school year. The dependent variable was consistent use of the Singapore Math program in the classrooms of the elementary students included in Cohort A and Cohort B. Cohort A was comprised of students in fifth grade in 2009. Most of these students did not have exposure to the Singapore Math program until 2008, unless they were in a classroom piloting the Singapore Math program. Cohort B included students who received curriculum from the Singapore Math program throughout their elementary years.

The Singapore Math program was developed by Singapore's Ministry of Education. It utilized a variety of strategies that were unique among other programs used in U.S. classrooms. The Singapore Math program contained fewer topics of study per year, focusing on mastery of the topics, with less time spent reviewing previously taught material. Its topics were intentionally sequenced in order to enhance the learning process and utilize an understanding of mathematics from a concrete-to-pictorial-to-abstract perspective, through a balance of problem solving and drill activities. Its flagship problem solving strategy, Model Drawing, provided students with a consistent visual model with which to solve problems and create a connection to algebraic thinking.

Triangulation of Results

This study did not definitively prove a correlation between the use of the Singapore Math program and mathematics achievement on the MAP test; however, trends did show the use of the program should be considered for public school elementary children. Table 37 indicated that despite the district growth of roughly 500 to 600 students per year, percentages of the population scoring Proficient or Advanced on the MAP continued to increase, as the Singapore Math program became instituted district-wide in elementary classrooms. Fifth graders in the district earning Proficient or Advanced in 2009 had increased 11% from when the students were enrolled in third grade to 57.1%. During 2012, fifth graders who had received Singapore Math teaching since the first grade had 55.1% of third graders scoring Proficient or Advanced, and increased by 14.7% to 69.8% by the end of fifth grade. A side-by-side comparison of the two study populations during the time of Singapore Math implementation showed students in Cohort B had a higher percentage of students scoring Proficient or Advanced in each of the grade levels and had a larger increase of proficiency when compared to Cohort A. This conclusion was not as apparent from the random sampling of students in this study. It should be noted, however, that the *z*-test for difference in proportions indicated the proportions of proficiency for Cohorts A and B were statistically different from one another. The third and fourth grade students of Cohort B had a statistically higher percentage of students who scored Proficient or Advanced, indicating growth during the time of implementation of the Singapore Math program.

Table 37

Population Proficiency Percentages: Grades Three, Four, and Five

Grade	**Population A (%)**	**Population B (%)**
3rd	46.1	55.1
4th	46.4	60.4
5th	57.1	69.8

Note. Population A = Population of students from which Cohort A was selected; Population B = Population of students from which Cohort B was selected.

When the subgroups of students (female, Black, and Free and Reduced Lunch) were compared to Cohort B, results were commensurate. This was particularly true with the female student group. In all tests applied, the female students scored statistically similar to the students of Cohort B. This indicated that while Singapore Math program was implemented district-wide, female, Black, and students with Free and Reduced Lunch status performed similarly to Cohort B on the math portion of the MAP test. It was noted, however, the *z*-test for difference in proportions showed a statistical deficit in the analysis. Fifth grade Black students and fourth and fifth grade students with Free and Reduced Lunch status scored statistically lower than students in Cohort B.

Personal Reflections

I wish that I could say that the implementation of the Singapore Math program had a positive correlation with student achievement on the mathematics MAP test. When dealing with a study that addressed student achievement, it was difficult to measure one component in isolation. There were several factors that played a factor in student progress: teachers' math competencies, teachers' effectiveness and fidelity in delivering the material, students' backgrounds, and students' confidence, among many others variables, helped determine how students learned, connected with, and performed mathematics tasks and assessments. Still, data gathered directly from MODESE, as well as the data in this study indicated that a program such as Singapore Math was beneficial for the students in the studied district.

Part of the reason my results did not yield a correlation was my sample size was relatively small. By limiting the sample to a maximum of 50 students, I did not see the type of growth I had expected through analysis of the proficiency rates provided from MODESE and the district. In this study, even a relatively small number of low achieving students could have influenced the outcome of the three tests applied to the data.

As an administrator in the studied district and former middle school math teacher, I found that students in my building seemed to enjoy and understand math concepts more than they did a few years previous to this analysis of mathematics performance. The components that made up a Singapore Math lesson kept the students focused on solving real-world problems and could be scaffolded for struggling students through the implementation of manipulatives and the Bar Model process. In general, problem-solving skills were increasing, but upper, elementary and middle school teachers still found that students did not quickly memorize their multiplication or division facts.

One of the benefits of the Singapore Math program was that teachers began to more deeply understand math. After new teachers attended the week-long summer training held at the school district, I was approached by a teacher who said, ‘If only I had been taught math this way,’ or ‘I would have understood math so much better if I had learned this way!’, or ‘Now I understand!’ Our teachers left the workshop with renewed vigor to teach mathematics.

While I am pleased that the training was enhancing our teachers’ depth of math understanding, I am disheartened at the number of teachers at the elementary level that did not have a solid foundation in math, yet were asked to stand in front of a class of students each day and pretend to know mathematics. If our teachers do not have an understanding of math, it is unreasonable to assume that our students will understand any better. If a week-long training gave valuable insight to teachers, what would a semester do? Our teachers need to have a clear understanding and confidence in math before our students can. I contend that teacher preparation programs in the United States should be reviewed in order to ensure that teachers (particularly at the elementary level) receive not only adequate training in math pedagogy, but also in math content. Countries, such as Singapore follow this model and their students have repeated international success in mathematics. We would then have highly trained teachers paving a clear path for our next generation of engineers, scientists, carpenters, and teachers, because of the confidence that a deep understanding of mathematics can provide in today’s economy.

Recommendations to the Program

Although quantitative data for the Singapore Math program were not positively correlated in all areas, the studied district has seen an increase of student achievement on the MAP since the implementation of the program. Percentages of the population indicated clear and consistent gains despite the sharp growth of student population the district experienced over the several years previous to this writing. While there were many variables, the Singapore Math program was one constant that was implemented in the district by all elementary buildings.

The program should continue to be offered to students on a daily basis for at least 60 minutes, uninterrupted, and utilize all five of the components of the lesson. The five components ensure that students are strengthening their math skills in a variety of areas: mental math, acquiring and applying new skills, and problem-solving. To consistently cut math time short would be a disservice to students, as the components were designed to intentionally support one another. Since data showed there was still a wide variance between high and low-performing students, built-in time to work with small groups or individuals should be examined as a viable option during independent work time. This strategy could be used to reteach the day's lesson, or give other supports to contribute to the close of the achievement gap.

The study-district program should continue to offer professional development annually, but expand the opportunity to attend to all teachers in the district, whether new or experienced, so they may continue to learn and hone math skills and pedagogical strategies. The district should also continue providing new teachers professional development throughout the year to ensure they are delivering the material with fidelity and confidence to the students. In the past, it cost the district roughly $60,000 to provide the annual training to new teachers. This covered the cost of the two presenters (at $10,000 each) and their accommodations, as well as teacher stipends for attending the training.

It would also be beneficial for middle school teachers to attend Singapore Math professional development, in order to bridge the gap between the Singapore Math program and the various math courses offered at the middle school level. In my experience, this was often a point of dissension with parents. Strategies that were taught in the elementary level were often ignored, and students were told to learn an entirely different method to solve a problem. By providing professional development to middle school teachers, incorporation of a smooth transition can take place and the students can continue to add to their learning in middle school, rather than unlearn what they were taught and then learn replacement strategies.

The program should reinstitute the use of Parent Nights at each of the elementary schools. Parent Nights would take place three times throughout the school year and were designed to assist parents in understanding the math program. Parents would break into groups, based on the grade level of their children, and the teachers would teach them math lessons utilizing Singapore Math's strategies and manipulatives. At the end of the evening, there was a question and answer session to address concerns of the parents. During the first few years of implementation of the Singapore Math program (2008 and 2009), Parent Nights were mandatory. As the program became a part of the district's culture, Parent Nights were discontinued. With the amount of growth the district was experiencing and the need for home support in mathematics, holding quarterly meetings to address math would be beneficial to student success. Additionally, new tutorial videos should be posted on the website, so parents can get an understanding of the methods used at school and help their children.

Finally, additional resources should be readily available to teachers in order to support the Singapore Math program. Often, it was reported that teachers needed to research or recreate simple tools because they did not exist or were not available in a digital format. Providing a wider range of resources for teachers could allow focus on the content they are teaching, rather than focus on creating visuals for students. Students would be able to see math in context more readily and connect abstract ideas to the world around them. Additionally, digital resources can easily be shared with parents to help support students at home.

Recommendations for Future Research

This study of the effectiveness of the Singapore Math program over time was not sufficient enough to truly evaluate the program's success. Follow-up studies should investigate cohorts of students from two similar school districts; one district that utilizes the Singapore Math program and one that does not, in order to determine how effective the program is when compared to a different group of students taking the same test. Future research could then determine if a relationship could be found with achievement on the MAP and the Singapore Math program. Another option would be to analyze a different set of data, such as common assessments, in addition to MAP data in order to triangulate results and compare the findings.

Finally, since attitudes regarding math play a vital role in achievement, it is recommended that future studies analyze students' and teachers' feelings of effectiveness of the Singapore Math program. The qualitative study could test data to see if there is a relationship between their attitudes about the program and student achievement.

Conclusion

In the study of the effectiveness of the Singapore Math program, the results were mixed. In the comparison of Cohorts A and B, there was no significant difference between the two groups on the math portion of the MAP test, despite state data that showed consistent improvement over a six year timeline. It was evident, however that the three subgroups compared to Cohort B performed commensurately at the time the Singapore Math program was implemented district-wide. Female students performed most equally, scoring statistically similar to Cohort B in all nine statistical comparisons performed. While Black students and students with Free and Reduced Lunch performed commensurate with Cohort B, it should be noted that three statistical tests yielded results which showed that the percentage of students achieving Proficient or Advanced scores was statistically lower than Cohort B.

Despite the success of the Singapore Math program in the district, not all students were able to graduate high school and meet the demands of the 21st Century workforce. Certainly, implementing a program such as Singapore would help to address gaps in content and introduce algebraic thinking at an earlier age and prepare students for Algebra courses. However, going beyond a math program to address issues such as race, socioeconomic status, and teacher preparation are key in finding a more proactive and lasting solution to America’s difficulty with mathematics. American educators should continue to research ways to support students in math, with proficiency as the ultimate goal in order to help students become successful in math and problem-solving skills, because these skills are necessities in today’s economy regardless of the avenues students pursue after high school.

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Vitae

Justin Reynolds was raised in Troy, Missouri. He graduated from Troy-Buchanan High School in 1997. He later attended Central Missouri State University and earned a Bachelor's of Science degree in elementary education in 2001. Upon graduation, Mr. Reynolds taught third grade at a rural school, and later, was a sixth grade math teacher in a suburban district. While teaching, he earned his master's degree in administration from Lindenwood University in 2007. In 2008, he became an assistant principal in the Wentzville School District. He continued his education courses at Lindenwood, earning a Specialist in Educational Leadership degree in 2011 and plans to graduate with a doctorate in educational leadership in December of 2014. He now resides in O'Fallon, Missouri, with his wife, Whitney, and his two children, Brock and Leila.