Fall 11-18-2019

Educators' Perceptions Concerning the Leading Factors of Mathematics Achievement in Top-Ranking Nations Around the World

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by

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November 18, 2019

A Dissertation submitted to the Education Faculty of Lindenwood University in partial fulfillment of the requirements for the degree of

Doctor of Education

School of Education

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Achievement in Top-Ranking Nations

Around the World

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This Dissertation has been approved as partial fulfillment

of the requirements for the degree of

Doctor of Education

Lindenwood University, School of Education

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Declaration of Originality

I do hereby declare and attest to the fact that this is an original study based solely upon my own scholarly work at Lindenwood University and that I have not submitted it for any other college or university course or degree.

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Acknowledgments

I would like to recognize and extend a heartfelt thank you to my dissertation committee, Dr. Julie Williams, Dr. Kathy Grover, Dr. Dennis Cooper, and Dr. Sherry DeVore for their unwavering support and expertise throughout the doctoral process. I want to acknowledge and express my gratitude to the extremely knowledgeable post-secondary professors who took part in this study; your time and insight concerning mathematics education are invaluable, and I will carry it with me in every educational endeavor. To my colleagues and administrators at Mansfield R-IV School District, your patience and encouragement have meant the world, especially Dr. Jamie Walker-Davidson, who provided enjoyable discussions and continuous motivation on the way to class and throughout this study. Also, to my mother, Alane Mitchell, and my mother- and father-in-law, Tammy and Verne Snow, I will always be grateful for you pushing me to be a better version of myself. My deepest appreciation goes to my husband, Brandon Snow, and my three beautiful children, Wyatt, David, and Isaac. Your time and sacrifice were immeasurable, along with your encouragement and inspiration; I will always be indebted to all of you. Finally, I would like to express my sincere gratitude to all who made this study possible, who include, but are not limited to those listed. Throughout time, from life's many adventures and trials, your immense support and wisdom have been of great value and will always be treasured.

Abstract

The purpose of this study was to examine the perceptions of post-secondary mathematics educators from countries considered to be highly successful in the area of mathematics based upon results of international assessments. Four instructors were randomly selected from a homogenous sample within the nations of Canada, China, Japan, and Singapore to contribute information about the phenomenon of mathematical achievement. Frameworks from SRI International (2009), Matthews (2013), and Schoenfeld (2014) were integrated to design this study focused on curriculum, pedagogy and instructional approach, teacher knowledge and expectations, and organizational and social climate as possible factors that support student mathematical proficiency. Interviews were conducted using a consistent set of open-ended questions based upon the conceptual framework. Data were examined and analyzed to extract commonalities and differences among responses. To bring about mathematical improvement for a collective population of students, the following variables must be present: (1) cohesive and coherent standards within a curriculum based upon a balanced approach to procedural knowledge and deeper learning within a collaborative setting; (2) local and federal initiatives fostering the concept and importance of students' maintaining a mathematical mindset; and (3) effective and frequent professional development grounded in data-driven goals, research-based pedagogy, and opportunities for reflection shared with experts in the field of mathematics.

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Chapter One: Introduction

To many around the world, it is believed a quality education will lead to economic prosperity and a competitive edge in today's 21st-century global markets (Ball, 2017; Enderson & Ritz, 2016; Leung, Leung, & Zuo, 2014; Sparapani, Callejo Perez, Gould, Hillman, & Clark, 2014). The National Research Council asserted many occupations require mathematics skills necessary to function in the workplace, and those skills correlate to the possibility of increased wages (as cited in Larson & Kanold, 2016). Hattie (2017) described the need for mathematics skills as a way to get through the significant milestones of life, such as graduating high school, earning a college education, and in general, "hav[ing] a higher quality of life" (p. 1). Hattie (2017) suggested research indicates many college programs with the greatest earning potential demand a strong understanding of mathematics.

Within the past decade, due to extreme fluctuations of economic stability, policymakers around the world have begun to shift their focus to promote preparedness of their graduates in the areas of science, technology, engineering, and mathematics (STEM) (Bell, 2016; Enderson & Ritz, 2016). According to Costa (2017), "Since 2007, computer and math jobs have grown by 21%, which is faster than any other educational category" (p. 32). In addition, "The United States STEM workforce has grown at more than four times the rate of total employment" (Hossain & Robinson, 2012, p. 443). Enderson and Ritz (2016) noted within a progressive global market, technological advances fuel economic prosperity, which requires an advanced workforce equipped with a sophisticated set of problem-solving skills.

Several researchers have noted a deficient number of students pursuing studies in the area of STEM, as well as a lack of instructors qualified to teach STEM-based courses (Bell, 2016; Enderson & Ritz, 2016; Hossain & Robinson, 2012; Julie, 2014). While the subject of mathematics is only one aspect of STEM education, Enderson and Ritz (2016) highlighted this area as a weakness among the college-bound and those in the workforce who often lack a basic understanding of mathematics due to the inability to problem-solve, rationalize, and synthesize real-world scenarios. Post-secondary educators and multiple researchers have noted the high incidence of high school graduates (over 50%) who require remedial courses to progress to college-level mathematics classes to earn necessary credits (Houston & Yonghong, 2016). Bell (2016) determined educators must become aware of their ability to integrate STEM education into multiple facets of learning to further engage students' intrigue and future dedication in these disciplines.

A background of mathematics teaching and reform is provided in Chapter One to provide a working knowledge of the changes in ideology that have influenced mathematics instruction throughout the past century. Discussed are the current state of mathematics achievement and the difficulty faced by many American children in the area of mathematics. The need for further research is addressed to encourage government leaders, educational administrators, and instructors to adopt a different approach to mathematics instruction based upon the methods of those countries proven to be successful in the art of cultivating mathematical knowledge. The research questions that pertain to this study are presented, the meanings of educational terms are defined, and the limitations of this study are explained.

Background of the Study

The learning of mathematics traces back to ancient Babylonian times (Kilpatrick, 2014). One of the first published works to outline a prescribed method for teaching mathematics in the United States was written by Nicolas Pike in 1788 and was titled *Arithmetic*; the book addressed the need for direct instruction, essential procedures, rote memorization, and repeated practice (Larson & Kanold, 2016). Throughout the 1800s, many works followed a model based on a rudimentary understanding of basic arithmetic at the elementary level (Larson & Kanold, 2016). It was not until the early 1900s when educational reform began to place a stronger emphasis on students' abstract understanding of mathematics at the secondary level (Larson & Kanold, 2016).

Mathematics education reform has been in flux throughout the past century, making another appearance during the Cold War era when competition to put the first satellite into space arose between the United States and Russia (Schoenfeld, 2016). The Russian launch of the satellite Sputnik into orbit in 1957 resulted in immediate educational reform, particularly in the fields of mathematics and science (Schoenfeld, 2016). At the time, Herbert Zelenko, New York Representative, stated, "Defense is no longer a matter of muscles and masses… Formulas and equations have taken the place of spears and guns" (as cited in Phillips, 2014, p. 458). He concluded, "Education is the true defense" (as cited in Phillips, 2014, p. 458).

This mantra of thinking brought about the new math era in the late 1950s (Larson & Kanold, 2016). Larson and Kanold (2016) explained the new math was a reform effort in which mathematics education evolved from rote learning and memorization to teaching in a manner to support discovery and a conceptual understanding of mathematics itself. The new math movement pushed through the 1960s, but in the early 1970s, critics of new math came out in force (Larson & Kanold, 2016).

Larson and Kanold (2016) mentioned an article published in the *Washington Post* in 1972 which detailed the story of a parent who became frustrated with new math as he tried to help his daughter with her elementary mathematics homework. The parent, who happened to be a chemist, found he did not understand the assignment and deemed it unnecessarily difficult (Larson & Kanold, 2016). Many politicians, educational leaders, and parents began to question the efficacy of the program, finding a higher priority was placed on abstract math skills, essentially replacing practical basic math skills necessary for daily living in and outside the home (Phillips, 2014).

As a result of the failed new math reform efforts in the mid-1970s, educational stakeholders went back to the old approach of procedural learning of basic math skills, acknowledging clear objectives, and direct instruction aimed at the proficiency of the standards (Larson & Kanold, 2016). This application of mathematics instruction continued until the 1980s when the National Council of Teachers of Mathematics (NCTM) made several attempts by publishing *An Agenda for Action* (1980) and *Curriculum and Evaluation Standards for School Mathematics* (1989) (Lester, 1994). Both of these were written to urge policymakers and educators to reform current mathematics standards and to stress the need for an intense emphasis on problem-solving (Lester, 1994). According to Kilpatrick (2014), these documents were noteworthy because they established the first attempt at a national curriculum in any subject at the elementary and secondary level and were not federally funded, allowing autonomy to proponents of standards-based reform.

However, similar to most mathematics reforms, there were opponents throughout the late 1990s who declared the standards set forth by the NCTM did not correctly emphasize procedural learning, memorization of basic math facts, and understanding of essential algorithms (McLeod, 2003). In contrast, they challenged the standards, which placed an unwarranted significance on areas such as data analysis and probability (Larson & Kanold, 2016). Once again, the battle ensued among two polarizing viewpoints: procedural learning, memorization, and direct instruction versus problem-solving, critical thinking, and conceptual understanding of mathematics (Larson & Kanold, 2016).

Conceptual Framework

This study was conducted to examine specific elements found in mathematics instruction by integrating the conceptual frameworks of SRI International (2009), Matthews (2013), and Schoenfeld (2016). Multiple frameworks were selected due to comparable components demonstrating significant results leading to mathematical improvement. By using collective frameworks, researchers have noted that an educator's pedagogy can be strengthened (Charalambous & Praetorius, 2018; Mincu, 2015). While many factors drive student achievement in the area of mathematics, the instruction provided by an expert instructor is a key component (Li & Kaiser, 2011). Initially, there were no specific guidelines for an intensification strategy for students performing below grade-level, although an increase of time or content is necessary for students to accelerate to the desired instructional level (SRI International, 2009).

To address the problem, SRI International (2009) developed a framework that includes five dimensions to be incorporated to improve mathematics instruction in a low-performing school. These five dimensions include the following:

1. Intensification Strategy
2. Curriculum
3. Pedagogy and Instructional Approach
4. Teacher Knowledge and Expectations
5. Organizational and Social Climate. (SRI International, 2009, p. 1)

According to Hunt and Little (2014), a program designed to utilize additional time to provide intense instructional interventions is referred to as Response to Intervention (RTI). Many educational researchers have prescribed methodologies for RTI programs, and RTI has been proven time and time again to be one of the most effective influences on increased academic success (Hattie, 2017).

When discussing an update to the curriculum, textbooks are considered an essential focus, as they are a reflection of the curriculum for all stakeholders in the learning process (Özer & Sezer, 2014). However, Castro Superfine, Marshall, and Kelso (2015) noted fidelity of the curriculum, as well as how the instructor implements the curriculum, are integral keys to determining the value of what students learn from the curriculum itself. In addition, SRI International (2009) placed an emphasis on measuring the value of content within the textbook and curriculum (e.g., mathematical problem-solving strategies and representations, pedagogical philosophies, and integration of technology).

As researchers continue to point out, an educator's instructional approach and the mathematical pedagogy he or she implements are also vital when educating young math minds (SRI International, 2009). Instructors should have the appropriate pedagogical content knowledge to provide intervention and remediate the struggles faced by students

(Depaepe et al., 2015). Matthews (2013) designed her framework around the hypothesis that an educator's expertise in pedagogical content knowledge has been proven to be an effective predictor of student achievement. The SRI International (2009) and Hattie (2017) outlined various pedagogical structures in mathematics, including cooperative skills, support for the use of cognitive skills, encouragement for students to explain their thought processes, and an increase in the implementation of formative assessments.

An educator's pedagogical content knowledge should not be confused with his or her content knowledge (Matthews, 2013). Depaepe et al. (2015) pointed out a teacher's content knowledge is a focus upon his or her procedural knowledge of the content, while pedagogical content knowledge is the understanding of how to prevent student misinterpretation of the content and procedures. Matthews (2013) cited multiple theories contrasting pedagogical content knowledge and content knowledge at the secondary level. Through Matthew's (2013) study, she found content knowledge reinforces pedagogical content knowledge, and the instructor's pedagogical content knowledge has a greater impact on student outcomes.

The SRI International (2009) determined an effective mathematics instructor must possess an extensive understanding of mathematics to facilitate tasks that allow students to dig deeper into the content presented. Students can only gain deeper, metacognitive thinking if the instructor expects the students to move toward greater learning outcomes (SRI International, 2009). Romberg (1984) addressed the fact educators who have lower expectations tend to focus on teaching remedial skills, while those with higher expectations move toward rich mathematical tasks that direct students to engage in deeper comprehension of the concept.

The final dimension noted for accelerating low mathematical performance was the need to improve the school's organizational structure and social climate (SRI International, 2009). This concept has gained considerable strength and inquiry among leaders in the fields of science and education (Dweck, 2014). Dweck (2014) defined growth mindset as a student's intellectual ability to develop over time; she cited two studies that encompassed the ideology of growth mindset with a group of seventh-grade math students that resulted in increased motivation among the students and higher scores on the following achievement assessment. Data also support the SRI International (2009) message that educators must be trained, and professional development should be implemented to provide educators with the understanding students' mathematical abilities are not static. Further, with proper motivation and support, students can find success and show considerable improvement in the area of mathematics (SRI International, 2009).

Statement of the Problem

It is evident progress can be measured by national and international mathematics assessments; however, the United States continues to demonstrate insufficient proficiency in the area of mathematics (Larson & Kanold, 2016). The National Center for Education Statistics (NCES) (2015) administers the National Assessment of Educational Progress (NAEP), which measures student progress in mathematics and reading in grades 4, 8, and 12 on a biannual basis in the United States. The results from the 2015 NAEP indicated while mathematics achievement has increased since 1992, only 40% of fourth-graders, 33% of eighth-graders, and 25% of 12th-graders scored at or above Proficient (National Center for Education Statistics [NCES], 2015).

In addition, the Organisation for Economic Co-operation and Development (OECD), who administers the Programme for International Students Assessment (PISA), reported in 2015, a sample of 15-year-olds who represented the United States scored below the international average in mathematics (OECD, 2016a). The OECD (2016a) claimed the PISA assesses 72 participating countries, which accounts for 540,000 students. It is worthy to note, since 2003 when the PISA mean scores of participating countries were made public, the United States has scored *below* the international average (NCES, 2017a). Furthermore, the countries examined in this current study (Canada, China, Japan, and Singapore) have scored *above* the international average of all OECD countries since 2003, unequivocally surpassing the student representatives of the United States in mathematics achievement (NCES, 2019b).

It is important to draw awareness to the 2015 Trends in International Mathematics and Science Study (TIMSS) since American eighth-grade students scored 10th out of 38 countries and ranked above the TIMSS scale centerpoint (Mullis, Martin, Foy, & Hooper, 2016). The United States (eighth grade) has scored slightly above the scale centerpoint in mathematics since the 1999 TIMSS; the only instance in which they scored slightly below the centerpoint was in 1995 (NCES, 2019b). Similar to results on the PISA, the United States achieved the lowest mathematics achievement scores since 1995, in comparison to the countries evaluated in this study (NCES, 2017a). It should be noted Canada did not participate in the 2007 or 2011 TIMSS (NCES, 2019b).

Purpose of the Study

The purpose of this study was to gain perspective on the crucial elements that contribute to students' mathematical achievement based upon a sample of countries consistently successful in the area of mathematics (OECD, 2016b; Tucker, 2011). A phenomenological study was conducted to examine the perceptions of post-secondary educators concerning the instruction provided by secondary instructors who lead in mathematics efficacy in the countries of Canada, China, Japan, and Singapore. Attention was focused on curriculum, secondary instructors' pedagogical knowledge, instructor content knowledge and expectations, and the organizational structure and social climate of classrooms.

By determining the attributes common among these countries, school systems, administrators, and educators can suggest strategies for further improvement in mathematics not necessarily dependent upon socioeconomic status or local culture (OECD, 2016b; Tucker, 2011). Educational leaders can then implement appropriate interventions that best meet the needs of their schools or classrooms to bring about more significant gains in mathematical achievement for students (Hattie, 2017).

Research questions. The following research questions guided the study:

1. What leading factors do post-secondary mathematics instructors attribute to the academic achievement of secondary students, based upon the performance of secondary educators within the countries studied?
2. How do post-secondary mathematics instructors, from the countries studied, describe the preparation for secondary instructors' pedagogical approach?
3. How do post-secondary mathematics teachers characterize the social climate of the typical secondary classroom, among the countries studied?

4. Among the countries studied, how do post-secondary educators depict the structure of the curriculum and additional materials used by secondary instructors in the mathematics classroom?
5. Among the countries studied, how do post-secondary instructors summarize the initial and ongoing professional development secondary instructors receive throughout their careers?

Significance of the Study

While some of the statistics derived from established national and international assessments show promise, others have caused alarm among educational leaders, prompting continuous research and legislation to promote the educational success of students in the United States and abroad (Kalaycıoğlu, 2015; Leung et al., 2014). Studies have been conducted citing multiple educational attributes are required for a country to achieve academic success in the field of mathematics (Areepattamannil, 2014; Callan, Marchant, Finch, & German, 2016; Li & Kaiser, 2011). Sparapani et al. (2014) concluded while there have been gains, the United States continues to be at a deficit with a growing need for skilled workers in a global market and an educational community struggling to meet those demands.

However, there is a deficiency in the literature comparing multiple factors contributing to academic proficiency in mathematics from countries that consistently rank in the top 10 on common assessment instruments, such as the PISA and the TIMSS. Also, past researchers have pointed to quantitative methods to show the relationship of singular variables leading to mathematics achievement. Further research relating to a qualitative method, such as a phenomenological study, would enhance and contribute to a more in-depth perspective of the attributes that allow these countries to gain such academic success, particularly in the subject of mathematics (Tucker, 2011). Bonner (2014) noted more research was needed in the area of mathematics, especially relating to effective teaching strategies leading to a decrease in achievement gaps.

Definition of Key Terms

For the purposes of this study, the following terms are defined:

Every Student Succeeds Act. The Every Student Succeeds Act (2015) was ". . . an act put forth on January 6, 2015, to reauthorize the Elementary and Secondary Education Act of 1965 to ensure that every child succeeds" (p. 1).

National Assessment of Educational Progress (NAEP). The NAEP is a project commissioned by Congress to construct a standard measurement of student learning to compare students' achievement in multiple areas and grade levels across the United States (Gorman, 2010).

National Center for Education Statistics (NCES). The NCES is a department branch of the Institute of Education Sciences of the United States Department of Education (Gorman, 2010). The NCES is accountable for "…developing test questions, administering the assessment, scoring student responses, conducting analyses of the data, and reporting the results" (Gorman, 2010, p. 4).

No Child Left Behind Act of 2001 (NCLB). The NCLB was bipartisan legislation enacted from 2002 to 2015 with the goal to lessen the gap in achievement among all students by holding districts accountable for demonstrating improvement and creating equitable learning opportunities (No Child Left Behind Act, 2002).

Organisation for Economic Co-operation and Development (OECD). The OECD (2017) is an independent organization drawing from evidence-based research to bring about recognition of global policies that lead to economic and social prosperity.

Proficient. For the purpose of this study, proficient is a term used to represent the mastery of educational standards and the ability to effectively perform challenging academic tasks (Gorman, 2010).

Programme for the International Assessment of Adult Competencies (PIAAC). The Survey of Adult Skills is administered as a part of the PIAAC (OEDC, 2016c). The PIAAC collects data to reach the ultimate goal of reliably predicting the abilities of an adult population in "… information-processing skills, but also to identify differences in proficiency between population sub-groups, to better understand how such skills are developed, maintained and used, and to determine the impact of different levels of proficiency on life chances" (OECD, 2016c, p. 13).

Programme for International Students Assessment (PISA). The PISA is an international, standardized assessment administered by the OECD to 15-year-old students, typically toward the end of the required number of years of schooling among many nations (OECD, 2016b). The PISA assesses the "…acquired key knowledge and skills that are essential for full participation in modern societies" (OECD, 2016b, p. 3).

STEM. A STEM program of study is defined by incorporating the disciplines of science, technology, engineering, and mathematics (Bell, 2016).

Trends in International Mathematics and Science Study (TIMSS). The TIMSS is an international comparative assessment that evaluates academic efficiency in the fields of math and science (Mullis et al., 2016). The TIMSS is administered to a select group of students in the fourth and eighth grades every four years, from over 38 countries around the world (Mullis et al., 2016).

Limitations

The following limitations were identified in this study:

Sample demographics. The purposive sample of participants included post-secondary mathematics instructors who taught at American universities and collegiate institutions from their respective native countries (Canada, China, Japan, and Singapore). A limitation of this study was the data may not represent all perceptions of the population of post-secondary mathematics teachers from the studied countries (Fraenkel, Wallen, & Hyun, 2015).

Summary

Mathematics instruction has been embedded into daily life throughout the centuries, and the great debate concerning the most advantageous mathematics paradigm continues today among politicians, educational leaders, parents, and students (Larson & Kanold, 2016). Currently, and in the past, students of the United States have demonstrated a deficiency in the area of mathematics as indicated by national and international comparative assessments (Boaler & Staples, 2008; Mullis et al., 2016; OECD, 2016d). This investigation was designed to lead to a further understanding of mathematics achievement in nations considered to dominate the core content of mathematics and to assist in the development of manageable interventions and reforms for school systems that struggle in America and abroad.

In the following chapter, research is introduced to compare and contrast the essential components of high-quality mathematics instruction and factors that contribute to raising achievement in low-performing schools. In addition, literature is presented citing the compulsory elements found in mathematics education from those countries that predominately excel in the respective field. Cultural elements and ideologies of mathematics instruction within these countries are also discussed.

Chapter Two: Review of Literature

There is an outcry among many in the educational community that the United States must take steps to improve mathematics education, and the education system in general, to maintain the nation's influence as an economic and innovative leader in the world (Boaler, 2016; Clements & Sarama, 2015; Larson & Kanold, 2016; OECD, 2016a; Tucker, 2011). National and international assessments have shown American students continue to demonstrate a lack of proficiency in the area of mathematics, while other countries have dominated the field (Mullis et al., 2016; OECD, 2016a). The disparity among countries leaves researchers questioning the fundamental differences of specific school systems and educational design, essentially having to rely on existing quantitative data to evaluate specific factors that have led to the success of other countries (Kalaycıoğlu, 2015; Li & Kaiser, 2011; Rasmussen & Bayer, 2014).

Hattie (2003) and Baete and Hochbein (2014) revealed the notion that teachers have the greatest impact on learning and can bring about the greatest positive effects on student achievement. These findings were echoed by William (as cited in Mincu, 2015):

> Teacher quality appears, therefore, to be a key variable at the classroom level. To sum up: (1) the most effective teachers are at least five times as effective as the least effective; (2) teacher quality may close the achievement gap in both primary and secondary schools; and (3) good teachers continue to benefit students for at least two years after they have stopped teaching them. (p. 256)

It is through this concept that further research should be conducted to determine the elements that compose a quality education facilitated by an expert instructor (Tucker, 2011). A review of the current research clarifying the relationship between the components of successful mathematics instruction and an increase in student achievement is presented in this chapter.

Conceptual Framework

Educational theorists have discussed the complexities of teaching mathematics, not only with students who show an aptitude or perform particularly well in the area, but even more so with students who find mathematics difficult for various reasons (Ernest, 2016; National Council of Teachers of Mathematics [NCTM], 2014; SRI International, 2009). The SRI International (2009) examined 17 math interventions to find a great deal of support was misplaced in one area or another when trying to improve the mathematics performance of secondary students. The framework suggests five areas must be acknowledged simultaneously for students to show growth in the area of mathematics: 1) intensification strategy; 2) curriculum; 3) pedagogy and instructional approach; 4) teacher knowledge and expectations; and 5) organization and social climate (SRI International, 2009).

In addition to this framework, other researchers have focused theories on the broader subject of mathematics education and processing. Matthews (2013) discussed the professional competence of teachers cognitively activating (COACTIV) the model in her framework, which drew parallels to the need for secondary educators to possess certain teacher qualities to foster constructive learning. The model includes the following elements: a) professional knowledge, b) motivation, c) beliefs, and d) self-regulation (Matthews, 2013). The Max Planck Institute developed the COACTIV model in Germany, and it was integrated into teacher and student questionnaires on the German 2003/2004 PISA (Bruckmaier, Krauss, Blum, & Leiss, 2016; Matthews, 2013). From the data, the researchers went further to notably acknowledge there are two distinct categories of professional knowledge: professional content knowledge and content knowledge (Matthews, 2013). As discussed in Chapter One, researchers found instructors must possess content knowledge to fortify professional content knowledge, which is a crucial element in increasing student mathematical growth (Matthews, 2013).

Schoenfeld's (2016) work was originally published in 1992, and his research was conducted to analyze the concept of "thinking mathematically," both from the educator's and student's perspective (p. 1). His framework centered around five central concepts relating to human cognition in the area of mathematics: a) the knowledge base, b) problem-solving strategies, c) monitoring and control, d) beliefs and affects, and e) practices (Schoenfeld, 2016).

Throughout each framework, the ability to problem-solve and to teach students problem-solving strategies is paramount to student success in mathematics through a well-planned curriculum, an educator's pedagogical approach, and students monitoring and self-regulating their steps through a problem (Matthews, 2013; Schoenfeld, 2016; SRI International, 2009). George Polya, a Hungarian Jewish mathematician, revolutionized the notion of teaching problem-solving in 1945 (Kilpatrick, 2014). He suggested multiple strategies such as simplifying the problem into manageable pieces, guess-and-check methods, building diagrams and models, visualizing the exercise, working backward, and organizing data in such a way to find patterns to assist in problem-solving (Kilpatrick, 2014). Kilpatrick (2104) believed these previously mentioned problem-solving methods, also known as heuristics, did not improve student mathematical growth in various studies; however, he noted typically these strategies were taught in one specific lesson and not generalized across multiple topics and scenarios within the mathematics curriculum.

Various researchers have argued while problem-solving is key to mathematics instruction, there continues to be value in procedural knowledge, which has led to a balanced approach (Hattie, 2017; Larson & Kanold, 2016). Munster, Stein, and Smith stated there are two types of instructional approaches:

> In the direct instruction model, when students have the prerequisite conceptual and procedural knowledge, they will learn from a) watching clear, complete demonstrations of how to solve problems, with accompanying explanations and accurate definitions, b) practicing similar problems sequenced according to difficulty, and c) receiving immediate constructive feedback. Whereas in the dialogic model, students must a) actively engage in new mathematics, persevering to solve novel problems; b) participate in a discourse of conjecture, explanation, and argumentation; c) engage in generalization and abstraction, developing efficient problem-solving strategies and relating their ideas to conventional procedures; and to achieve fluency with these skills, and d) engage in some amount of practice. (as cited in Hattie, 2017, p. 23)

Hattie (2017) explained through his research that both instructional styles have a significant impact on student achievement, which led him to develop the concept of precision teaching.

Precision teaching is the ability to determine what method of instruction is appropriate at a specific point in the students' learning process (Hattie, 2017). When evaluating Larson and Kanold's (2016) suggestions, satisfactory K-12 math programs are comprised of a balance of procedural knowledge and conceptual comprehension, collaboration among students, persistence in mathematical exercises, feedback from instructors and pupils, and the incorporation of technology to reinforce concepts. Similarly, Codding, Mercer, Connell, Fiorello, and Kleinert (2016) reiterated the importance of procedural knowledge by noting students must obtain whole-number fluency to grasp mathematical concepts and correctly adapt algorithms to apply problem-solving measures to real-world scenarios. After reviewing multiple studies, Codding et al. (2016) determined this practice produced higher mathematics proficiency at the elementary and high school levels.

The literature in this review was chosen to draw upon particular components embedded in the conceptual frameworks which correlate with mathematical achievement, including, but not limited to curriculum, pedagogy and instructional approach, teacher knowledge and expectations, and organizational and social climate. An evaluation of the content currently available concerning the educational practices of specific countries is presented in Chapter Two. This research aided in the investigation of Canada, China, Japan, and Singapore's secondary mathematics programs to find similarities and differences among the countries.

Curriculum

Curriculum is considerably relevant to increased student performance, as it is a pillar the instructor relies upon to support instruction and curricular goals (Castro Superfine et al., 2015; Fan, 2014). Hodges and Jong (2014) stated, "The relationship between the teacher and the curriculum materials is seen as dynamic and interactive, shaping how the teacher identifies with mathematics teaching and learning" (p. 25). In a study conducted by Reys, Reys, and Chavez (as cited in Jung Kang, 2014), 90% of kindergarten through eighth-grade teachers relied on the textbook 90% of the time through three-fourths of the class period. However, the instructor has a responsibility to present the curriculum in a meaningful way and to adapt the activities within the curriculum to bring about the greatest potential for student growth (Dietiker, 2015).

Researchers have found in the United States that textbooks at the elementary level introduce a broad range of topics repeatedly, leaving less time for students to explore fundamental concepts more deeply (Jung Kang, 2014). Baete and Hochbein (2014) noted within their study of a multi-faceted reform, a “narrow and focused” curriculum was one aspect implemented so assessments could be aligned, and educators could recognize achievement among students in the same grade level. Through this study, the proficiency of secondary students in mathematics increased, and the variation among achievement levels decreased (Baete & Hochbein, 2014).

Hyun Jung Kang (2014) established in America those who govern at the state and district levels often determine the curriculum and texts used within the classroom. Several companies publish texts for classroom use, which leads to vast differences in the presentation and organization of material (Jung Kang, 2014). In addition, Özer and Sezer (2014) believed questions in American textbooks tend to be less-demanding, with a variety of mathematical concepts introduced to students years later than in textbooks from Eastern Asian publishers.

In the case of Railside High School, there was a large disparity in mathematics achievement among ethnic groups within the school; however, within two years, the variance among the performance of the students was obliterated on standardized assessments (Boaler & Staples, 2008). While Railside High School enacted a multi-reform approach, the researchers noted the educators' desire to collaborate to build a curriculum that integrated rigorous material and exercises to promote critical thinking among all student groups, including those with mixed abilities, and this was a key component of their success (Boaler & Staples, 2008). However, Smith and Morgan (2016) revealed throughout a multi-cultural study of secondary curricula across the globe, including the United States and the aforementioned top-ranking nations, every jurisdiction offers flexibility within the curriculum for pupils seeking practical and applicable real-world solutions to mathematics.

Researchers have noted many top-ranking nations implement clear national frameworks and standards, and some even have a national curriculum and government-issued textbooks (Li & Kaiser, 2011; OECD, 2016a; Tucker, 2011). Efforts were made at the federal level in 2010, to deepen students' problem-solving and critical thinking skills throughout the United States, by introducing rigorous standards and objectives known as the Common Core State Standards of Mathematics (CCSSM) (Larson & Kanold, 2016). With various levels of proficiency and mathematical standards taught among the states in the past, the bipartisan endeavor was to ensure students across kindergarten through 12th grades would equivocally and coherently learn and address the same mathematical goals by grade level, no matter the district or state where students attended public school (Larson & Kanold, 2016; Schoenfeld, 2014).

However, Schoenfeld (2014) specified the standards were never considered to be a national mathematics curriculum and were not adopted throughout the United States as a whole. Initially, 45 states agreed to the implementation of the CCSSM, but due to a lack of preparation for educators, administrators, and parents, copious amounts of funding needed to redesign state assessments to reflect the standards, and confusion among the ideology of the states' control of education, the Common Core Initiative eventually faded (Larson & Kanold, 2016; Schoenfeld, 2014). Schoenfeld (2014) noted companies rushed to distribute textbooks with updates that aligned with the CCSSM. Polikoff (2015) discussed the mathematics textbooks studied were not aligned with the standards as stated but continued to emphasize memorization and mathematical procedures.

Curriculum in Canada. According to Tucker (2011), Canada is relatively new to the forefront of international education rankings, which has led to a deficit in scholarly research describing the reasons for Canada's mathematics achievement. Canada is unique in the fact that the education system is not overseen by the federal government but rather at the provincial level, unlike most countries that have found success on international assessments (OECD, 2016a). The 13 provinces and territories of Canada individually dictate curriculum, funding, teacher preparation, and educational policies (Tucker, 2011). Similar to the United States, but garnering superior educational outcomes on international tests, by comparison, Canada's populations are culturally and linguistically diverse, and school systems are divided into districts led by school boards at the local level (Tucker, 2011).

However, multiple researchers and authors have noted Canada's provinces and territories have demonstrated scores on international examinations with considerable variations; Quebec produced significantly better scores than Canada's overall average on the 2012 PISA in the area of mathematics, with British Columbia, Alberta, and Ontario scoring similar to the national average (Tucker, 2011; Vashchyshyn & Chernoff, 2016). Also, results on the PISA 2015 indicated a significant decrease in mathematics achievement in the provinces of Quebec and Ontario, leading education officials to question specific aspects of the provincial education system, especially when comparing the consistently high mathematical success of many Eastern Asian nations (Reid & Reid, 2017; Stokke, 2015). According to Smith and Morgan (2016), in a study comparing mathematics curriculum across various nations, the provinces of Ontario and Alberta were found to have secondary curriculums with a strong focus on problem-solving embedded in real-world contexts. Also, the study revealed within the province of Ontario students are allowed to utilize alternative pathways at the secondary level in the area of mathematics that primarily center on practical applications for those who struggle and a more abstract focus on mathematics for accelerated learners (Smith & Morgan, 2016).

Canada is known to have the highest proportion of immigrants around the world (Tucker, 2011). Despite high immigration, the OECD (2016a) noted on the PISA 2015, no mathematical achievement discrepancies were found between students who immigrated to Canada and those native to the country. Researchers have attributed this phenomenon to a highly equitable system within the Canadian education system and a migrant policy that actively seeks skilled professionals and scholars (OECD, 2016a). It is important to note multiple provinces within the nation have boasted reforms centered upon improving teacher quality, increasing collaboration among leadership, promoting initiatives reinforcing indigenous and immigrant pupils, and developing strategies to increase students’ learning outcomes with the use of formative assessment (OECD, 2016a).

Curriculum in China. China's success in the academic sector is rooted in an extensive history and emphasis placed on education (Wang, Liu, Du, & Liu, 2017). From a curricular perspective, Chinese teachers often provide a rich historical background when introducing new mathematical concepts to engage students and provide a strong conceptual context (Yang & Wang, 2015). According to Tucker (2011), high-stakes examinations and an appreciation for quintessential works such as *Four Books* and *Five Classics* date back to 603 A.D. While prominent philosophers place a significant amount of weight on high-stakes exams and classic works, China has moved its testing practice toward open-ended scenarios with real-world applications and has practically eliminated multiple-choice exams (Tucker, 2011).

During the past four decades, China's educational reform has evolved to target public education for all and holds the record for the world's largest population of higher education students (OECD, 2016a; Tucker, 2011). During the late 1980s, legislation titled *Law of Compulsory Education* mandated every child complete six years of school at the primary level and three years at the junior secondary level (Tucker, 2011). A few years later, textbook production was decentralized and published at the regional level, yet still aligned to national standards and required state approval (Tucker, 2011).

Throughout the late 1990s, post-secondary institutions were required to make significant increases to their student bodies, making a college education available to a greater percentage of China's population (Tucker, 2011). In the early 2000s, China developed the Mathematics Curriculum Standards for Full-time Compulsory Education and the Mathematics Curriculum Standards for Secondary Education (Wang et al., 2017). The Mathematics Curriculum Standards for Secondary Education integrated traditional

Chinese mathematics standards of foundational skills with 21st-century learning skills, incorporating active learning, mathematics literacy, and critical thinking skill; establishing the need for mathematical modeling and information technology; and promoting a mathematics culture (Wang et al., 2017). According to Tucker (2011), a key slogan in China's realm of education remains "to every question there should be more than a single answer," which highlights the need for integrating 21st-century skills (p. 35).

Throughout the reforms, Wang et al. (2017) revealed educators at the public-school and post-secondary levels developed curriculum standards which were field-tested by multiple provinces, revised by expert panels collectively seeking the opinions of educators, and implemented over 10 years. Tucker (2011) highlighted current reforms to China's educational system include developing financial and instructional equality among schools and moving toward student-centered learning approaches. At the secondary level, the need for mathematics as a part of Chinese culture is accentuated, which requires students to take the same amount of coursework, although additional courses are offered to accelerate student knowledge (Smith & Morgan, 2016).

According to Jiang, Hwang, and Cai (2014), a great deal of importance is placed on the relationship between the educator and the textbook. Prompts in the textbooks in China frequently direct the students to reflect upon the reasonableness of a solution and the problem-solving methods used (Jiang et al., 2014). While China's secondary mathematics curriculum has transformed throughout the past century, Lv, Chen, Peng, and Wang (2015) stated, "The mathematics curriculum was an integrated curriculum following a spiral structure, that connected mathematical content to students' life experience, and tied together mathematics and reality" (p. 201). Cai and Hwang (2015) added the mathematics curriculum is considered to be narrow but deep in its approach to higher-order thinking, introducing algebraic principles and problem-solving at the first-grade level.

Curriculum in Japan. The rigorous Japanese curriculum is often seen by leaders in education to be the catalyst for Japan's academic success, especially in mathematics (Tucker, 2011). Japan provides a free, compulsory education system, which consists of six years for elementary school and three years of lower-secondary school and boasts a 98% attendance rate in upper-secondary school (Tucker, 2011). The national standards-based curriculum of Japan, also known as a Course of Study, includes textbooks reviewed and chosen by the government (Archer, 2016). Most chapters of Japanese mathematics textbooks begin by focusing on problem-solving as an introduction to new topics (Takahashi, 2011). The curriculum is "highly coherent" and presented in a rigid sequential order, with sufficient time given for each carefully crafted standard to be mastered and lead students to a deep understanding of the content (Tucker, 2011, p. 87). At the secondary level, a standard sequence of mathematics courses is offered, with the option given to students to further their learning with additional coursework (Smith & Morgan, 2016).

Throughout the curriculum reform in 2008, Takahashi noted experts conferred with instructors to write new versions of textbooks, which were later introduced in April of 2011, leaving plenty of time for a careful study of the reliability and validity of the texts (as cited in Archer, 2016). As clarified by Archer (2016), educators see textbooks as the beginning point to instruction and as educational guides stimulating deep thinking, while steeped in mathematical pedagogy that took years to develop. Japanese educators believe while anyone can teach from the textbook, a true instructor must use the textbook to enhance the lesson, having a deep comprehension of the material that lies within and being able to guide students toward a greater learning experience (Takahashi, 2011). Tucker (2011) noted compared to other industrialized nations, textbooks from Japan tend to be reasonably affordable, concise, and to the point; teachers are expected to go over all of the material within the text thoroughly.

Özer and Sezer (2014) stipulated Japan's success, specifically when compared to America's education system, stems from the introduction of mathematical concepts taught to Japanese students earlier in their educational careers. Takahashi (2011) indicated most textbooks include problem-solving at the beginning of each chapter as a way to introduce new topics. Japanese curriculum highlights that mathematics brings excitement and fulfills a fundamental need in one's life (Smith & Morgan, 2016).

Curriculum in Singapore. In the past 50 years, after gaining its independence, Singapore has metamorphosed their infrastructure to become a symbol of success within the Asian community (Tucker, 2011). The federal government of Singapore mandates compulsory education, where students typically attend school for at least 10 to 12 years (Tucker, 2011). Due to Singapore's religious and ethnic diversity, a key feature of their school system is inclusivity, which is demonstrated by their requirement for all students to be bilingual (Ministry of Education: Singapore, 2015).

According to Tucker (2011), schools in Singapore have transitioned from a nationally controlled system of education to a more autonomous approach, where some jurisdiction is given to schools grouped by clusters to invite creativity and innovation within their systems. Due to the relatively small size of Singapore and control at the federal level, Chan (2015) recognized the ease of implementing a highly coherent and cohesive curriculum. Within the national curriculum, during the first four years at the primary level, a heavy emphasis is placed on the subjects of English, Malay the "mother-tongue," and mathematics (Tucker, 2011). After completing year six in primary school, Singapore students complete the Primary School Leaving Examination through which science, English, mathematics, and Malay are assessed (Tucker, 2011). From there, about 60% of students are placed in an express academic track, 25% follow a normal academic track, and 15% are admitted to technical courses (Tucker, 2011). Smith and Morgan (2016) noted at the lower secondary level students are placed into educational pathways; although all tracks integrate mathematical courses based on applying mathematics from a practical approach to real-world contexts, higher-attaining students gain a more abstract perspective, while lower attainers gain a hands-on approach.

Those within the Singaporean education system and researchers conducting studies in Singapore have been consistent in pointing to the cohesive nature of the system to explain the nation's success (Tucker, 2011). Tucker (2011) directed attention to a particular dynamic delineating the United States and Singapore by suggesting loose and tight alignment and consistency, respectively, differs a great deal in the way the nations approach curriculum reforms, national assessments of learning at the primary and secondary levels, encouragement of student achievement, and measures of accountability for educators and administrators. Mathematics education in Singapore leans toward a spiral approach, where similar topics are introduced year after year, but the exercises and content grow in depth and complexity as students advance by grade level (Chan, 2015).

Schoenfeld (2014), an American professor, stated, “Singaporean teachers are deeply versed in their curricula and have been prepared to get the most out of the problems in their text” (p. 741).

Throughout the past two decades, Singapore has made a shift within the curriculum to explicitly state mathematical problem-solving methods and applications in real-world scenarios connect the curriculum to true learning endeavors to further society and compete globally in the field of STEM (Smith & Morgan, 2016). Özer and Sezer (2014) linked the visual aids in Singaporean mathematics textbooks to the strengthening of students’ conceptual understanding in their responses to questions. This is further evidenced by Singapore’s initiative to inform others of their curricular goals and objectives through the transparency of visual aids outlining their curriculum and pedagogy, placing an emphasis on drawing an understanding based on previous knowledge, positive outlooks toward education, and application to everyday life (see Appendices A and B) (Kaur, 2019; Ministry of Education: Singapore, 2018; Smith & Morgan, 2016).

Pedagogy and Instructional Approach

Many experts have determined an extensive correlation between an educator’s application of pedagogical philosophies, or teaching quality, and student achievement (Basque & Bouchamma, 2016; Pepin, Xu, Trouche, & Wang, 2017; Vashchyshyn & Chernoff, 2016). Researchers have expressed concern about the pedagogical approach of instructors in the United States, especially in the subject of mathematics (Tucker, 2011). The SRI International (2009) referenced educators in the United States often fail to offer mathematical experiences that allow students to explain their thought processes and develop a conceptual understanding of the material. Ing et al. (2015) found the more students engage in a discussion of their mathematical perceptions of a problem, the more likely the students are to demonstrate higher achievement. When determining the most advantageous pedagogical approach to bring about the desired learning outcomes, Charalambous and Praetorius (2018) unveiled the concept of using different frameworks, that often exhibit similar and contrasting applications to one another, to reach the teachers' and students' educational goals centered upon these interactions. Similarly, Mincu (2015) asserted to affect student learning an educator must have a plethora of pedagogical approaches to glean from to meet the individual needs of the learners and cohort.

One such pedagogical approach is known as cooperative learning. Cooperative learning is a teaching method which promotes social interaction among members of a small group to meet a specific objective (Chan & Idris, 2017). A meta-analysis completed by Capar and Tarim (2015) revealed shared learning, or a cooperative learning experience, has a greater impact on student achievement in mathematics. This educational outcome mirrored a subsequent meta-analysis completed by Turget and Gülşen Turget (2018), who found cooperative learning positively affected learning to a moderate extent. Chan and Idris (2017) reviewed multiple scholarly works across many countries, only to find a student's ability to problem-solve and perform complex mathematical exercises improved with the implementation of cooperative learning.

However, there is a delicate balance within cooperative learning that stems from student-teacher practices; an instructor must support student interactions to determine the level of understanding of the students (Ing et al., 2015). Also, the students must be willing to interact with their instructor so the instructor can determine the level of detail needed to reach a conceptual understanding of the material (Ing et al., 2015). The NCTM (2014) suggested instructors design or provide students with tasks and exercises that promote deep thinking and problem-solving with multiple points of entry and various pathways to solutions. The foundational principles of students interacting with rich mathematical tasks, finding multiple approaches to problem-solving, and articulating their logic and reasoning within an egalitarian group cited within the NCTM (2014) study were congruent with SRI framework (2009) and the work of Knudsen, Stevens, Lara-Meloy, Kim, and Schechtman (2018). Silver and Mesa (2011) noted students in the United States have difficulties using cognitively demanding strategies, including logically working through multiple steps to find a reasonable solution.

Various other mathematical studies and reforms favor constructivist pedagogy (White-Clark, DiCarlo, & Gilchriest, 2008). Constructivism is a form of teaching centered on the student deriving meaning from a topic by participating, discovering, and questioning others within the environment (White-Clark et al., 2008). Piaget and Vygotsky both endorsed the constructivist theory with a slightly different view; Piaget believed in cognitive constructivism, while Vygotsky accepted social constructivism (Schcolnik, Kol, & Abarbanel, 2016). While both ascertained the theory hinged on a student building knowledge, Piaget emphasized knowledge was built based on the student’s current cognitive structures and interactions within the environment. On the other hand, Vygotsky revealed learning stemmed from social encounters (Schcolnik et al., 2016).

In *Visible Learning for Mathematics,* Hattie (2017) referenced Vygotsky's and Piaget's works, along with others, depicting deep thinking processes and social interactions as necessary to derive meaningful learning endeavors in the area of mathematics. He elaborated once a student has moved through the "surface learning" phase, specifically focusing on the introduction of new concepts and procedures, the instructor plays an integral role in providing students the opportunity to collaborate with peers and make "…deeper connections in order to consolidate their understanding of mathematical concepts and procedures" (Hattie, 2017, pp. 29-30). Hattie (2017) referred to the student's progression of thinking from the "surface learning" phase to the "deep learning" phase (pp. 29-30). He went further to describe the greatest goal for the learner is the final phase known as "transfer learning;" this aspect of learning allows students to be leaders in the academic process and "…apply their thinking to new contexts and situations" (Hattie, 2017, p. 32).

Many members of the academic community, including the NCTM (2014) and Boaler and Staples (2008), highlighted the need to deepen student understanding and to motivate learners to take an active role in transitioning their learning across multiple scopes and applying knowledge to real-world situations (Hattie, 2017; SRI International, 2009). Consequently, deeper thinking and problem solving require students to rely on metacognitive strategies (Schoenfeld, 2016). Researchers explained these strategies comprise two categories known as "metacognitive knowledge" and "metacognitive control," which are defined as follows:

> Metacognitive knowledge, in one case, refers to one's knowledge and beliefs in his mental resources and his awareness about what to do. It also mathematically

> refers to the mathematical processes and techniques students have and their ideas about the nature of mathematics. Metacognitive control/regulation is considered as the ability to use knowledge to regulate and control cognitive processes. Metacognitive control is related with metacognitive activities that help to control one's thinking or learning. (Özsoy & Ataman, 2017, p. 68)

Within Özsoy's and Ataman's (2017) study of fifth-grade students, the implementation of metacognitive skills instruction was shown to significantly improve mathematics achievement in the area of problem-solving when compared to the control group. Also, the survey and examination completed during the administration of the PISA 2009 assessment revealed the use of metacognitive strategies correlated to a higher rate of success and was a reliable predictor of academic achievement (Callan et al., 2016). Evidence has demonstrated in multiple cases that the pedagogical implementation of metacognitive strategies in the classroom has a significant and desirable influence on educational outcomes, as cited by Hattie (2017) and Mincu (2015).

In addition to centering upon profound mathematical thinking and regulation of one's thoughts, the SRI International (2009) and Hattie (2017) suggested formative assessment plays a crucial role in mathematical pedagogy. Hattie (2017) defined the concept of formative evaluation as gathering data in real-time to guide and plan instruction and listed formative evaluation as one of the top five influences on student academic growth and achievement. The SRI International (2009) was clear to point out for formative assessment to be effective assessments must tie to learning goals and objectives. Parallelism allows for optimal feedback from the instructor to the student and builds the learner's self-efficacy and interest in mathematics (Rakoczy et al., 2019).

Leung et al. (2014) added a link between performance assessments and curriculum perpetuates validity of the instructor's pedagogy and educational approach.

Pedagogy and instructional approach in Canada. According to Vashchyshyn and Chernoff (2016), problem-solving is a major pedagogical foundation for the learning process within the Quebec education system, which is known for its high achievement scores in the area of mathematics on standardized international assessments. The practice and significance of problem-solving date back to the 1970s, with texts published by the Quebec Ministry of Education stating instructors should be considered facilitators of solving problems, rather than merely demonstrating how to solve problems (Vashchyshyn & Chernoff, 2016). Lajoie and Bednarz (as cited in Vashchyshyn & Chernoff, 2016) added an instructor's role is not reduced to helping students find solutions but includes generating mathematical exercises, aggregating data, and determining which problems best suit students' needs.

Smith and Morgan (2016) revealed the Canadian provinces of Ontario and Alberta often practice problem-solving within a constructivist pedagogy to "provoke curiosity and frame learning" (p. 39). However, due to declining scores on international assessments, a war currently rages over mathematical pedagogy in Canadian provinces pitting "rote-learning" and recall of basic arithmetic against "discovery math" or problem-solving (Ansari, 2016, pp. 4-5). Based on empirical data, Ansari (2016) concluded integration of the previously mentioned learning strategies must exist concurrently to increase achievement in the area of mathematics.

Pedagogy and instructional approach in China. While heuristic strategies do not exist in the Chinese curriculum, authentic problem solving is heavily emphasized among instructors (Jiang et al., 2014). Cai and Hwang (2015) revealed challenging tasks are often embedded in classroom instruction due to the heavy influence of Confucian principles of obtaining knowledge through hard work. According to a comparative study by Jiang et al. (2014) relating to mathematical strategies used by sixth-grade students in China and Singapore, both sets of students relied heavily on arithmetic strategies; the second-leading strategy for Chinese students was algebraic strategies. Contrary to the popular belief of Chinese students being rote learners, Liu, He, and Li (2015) discovered creativity and critical thinking were often embedded into classroom instruction when middle school mathematics classrooms were studied.

Li, Li, and Zhang (2015) outlined the basic structure of a typical mathematics lesson at the secondary level, noting due to large class sizes of 40 to 50 students, it is imperative the instructor begins class with a well-crafted introduction that extends prior knowledge to the current topic of discussion. Instructors typically use heuristic strategies throughout the exercises mentioned previously but extend those strategies to lead students to "…experiment, discover, and generalize and transfer knowledge and skills learned from these examples to a larger family of cases" (Li et al., 2015, p. 83). Throughout the problem-solving process, basic mathematical skills and knowledge are stressed as a practical way to move forward to a solution and show logical reasoning (Li et al., 2015). Chinese educators typically consider students proficient in mathematics when students are capable of utilizing mathematical procedures to solve problems and make connections among a set of scenarios with similar conceptual underpinnings (Li et al., 2015; Liu et al., 2015).

Due to the limited amount of time and immersive nature of complex problem-solving within the classroom, homework continues to be essential in the highly competitive structure of China's education system so students can reach greater heights through admission into top-ranked schools (Li et al., 2015; Tucker, 2011). Homework requirements are not surprising, as the culture of China is dictated by a strong work ethic that prizes attention to detail, rigor, and immense regard for education (Li et al., 2015). Due to the extreme emphasis placed on homework by educators, parents, and even students, China's Ministry of Education sets limits on after-school workloads (Tucker, 2011).

Pedagogy and instructional approach in Japan. After completing multiple classroom observations, Archer (2016) found Japanese instructors adhere to similar instructional approaches centered on whole-group interactions, typically without the use of technology. Stigler and Hiebert (as cited in Hino, 2015) discussed the typical lesson plan follows a similar structure comprised of five basic steps: 1) discussing the past lesson; 2) revealing the day's mathematical exercise; 3) pupils work independently on their own or in groups; 4) exploring various methods to solve the exercise; and 5) highlighting and recapping essential points. After evaluating a plethora of eighth-grade mathematics lessons in Tokyo schools, Hino (2015) found pressure was not on finding a solution to a problem but rather to coax students in the art of problem-solving, which revolves around forming conclusions and increasing their comprehension of the subject matter. Takahashi (2011) added during the problem-solving process instructors typically avoided telling students the correct solution to encourage them to think carefully about the solution and reflect to explain their reasoning.

Various researchers including Takahashi (2011); Schukajlow, Krug, and Rakoczy (2015); and Hino (2015) implied a key principle attributed to Japan's high mathematical achievement lies in the approach of having students derive multiple solutions to a singular exercise, which leads students to develop a sense of empowerment when discovering new approaches to problem-solving. The National Association of Mathematics Advisors (2015) also stipulated encouraging students, especially at the elementary level, to use multiple representations to support problem-solving is an efficient approach to teaching mathematics. After time is given for the students to problem-solve and utilize critical thinking skills independently, students are asked to compare and contrast their solutions to engage in thought-provoking small group and whole-class discussion (Schukajlow et al., 2015).

Takahashi (2011) explained the technical term for whole-group collaboration is typically described as *neriage,* which translates to "polish up" (p. 199). Neriage is seen as the "heart of teaching mathematics through problem-solving," and the instructor is instrumental in supporting student ideas as a way to begin solving problems and eventually finding their way toward a solution (Takahashi, 2011, p. 199). Throughout the problem-solving activity of neriage, educators have the opportunity to shed light on important concepts that bring clarification to the objective of the lesson and allow students to struggle, while eventually building a link between prior knowledge and the current lesson (Takahashi, 2011). In some Japanese classrooms, Archer (2016) observed instructors struggled with implementing neriage, and students often had difficulty with beginning the problem-solving process.

Even with class sizes ranging from over 30 to 40 students, Tucker (2011) disclosed student engagement is the ultimate objective and is not impacted by class size. It is imperative to mention within these larger classrooms Japan did not adhere to tracking students based upon their cognitive or academic abilities; notably, students who would often receive special education services in the Western setting received instruction in the regular education setting within the Japanese educational system (Tucker, 2011). While there is no proven correlation, many students sought *juku*, a form of private tutoring to supplement students' needs that is widely popular and utilized within the Japanese culture (Yamato & Zhang, 2017).

Pedagogy and instructional approach in Singapore. While Singapore is a relatively new nation from a global perspective, the world of mathematics education has recognized Singapore's substantial gains and successes through observation and international examinations (Chan, 2015). Pedagogical practices in the classroom remain at the heart of Singapore's philosophy of education (Ministry of Education: Singapore, 2015). Instructors are geared toward fostering a sense of mathematics within the student population (Tucker, 2011).

Jiang et al. (2014) noted the greatest concentration was problem-solving within mathematics education, and a plethora of specific problem-solving strategies exist within the national syllabus. Referencing their comparative study between sixth-grade mathematics students from China and Singapore, Jiang et al. (2014) found while particular emphasis was placed on arithmetic strategies by Singaporean students, the next-leading strategy was drawing models, followed with guess-and-check procedures. With this in mind, Tucker (2011) mentioned the goal is not pushing students toward the right answer but teaching them how to logically work through a series of steps to understand how mathematics applies to a situation.

Singapore's mathematics curriculum framework highlights the following five elements which encapsulate the prominent concept of problem-solving: concepts, skills, processes, metacognition, and attitude (Chan, 2015). For the subcategory metacognition, students are required to self-regulate mathematical problem solving and attend to comprehension during each step of the process when solving open-ended exercises (Chan, 2015; Kaur, 2019). Under the subheading processes, recognition and validity gave rise to connecting ideas and unveiling the reasoning of students' 21st-century mathematical thinking (Chan, 2015).

Teacher Knowledge and Expectations

The ability of an educator to increase student proficiency correlates with the educator's possession of highly developed cognitive skills, especially in the area of numeracy skills (Basque & Bouchamma, 2016; Goldhaber & Walch, 2014; Hanushek, Piopiunik, & Wiederhold, 2014; Reckase, McCrory, Floden, Ferrini-Mundy, & Senk, 2015). Researchers have proven the United States educational system does not draw its teaching candidates from leading academic performers (Goldhaber & Walch, 2014). Even more detrimental, Richey (2015) discussed the field of education as a historically female-dominated entity, but due to a historical rise of the female labor force, many women with exceptional abilities are seeking opportunities in other industries offering higher pay, which leads to a depletion of those who exhibit the traits necessary to be an asset in the academic sector. In contrast, Hanushek et al. (2014) and Tucker (2011) discussed the top-performing countries around the world acquire their educators from the top one-third of their academic classes.

An instructor's content knowledge is considered to be the instructor's comprehension of the content presented (Kleickmann et al., 2015). Rasmussen and Bayer (2014) declared, "It is likewise well documented that teaching content in teacher education programmes plays a crucial part in the development and composition of the knowledge base which teachers can draw upon when performing their profession" (p. 799). Evidence supported this line of thinking when Mincu (2015) documented that increasing the caliber of teaching programs at the collegiate level has been a significant component of high-performing countries throughout the global education system.

Mincu (2015) discussed two common factors that advance instructors within their professional careers—their studies at the post-secondary level and their development as educators in the field. Some believe majoring in a mathematics program is simply not enough, as an educator requires a specialized form of applied mathematics typically ignored in many undergraduate mathematics courses (Matthews, 2013; Reckase et al., 2015). Building upon this concept, Vashchyshyn and Chernoff (2016) posited while mathematics education programs at the post-secondary level often require advanced mathematics coursework, teaching pedagogy is overlooked as students integrate from applied mathematical sciences. Simmt (2011) extended her thoughts on this subject by suggesting a close examination of current mathematics education coursework at the post-secondary level; alterations to current course offerings must develop the appropriate expertise of future math instructors.

Fan (2014) reflected upon the growth of mathematics educators throughout their careers in his comparative study examining the mathematical pedagogical philosophies found in schools in the United States (Chicago, Illinois) and Singapore by posing three questions: “1) What knowledge do teachers need? 2) What knowledge do teachers have? 3) How do teachers develop their knowledge?” (p. 9). While there are minor differentiations among the definitions, many researchers, including Fan (2014), cited the need for pedagogical content knowledge, but he went further to include pedagogical curricular knowledge and pedagogical instructional knowledge (Kleickmann et al., 2015; Koponen, Asikainen, Viholainen, & Hirvonen, 2016; Matthews, 2013). Researchers were relatively concise to define pedagogical content knowledge as possessing the ability to present conceptual and procedural knowledge of mathematics (Fan, 2014; Kleickmann et al., 2015). In the Chicago study, Fan (2014) noted educators often advance their pedagogical content knowledge using various sources, but the main agencies were shown to be through “own teaching experience and reflection” and “informal exchanges with colleagues,” with the least important being “pre-service training” (p. 153).

To improve instructors’ knowledge in mathematics, educators should take part in consistent opportunities for professional development (Julie, 2014). In a project found to increase student proficiency in mathematics through multiple reforms, Baete and Hochbein (2014) revealed a factor of an educator’s effectiveness could be attributed to professional development at the district level when educators focused on actively engaging in professional learning communities. Researchers continue to encourage the use of professional learning communities integrated with oversight by expert teachers as a support system for novice teachers to increase student performance (Basque &

Bouchamma, 2016; Boaler & Staples, 2008). Schoenfeld (2014) added the philosophy that the greatest investment to be made in the United States educational system is the support of teachers to grow into their chosen profession and the ability to provide opportunities for said growth regularly.

Schmoker (2006) made an explicit effort to differentiate between "true learning communities" and "traditional staff development" found in many schools around the country (p. 106). Schmoker (2006) referenced continued research from Little, Gearhart, Curry, and Kafka and stated, "…Teams continue to discuss wide-ranging issues instead of looking closely and analytically at teaching and at *how their teaching affects* learning on an on-going basis" (p. 108). Beginning and experienced educators gave many reasons to support professional learning communities, including the following: engaging in a reflective process as a means of improving instruction, collaborating to build common initiatives, constructing curriculum and assessments, and forming accountability among educators and pupils (Schmoker, 2006). Popp and Goldman (2016) noted the teachers' knowledge base expands, and they realize greater gains when matters concerning assessments and data, rather than instruction, are discussed.

Hattie (2003) revealed there are direct differences between expert teachers and experienced teachers; expert teachers challenge their students with clear goals and engaging tasks while eliciting a high degree of critical analysis and metacognitive skills. Mincu (2015) added educators with the highest qualities continue their research and investigation within their field, especially from expert researchers at the post-secondary level or other leading authorities. However, expertise is not limited to these factors, as varied educational systems place divergent values on different aspects of teachers' qualities displayed in the classroom (Li & Kaiser, 2011).

Schoenfeld (2014) made the delineation between the success of American education systems and those of top-ranking countries as derived from the base of support given to teachers, plus the access and encouragement to continuing teacher development as life-long educators. Schoenfeld's (2014) thoughts were mirrored in Cai and Hwang's (2015) commentary about the Chinese education system, as they claimed, "Chinese teachers' knowledge does not appear to be as much a function of teacher preparation through college courses as it is to the ongoing process of professional learning practice" (p. 17). For professional development to be a constructive practice that transforms instructors' methodology, teachers must receive reinforcement throughout the instructional process, including during planning and reflection (Pepin et al., 2017).

Teacher knowledge and expectations in Canada. Tucker (2011) revealed Canada is like other top-performing countries; Canada draws teaching candidates from the top 30% of high school graduates. In the case of post-secondary institutes in Quebec versus other Canadian provinces, Vashchyshyn and Chernoff (2016) divulged students enrolled in post-secondary mathematics courses are exposed to an emphasis on both mathematics knowledge and mathematics pedagogy and are taught by exemplary mathematics practitioners previously or currently in the field of public education. According to Bednarz, the University of Quebec at Montreal created a revolutionary course to transform prospective educators at the secondary level to redirect their attention from academic mathematics (as cited in Vashchyshyn & Chernoff, 2016). The course requires pre-service secondary mathematics instructors to merge mathematics content knowledge and pedagogy by solving problems typically given to high school students in multiple ways and to pinpoint where potential mistakes may be found (Bednarz as cited in Vashchyshyn & Chernoff, 2016). Furthermore, the University of Quebec at Montreal also promotes learning from real-world situations for post-secondary students by offering examples of student work at the secondary level so pre-service teachers can examine student thinking and processing of the topic and understand typical difficulties under the tutelage of a mathematics didactician (Bednarz, as cited in Vashchyshyn & Chernoff, 2016).

Teacher knowledge and expectations in China. In China, a teacher's educational practices are consistently observed and open to scrutiny by the public; if deemed worthy, the educator's practices are readily shared and replicated (Li, Huang, & Yang, 2011; Tucker, 2011). Tucker (2011), among other researchers, noted an economical and functional disparity between rural and urban schools, which led to educational reforms in 2006. Essentially, rural schools were found to be lacking funds and high-quality educators, while urban schools had both in abundance; this led to the formation of an exchange program among educators (Tucker, 2011). The principle was derived from the notion to move rural instructors to teach in the urban sector so the instructor could take back what was learned to enhance teaching in the rural school; likewise, high-quality urban instructors and administrators were sent to rural districts to share their curriculum and best practices to enlighten rural educators (Tucker, 2011).

Tucker (2011) discussed professional development beginning at the "grassroots level" using subject-based teaching groups (p. 29). Pepin et al. (2017) elaborated upon these teaching research groups and noted the groups typically meet on a monthly to weekly basis. As instructors typically teach one to three repetitions of the same lesson per day within a specific content area, this allows time to be spent with colleagues collaborating, researching, and perfecting lessons for the next day (Mincu, 2015; Pepin et al., 2017). Also, Mincu (2015) noted in China, specifically Shanghai, there is a great deal of emphasis placed on the instructor taking the form of an active researcher, ensuring students are met with the best practices in the classroom to suit their needs.

Pepin et al. (2017) cited a study conducted by Yang and Wang (2015) summarizing the views of Chinese mathematics instructors about the definition of what it means to be an expert educator. Pepin et al. (2017) found to be a leading authority in education one must play multiple roles including those of a published researcher, a mentor to novice educators, a scholar in educational disciplines, and a pedagogical archetype among pupils and instructors. Similar to Yang and Wang (2015), Li et al. (2011) surmised in China, an expert mathematics teacher encompasses the following qualities:

> 1) having sound subject content knowledge of teaching concepts; 2) appropriately identifying and dealing with difficult content points in students' learning; 3) emphasizing the development of students' mathematical thinking and ability; 4) using mathematics problem solving and problem posing for developing effective classroom instruction; 5) emphasizing and practicing student-centered instruction; and 6) motivating students. (p. 176)

Within the Chinese school system, educators are commonly promoted in rank as exemplary educators; this is typically done through continuing teacher training, receiving prizes or honors in teaching competitions at the regional level, taking part in mathematics education research, and enhancing the instructional abilities of colleagues (Li et al., 2011).

Teacher knowledge and expectations in Japan. In the Japanese education system, instructors have time to transform their craft of teaching, taking part in professional development that pertains to their classroom every week with highly qualified colleagues who have proven their expertise over time (Schoenfeld, 2014). In the Western world, this is often referred to as lesson study, also known as *jyugyo kenkyu* in Japan, which is a core principle of professional development in the Japanese education system (Mincu, 2015; Takahashi, 2015). While lesson study is used to enhance pedagogy, it more broadly advances student learning and teaching in the classroom and the district (Archer, 2016).

According to Archer (2016), lesson study in Japan is inquiry-based in nature, centering around instructors taking on dual roles of educators and researchers. Commonly, the lesson study's focus is concentrated on arduous topics to indoctrinate or pre-determined areas of weakness driven by school data (Archer, 2016). Takahashi and McDougal (2016) elaborated on the structure of lesson study to describe it as a careful study of a standard within the national curriculum, followed by extensive scholarly research over the specific unit, and leading to a close examination of the curriculum and any supplemental materials. In addition, Takahashi (2011) cited the importance of preparing for a variety of solutions ranging in complexity and understanding, noting by doing so, educators are poised to discuss the topic with flexibility and ease. The aforementioned process is carried out by the planning team, which sculpts the lesson, and one person from this team is chosen to teach to the class while colleagues from within and outside the planning team observe (Takahashi & McDougal, 2016).

The crucial facet of lesson study unique to Japan's philosophy is the collective group of instructors who must be comprised of *koushi* or a "knowledgeable other," considered to be highly experienced and specialized in the core subject (Takahashi & McDougal, 2016, p. 515). Lesson study consists of a three-pronged process that includes designing the lesson, finding research to best support teaching methodology, and a follow-up discussion after the presentation of the lesson, all including the expertise of the *koushi* and other colleagues throughout each phase (Simmons, 2016). Experts have stated the inherent value of lesson study lays in the constructive collaboration after observation of the lesson among colleagues including the knowledgeable others (Archer, 2016; Schoenfeld, 2014; Simmons, 2016; Takahashi & McDougal, 2016). Archer (2016) noted educators observed from Japan did not strive to teach the perfect lesson, but at the heart of their intent was to learn from each other and grow educationally and professionally.

Teacher knowledge and expectations in Singapore. Like many other Eastern Asian nations, Singapore places significant importance upon teacher selection, granting admission to initial education majors from the top one-third of secondary graduating classes, and instructors receive training specifically over the nation's course of studies in the sole teacher preparation program at Nanyang Technological Institute (Tucker, 2011). In a perceptual study comparing the pedagogies of mathematics in Chicago and Singapore, Fan (2014) concluded both sets of educators rely on their experience and reflections as instructors. Within the same study, American educators ranked their pre-service, post-secondary training to be least important, while Singaporean educators linked their training as being remarkably more essential (Fan, 2014).

A key facet of Singapore's success focuses on how the country encourages educators to grow professionally, even providing funds at each school for teachers to travel to various countries to view and research sound educational practices different from their own (Tucker, 2011). Singaporean educators continue their professional work in professional learning communities, giving credence to scholarly works and in-service training focused on mathematical pedagogy (Kaur & Wong, 2017). In the school setting, Kaur and Wong (2017) also discovered Singaporean professional development included instructors conducting research projects and taking part in the practice of lesson study.

Organizational and Social Climate

White-Clark et al. (2008) stated, "Teachers' beliefs, behaviors, and attitudes are invaluable variables to student learning" (p. 40). Educators' epistemological views could be worrisome due to the fact Boaler (2016) reviewed a study that annotated the beliefs of college professors; researchers found the subject of mathematics is where most collegiate educators held the perception that only a certain group of students could perform mathematics. The idea a person is born with a natural aptitude for mathematics and is only able to comprehend the subject matter at a certain level is a "fixed mindset" (Boaler, 2016, pp. 5-6).

Boaler (2016), author of *Mathematical Mindsets*, was very clear to state anyone can become a mathematical thinker if he or she is willing to engage and persevere in mathematical exercises. Ericsson, Charness, Feltovich, and Hoffman (as cited in Dweck, 2014) conducted research relating to "geniuses," or those who exhibit substantial creative solutions when compared to various talented peers. The researchers found what delineated genius accomplishments from other gifted persons was simply the continued effort and practice applied to their discipline (Dweck, 2014). Several researchers have stipulated it is imperative and beneficial for those who find mathematics difficult to transition toward a positive or growth mindset to further their success in the area of math (Kalaycıoğlu, 2015; Westenskow, Moyer-Packenham, & Child, 2017).

Furthermore, Boaler (2016) and Zoido, an analyst with the OECD, dissected data from the PISA 2012 assessment and revealed, "The highest-achieving students in the world are those with a growth mindset, and they outrank the other students by the equivalent of more than a year of mathematics" (p. 7). In a study conducted in the United States, students with increased confidence and motivation were found to interact more within the academic realm and take part in school-based activities more often than their same-aged counterparts with lower confidence and motivation (Areepattamannil, 2014). In populations of minority students, when these students prescribe to a positive opinion concerning school and their ability in the subject of mathematics, there are significant, documented gains in their mathematical performance (Bonner, 2014). Leading neuroscientists from Stanford University have corroborated similar findings and reported students' brains function more effectively during math exercises when said students have a positive attitude toward mathematics (Sparks, 2015).

Researchers have not only pointed to the social climate brought about by the students but by the instructors. As Boaler (2016) pointedly remarked, it is essential for educators to reinforce positive thoughts and lofty goals for students who appear to lack motivation and who arduously complete mathematical tasks. Researchers have mirrored these thoughts by elaborating upon specific goals for educators:

> Mathematics instruction should provide students with a sense of discipline—a sense of its scope, power, uses, and history. It should give them a sense of what mathematics is and how it is done, at a level appropriate for the students to experience and understand. As a result of their instructional experiences, students should learn to value mathematics and to feel confident in their ability to do mathematics. (Schoenfeld, 2016, p. 12)

Katz and Stupel (2016) found while studying educators at the elementary level, that their beliefs concerning their ability to teach students mathematics was lacking; after instructors attended a seven-month workshop, instructor self-efficacy improved and led to an increase in students' mathematical success and motivation (Katz & Stupel, 2015). Various studies have demonstrated instructors who exude a love of teaching and motivational behavior often perpetuate the belief students can meet rigorous objectives and standards (Usta, 2016; You, Dang, & Lim, 2016), even in populations of underserved students (Bonner, 2014). Students gain the ability to engage in discussion when support and motivation from instructors are available (Kelly & Yuan, 2016).

When reviewing literature pertaining to a school system's organizational and social climate, equity was highlighted time after time as being a pivotal determinant of a country's mathematical success, including schools within the United States (Clements & Sarama, 2015; Nasir, Cabana, Shreve, Woodbury, & Louie, 2014; OECD, 2016a). Once again relating to the achievement of *Railside High*, Boaler and Staples (2008) noted a great significance of the approach of educators to prevent and discourage social differences among student groups by promoting the tenet students have various abilities the group can draw upon to be successful. A key principle of *Complex Instruction*, originated by Cohen and Lotan (as cited in Boaler & Staples, 2008), was used throughout the study to encourage equity among student groups by promoting multidimensional classrooms and various approaches to teaching practices.

The OECD (2016a) defined equity as "…ensuring that all students, regardless of their background, have the opportunity to obtain a quality education and reach their full potential" (p. 42). Multiple reforms should be considered to provide equity within a school system and enhance mathematics instruction, including the following: 1) commensurate funding and access to resources, 2) concise and rigorous standards with congruency across curriculums, 3) identifying a school's needs and appropriately monitoring for improvement, 4) building interpersonal relationships among students to foster a love for mathematics, 5) providing quality mathematics instructors who are willing to collaborate and grow as educators, and 6) teachers who prescribe to the same philosophical methods and pedagogy of mathematics instruction (Clements & Sarama, 2015; Nasir et al., 2014; NCTM, 2014; OECD, 2016a). Reforms centered around promoting equity among school systems have been cited in multiple instances to bring about and enhance mathematical achievement, even with vast disparities in student socio-economic background and immigrant populations (Boaler & Staples, 2008; Gustafsson, Nilsen, & Hansen, 2018; Nasir et al., 2014; NCTM, 2014; OECD, 2016a; Tucker, 2011).

Organizational and social climate in Canada. According to Vashchyshyn and Chernoff (2016), a leading factor of Quebec's success stems from the view framing mathematics as a recreational activity for students. Quebec has a long history of educators and various associations hosting mathematics competitions, including riddles, “magic tricks,” and logic puzzles to engage over 18,000 students from multiple ability levels, not just the elite (Vashchyshyn & Chernoff, 2016, pp. 5-6). In a study completed contrasting intrinsic and extrinsic motivation factors, Areepattamannil (2014) found among Indian immigrants in Canada and their peers in India, intrinsic motivation was higher and led to more favorable views of mathematics. Divergent from these results, Indian teens from India tended to display statistically higher rates of extrinsic motivation, which negatively impact mathematical success (Areepattamannil, 2014). The study detailed Canada’s “individualist” culture and made the conjecture individualism creates autonomy, autonomy leads to intrinsic motivation among students, and intrinsic motivation incites self-learning and greater academic achievement (Areepattamannil, 2014).

Organizational and social climate in China. Researchers have pointed to the heavy influence education plays in Chinese society, especially education’s centrality to moving up the social ladder (Tucker, 2011). Teachers are seen as examples of morality and are often respected as experts in their chosen fields (Li et al., 2011). Tucker (2011) relayed the most common belief among the Chinese revolves around effort and its ability to compensate for one’s natural ability. In a multi-grade-level comparative study of children in China and the United States, Bear et al. (2018) determined Chinese students had a higher opinion of the school climate within their buildings prior to elementary school; researchers found this construct did not impact the students’ engagement within the classroom setting.

Organizational and social climate in Japan. Due to Japan having a mountainous terrain, a lack of natural resources, a world-renown population density, and a geographical location exposing them to relentless natural disasters, inhabitants have continuously relied on problem-solving and critical thinking skills to see them through difficult situations (Tucker, 2011). Not only do these factors contribute to their dedication to education to survive problematic instances, but it has also led them to rely on each other—providing for a group-oriented, or unified, nation instead of one solely based on individualistic interests (Bjork, 2015; Tucker, 2011). Tucker (2011) revealed in the Japanese society, high-stakes examinations are commonly seen as the path to display a student's fervent dedication to education, along with the responsibility to earn respect and demonstrate success to elders (e.g., parents, family members, teachers, and administrators). In Japan, a child's academic success is often seen as a reflection of parenting ability and familial stability; to not disrespect family honor within the community, students take all examinations very seriously (Tucker, 2011).

A central principle within Japan's educational system remains a student's effort is the determinant of academic success, not inherent ability (Tucker, 2011). This belief may help explain why Japanese educators and parents place a large emphasis on after-school and private tutoring, along with considerable parent involvement and contact to prevent a student from falling behind (Tucker, 2011). Results from a 2011 TIMSS assessment found eighth-graders in Japan continued to produce similar outcomes in the area of mathematics when tracing the link between confidence and achievement (House & Telese, 2014). Much like other students around the world, students who demonstrated high self-efficacy skills in the area of mathematics tended to earn higher achievement scores, while those who rated themselves as having difficulty in mathematics tended to score lower (House & Telese, 2014).

Organizational and social climate in Singapore. Singapore takes education quite seriously with a holistic approach to mathematics education by believing everyone is capable of learning no matter their current cognitive level, while continuing to support the individual child's growth as a student through multiple pathways (Kaur, 2019; Ministry of Education: Singapore, 2015; Tucker, 2011). Chan (2015) observed throughout the revisions of the mathematics curriculum in 2000, "perseverance" was added under the category of "attitudes" to suggest to students the need to strive toward solutions to "non-routine" and "open-ended" exercises (p. 935). Furthermore, in a 2009 revision of the curriculum, the sub-heading "beliefs" was indoctrinated under the heading of "attitudes," indicating a desire for students to reflect on their sense of place within the realm of mathematics (Chan, 2015).

Luo (2017) noted little research had been conducted to determine the impact of motivational behavior in the Singaporean classroom. In the previously mentioned study concerning secondary math students' engagement, the researcher determined educators who approached learners from the perspective of wanting to improve students' skills, create a relatable learning environment, and give students detailed feedback when struggling tended to have higher rates of engagement (Luo, 2017). In contrast, those instructors who continually valued performance based on classroom assessments saw lower rates of engagement with students (Luo, 2017).

Summary

When determining the best approach and system of reform to increase achievement in mathematics, it is essential to review multiple aspects that contribute to student performance (SRI International, 2009; Tucker, 2011). The essential focus of the literature mentioned above centers upon the need for a concise, standards-based curriculum; a pedagogical approach centered on research-based methods; collaboration and continued professional development on the behalf of the educators and school leaders; and instructors exuding a positive instructional philosophy that incorporates continued growth and motivation of students (Baete & Hochbein, 2014; Boaler, 2016; Hattie, 2003, 2017; Schmoker, 2006). For the United States to improve their standing among global front-runners in mathematics, and to allow students to one day be competitive and successful with a higher quality of life, significant changes must be made at the state and federal levels (Hattie, 2017; Larson & Kanold, 2016; OECD, 2016a; Tucker, 2011).

In Chapter Three, the methodology of this qualitative study is presented and justified. The problem and purpose of this study are briefly examined, along with questions that have evolved through the research. The population from which this sample was derived is discussed, as well as the instrumentation used to collect the data. Finally, the implementation and ethical considerations of the data analyzed are explained thoroughly.

Chapter Three: Methodology

This study was conducted to unveil specific strategies and common occurrences found in teachers' mathematical practices from top-ranking countries around the world. A qualitative method was utilized to reveal a rich perspective of multiple variables that could point toward mathematical achievement. Within this chapter, the problem and purpose of this body of research are reviewed, and the research questions are presented to guide the investigation. A close examination of the research is laid out, with a primary focus on the research design, extraction of the data from the population and sample, and the instrumentation utilized. Particular attention is directed toward the ethical considerations and processes with which the data were collected and analyzed.

Problem and Purpose Overview

National and international assessment results revealed a significant issue relating to a lack of achievement in mathematics in the United States when compared to Canada and Eastern Asian nations (OECD, 2016b; Tucker, 2011). The United States ranked 38th out of 71 countries assessed in the area of mathematics on the 2015 PISA Assessment; out of the 35 countries that participated in the OECD initiative, the United States ranked 30th (DeSilver, 2017). However, it is imperative to note the 2015 TIMSS assessments demonstrated eighth-graders in the United States ranked eighth in mathematics proficiency out of 37 participating countries (DeSilver, 2017).

While certain educational statistics concerning mathematics performance show promise, researchers continue to probe for various strategies that can further improve the quality of mathematics instruction now and in the future (Larson & Kanold, 2016; Tucker, 2011). There is an abundant amount of research related to improvement in mathematics, but few researchers have examined multiple factors that contribute to high mathematics performance. There have been a limited number of investigations into the success of multiple countries concerning a sustained pattern of mathematical proficiency (Tucker, 2011).

Further research and inquiry are required to discover key elements that can lead to greater academic achievement in the United States, specifically concentrated on mathematics (Hattie, 2017; Tucker, 2011). The purpose of this phenomenological inquiry was to discover the central "essence" of mathematical success among students who excel in mathematics in the top-ranking countries of Canada, China, Japan, and Singapore (Fraenkel et al., 2015, p. 430). Success in mathematics is defined as a continuous pattern of achievement on all international assessments.

Research questions. The following questions guided the study:

1. What leading factors do post-secondary mathematics instructors attribute to the academic achievement of secondary students, based upon the performance of secondary educators within the countries studied?
2. How do post-secondary mathematics instructors, from the countries studied, describe the preparation for secondary instructors' pedagogical approach?
3. How do post-secondary mathematics teachers characterize the social climate of the typical secondary classroom, among the countries studied?
4. Among the countries studied, how do post-secondary educators depict the structure of the curriculum and additional materials used by secondary instructors in the mathematics classroom?

5. Among the countries studied, how do post-secondary instructors summarize the initial and ongoing professional development secondary instructors receive throughout their careers?

Research Design

A qualitative study was conducted that incorporated a phenomenological design, which constructed a comprehensive explanation for the success in mathematics of specific nations around the world (Creswell, 2014). Due to the fact mathematics instruction is complex in nature, and a multitude of factors were addressed to increase achievement (SRI International, 2009), a qualitative study was required to expose a "holistic" account after information was gleaned from "…multiple perspectives, identifying the many factors involved in a situation, and generally sketching a larger picture that emerges" (Creswell, 2014, p. 186). A qualitative investigation required flexibility when delving deeper through the participants' responses, which allowed the researcher to shift the focus to follow a meaningful path of enlightenment as various patterns and themes were revealed (Fraenkel et al., 2015).

A standardized open-ended interview was utilized to extract factors which contributed to academic success from post-secondary mathematics instructors currently teaching in the United States and who have taught abroad from collegiate institutions in Canada, China, Japan, and Singapore (see Appendix C). The sequencing and wording of the questions were exactly the same across all participants, which improved the likelihood for responses to be compared and allowed for recurrent conclusions to be drawn from the data (Fraenkel et al., 2015). Various perspectives were gained from post-secondary mathematics instructors, which led the researcher to determine, comprehend, and describe commonalities from the responses which supported key components of mathematical success (Creswell, 2014). From the commonalities and construction of themes, an explanation was derived from the data (Fraenkel et al., 2015).

Population and Sample

The population included mathematics instructors who currently teach at American universities and have also taught abroad in Canada, China, Japan, and Singapore; no relationship existed between the participants and the researcher. Various post-secondary instructors were selected from each of the designated countries. The educators who participated in the research were derived from a homogeneous sample, which is a type of purposive sampling found in qualitative research (Fraenkel et al., 2015). In this instance, the homogeneous sample contained instructors from the previously mentioned countries considered "experts" at the collegiate level. Expert mathematics instructors are defined by the following attributes put forth by Li and Kaiser (2011): implementing activities that elicit critical and conceptual mathematical thinking, being prepared for multiple outcomes of students' understanding and solutions, and providing quality feedback and timely questioning to provoke student comprehension.

Using professional judgment, mathematics expert educators at the tertiary, or university, level were contacted through the use of electronic mail and asked to take part in this study (see Appendix D). All willing participants were interviewed from this sample. Individual participant consent from the post-secondary mathematics educators was obtained once approval from the IRB Committee was acquired (see Appendices E, F, & G).

Instrumentation

An interview protocol comprised of open-ended questions was utilized within this study to provide consistent procedures among the interviewees (Creswell, 2014). The researcher acquired responses using a recording device and handwritten notes to allow for accuracy and because of the possibility of technological failure (Creswell, 2014). Qualitative research utilizing open-ended questions allows for rich, invaluable responses which contribute to the product of success (Fraenkel et al., 2015). Interviews tended to include follow-up questions asked to expand on the interviewee's thoughts (Fraenkel et al., 2015). A major concern was bias of information due to the lack of presence of the interviewer and to the perceptions of the interviewees who relayed information not directly observed or collected by the investigator (Creswell, 2014).

The researcher developed the instrument. While this is not the ideal method, as cited by Fraenkel et al. (2015), due to the time and energy required, it was an advantageous endeavor to obtain specific perceptions to match the existing comparative variables of this investigation. Before the interviews were conducted, the instrument was field-tested by 10 local instructors, some from Eastern Asian countries, to improve and format questions for increased clarity and to definitively allow for specific variables to be measured (Creswell, 2014; Fraenkel et al., 2015). A standardized, open-ended interview allowed the researcher to explore the data to find certain comparable themes that emerged from the responses (Fraenkel et al., 2015). Further information was necessary, and participants were asked to provide an additional amount of time for continued inquiry.

Data Collection

Once approval from the Institutional Review Board (IRB) was acquired, the selection of participants began. Four post-secondary professors from each country (Canada, China, Japan, and Singapore) were randomly selected. A recruitment letter was sent to the participants, identifying the nature of the study, and the criteria classifying the desired candidates (Creswell, 2014). Once the letter of consent was read and agreed to by each participant, an interview was conducted through Skype, Google Meet, or by phone, dependent on the participant's preference. After completion of interviews, a third-party source gathered and removed all identifiable instructor information from the data obtained from the document (Fraenkel et al., 2015).

Data Analysis

The data from the interviews were collected and analyzed using content analysis to find reoccurring themes within the data (Fraenkel et al., 2015). Content analysis has been defined as a way to investigate various human experiences through a close examination of communication (Fraenkel et al., 2015). Krippendorff (2018) validated the use of this technique to identify common practices within different schools and to extrapolate certain phenomena among educational institutions. As the researcher dissected the descriptive information, coding categories, or themes, emerged to create a narrative illustration of the findings (Fraenkel et al., 2015).

Fraenkel et al. (2015) noted the use of content analysis can be limiting, as the researcher typically relies on data recorded or collected by the researcher. The use of content analysis in a study can make validity difficult to maintain, as categorizing data may be considered subjective among researchers (Fraenkel et al., 2015). The validity of the research was increased by triangulating the interviews to grow justification for the prescribed themes, utilizing peer debriefing to ensure the accuracy of the description, and using an external auditor to provide a review of the study as a whole (Creswell, 2014).

Ethical Considerations

Before the data were collected, each participant was informed of the purpose of the study and how it contributed to the field of educational research. Those taking part in the study were required to permit the researcher to use the data each provided. There was no possibility of harm or risk to the participants as anonymity will be protected; however, if at any time participants wished to discontinue the study, they were allowed to do so with no obligation to the researcher (Fraenkel et al., 2015). During the study and data gathering stage, all data were secured on a pass coded desktop computer for the extent of the study. No other party had access to the researcher's storage media or equipment. Removable backup of data was created and secured in a locked file under the supervision of the researcher. All information was kept locked and secured throughout the study and will be destroyed after five years of the completion date (Creswell, 2014).

Summary

In this study, the researcher discovered and defined the attributes that led to the continuous occurrence of mathematical success among specific nations. The purpose of this body of research was to unveil possible links to increased mathematical abilities to implement reasonable interventions in the future within schools demonstrating low achievement. Qualitative data were collected, and content analysis was conducted to reveal the phenomenon of heightened mathematical performance among the countries. Ethical considerations were examined in this study. The results of this qualitative study are presented in Chapter Four. The analyses of the interviews are described in a narrative format, along with how the data related to the research questions.

Chapter Four: Analysis of Data

Larson and Kanold (2016) emphasized a variety of components within mathematics instruction in the United States have not changed a great deal over time, while students' needs and standards for higher-order problem solving and abstract thinking have significantly increased. Many would be dissatisfied to continue the practices of centuries past within a multitude of professions; however, as a culture, the United States education system continues to carry on those very same practices today (Larson & Kanold, 2016). Researchers within the United States and abroad are calling for action and reform to reinforce mathematics education founded upon the 21st-century goals of technological innovation, globalization, and economic development (Bell, 2016; Costa, 2017; Enderson & Ritz, 2016; Hattie, 2017; Larson & Kanold, 2016).

Currently, there are countries in the world that have found particular and continuous success in the area of mathematics based upon results of international comparative assessments (OECD, 2016a; Tucker, 2011). According to the NCES (2017a), the United States has fallen below the international average of OECD countries on the PISA since 2003. Canada, Japan, and China have always scored above the international average on the PISA; Singapore did not test their student population in 2003 or 2006 but has scored above the international average since beginning testing in 2009 (NCES, 2017a). The United States has scored below Canada, China, Japan, and Singapore on the TIMSS since 1995 (Mullis et al., 2016; NCES, 2017b). With regard to the countries mentioned, educational leaders and researchers have questioned the core practices and impetus behind their mathematical success (OECD, 2016a; Tucker, 2011).

The purpose of this study was to formulate a deeper understanding of the contributing factors that lead to mathematical success in the classroom. Literature was reviewed to distinguish dominant factors which contribute to mathematical success. Frameworks from the SRI International (2009), Matthews (2013), and Schoenfeld (2016) contributed to the research questions which guided this study. Multiple researchers in the field of mathematics education point to key areas to be addressed concurrently for a student's mathematical growth to take place. Those areas include curriculum, pedagogy and instructional approach, teacher knowledge and expectations, and organizational and social climate (Matthews, 2013; Schoenfeld, 2016; SRI International, 2009). By exploring the perceptions of secondary mathematics educators from the countries of Canada, China, Japan, and Singapore, similarities and differences emerged within the structure and pedagogy of their educational systems. A closer examination could lead to further awareness and analysis of educational strategies that could be implemented by instructors and educational leaders to increase efficacy in the area of mathematics.

Using a phenomenological study to gain the perspectives of secondary mathematics instructors through the use of interviews, responses were examined and analyzed to find the "essence" of superior mathematical instruction delivered by educators who are considered to be highly qualified in the field of mathematics (Fraenkel et al., 2015). An interview with open-ended questions was conducted with a post-secondary mathematics education instructor from each country included in this study (Canada, China, Japan, and Singapore). The interviewees responded freely based upon their experiences and expertise. All responses were digitally recorded and transcribed verbatim (Fraenkel et al., 2015). All information used to identify the participants was redacted, and anonymity was assured in this study (Creswell, 2014).

As suggested by Fraenkel et al. (2015), the data were coded by analyzing the manifest and latent content—a respective focus on what was disclosed or stated on the surface and the underlying meanings discussed throughout the interviews. Content analysis was utilized to find reoccurring themes within the educators' feedback to glean a further comprehension of the essential qualities of effective pedagogy and other factors that lead to mathematical success in top-performing countries (Fraenkel et al., 2015). The following themes were revealed through the interviews: mathematical mindset, professional growth, cohesiveness, foundational skills, deeper learning, and student engagement.

Demographic Analysis

A homogenous sample of post-secondary educators who taught at the post-secondary level in the area of mathematics education from the countries of Canada, China, Japan, and Singapore were the participants for this study. Four professors (one female, three male) with doctorate degrees were interviewed and had taught from one to five years at the graduate level in the respective countries. All participants had taught or continue to teach mathematics education in the United States at the collegiate level, and their cumulative teaching experience ranged from 13 to 24 years. The instructors are considered to be experts in their field based on the criteria put forth by Li and Kaiser (2011) and have published extensive studies or literature continuing to contribute to the field of mathematics education in the United States and abroad. All participants were fluent English-speakers; translators were not necessary for the completion of the interviews.

Responses to Interview Questions

This chapter includes an analysis of the perceptual data collected from post-secondary instructors related to multiple factors contributing to mathematical success in specific countries. Four educational leaders were interviewed and identified as Professor A from Canada, Professor B from China, Professor C from Japan, and Professor D from Singapore. As noted by Creswell (2014), codes were determined by examining the responses that ranged from the expected to conceptually relevant data. From these codes, comprehensive themes emerged and were clustered by highlighting the "essential structure" related to the phenomenon of mathematical superiority found in the corresponding countries (Fraenkel et al., 2015, p. 431). Creswell (2014) suggested the use of five to seven themes in a qualitative study. The following themes identified throughout this chapter include mathematical mindset, professional growth, cohesiveness, foundational skills, and deeper learning. Within these five major themes, a subtheme was identified and is discussed within this chapter. Each theme is classified with an acronym, and the interviewees' responses are organized into the following themes:

- Mathematical Mindset (MM)
- Professional Growth (PG)
- Cohesiveness (C)
- Foundational Skills (FS)
- Deeper Learning (DL)

Analysis of Interview Questions

Interview question one (FS, DL, & MM). As an expert in your field, what characteristic(s) can be attributed to the academic achievement of secondary students in the area of mathematics in your native country?

Responses to question one varied among professors, with instructors often citing multiple factors which could be attributed to their countries' success. However, when dissecting the data, commonalities could be found. Professor A from Canada attributed success to the repetition of foundational skills within the textbooks used by the school system, particularly at the middle school level. Professor A stated:

> I have found looking at textbooks, in middle school in particular, that they just kept revisiting, revisiting, and revisiting that same content over those three years, so I like that they provided those students with a much stronger base of mathematical knowledge than students get in the States. Years ago, if I were to compare a sixth-grade math book with a seventh-grade math book, then an eighth-grade math book, the chapter hierarchies were the same, Chapter 1 would be pretty much what you see across all the grade levels.

Professor B from China highlighted the need for a firm knowledge of the underpinnings of mathematics, but not necessarily from textbooks. Professor B noted, "A teacher has to know everything and be fully prepared. Math teachers have to be experts in math in the first place, so that affects how we train teachers, and how they teach and plan."

Professor D from Singapore also mirrored the relationship between the expertise of the teacher and quality instruction. Professor D elaborated:

> Their teachers are also very knowledgeable in the area of mathematics, continuing their research. The quality of mathematics education is superior. I mean they sometimes discover their own mathematical theorem, solve very sophisticated mathematical situations, but you know for the largest skill all teachers should be very qualified. Teachers from Singapore also have very strong mathematical foundations. I'm not saying that teachers without mathematical qualifications cannot be good teachers, but I think in terms of policy, teachers need to have a strong mathematical background.

Professor B from China was clear to state this was not the only characteristic that can be attributed to mathematical success but relayed the very nature of mathematics instruction and learning is multi-faceted. Professor B discussed, "The curriculum and standards in China are very rigorous from a mathematical perspective." He noted certain geographical areas are tested using different entrance examinations due to the federal government acknowledging one size does not fit all. However, he adamantly stated, "Overall, I think all of them have the same focus on rigor, coherence, and the logic of mathematics." Professor C from Japan also pointed toward curriculum and standards as being the main reason for the country's profound mathematical achievement. He revealed Japan goes deeper into their learning and specified, "During the '80s and '90s there was a shift from teaching by tens to a problem-solving art, and that's a characteristic of Japanese mathematics education."

Professor D from Singapore went further to formulate not only did Singaporean teachers have a greater depth and pedagogical knowledge of teaching mathematics but the society as a whole encouraged education. Professor D explained, "I think that

Singapore students learn from the heart, they have good overall learning attitude. Of course, there are exceptions. But overall, students learn mathematics quite seriously; they put a lot of emphasis on mathematics." Professor B from China mirrored this sentiment as well when elaborating on the rigor of the Chinese educational system:

> It's very focused, I mean math-focused, the teaching you know, is that a kid can learn whatever, whatever the objectives are being taught. On the student side, the students over there are more devoted, and they know that they have a lot of expectations on them from the parents, from teachers, from society.

Professor B from China went further to expand and formulate the causes of such a mathematic-centric society. Due to intense competition among the students concerning national exams and college admissions, students must focus on getting all the practice they can in and outside of the classroom. He explained:

> I sometimes make jokes about this, but no it goes all the way back—there's a saying in China, "Do not lose at the starting point." You would think as a schooling that it is a life-long competition; it is a competition, you can lose your way in the middle, and even at the end. However, the Chinese mentality is don't lose your way at the starting point… You try to win at the starting point. What do you mean by that? You try to do all the good things at the very beginning when they are born and learn all good stuff from a very young age, then continue this from that point on… The expectations are very high. There's a lot of pressure for the kids, parents, and teachers to do well, and obviously math as we all know, is one of the few most-important subjects. How can you wait? How

> can you stand out among millions of other kids? You've got to be good at math, even in Chinese language and other subjects.

While all professors from Eastern Asia discussed the focus on national examinations at one or multiple points throughout the interviews and how it ties to the importance of mathematics education, Professor A from Canada diverged from this ideology and spoke about the overall perceptions of Canada's mathematical mindset. She discussed, "Teachers were not teaching to the test, they were all about math is awesome, math is important, and it makes you think really deeply about the world." This aforementioned mentality refers to a sense of creativity and engagement the Western world exhibits, and Eastern Asia is trying to work toward and emulate this mentality.

Interview question two (DL & FS). As a post-secondary instructor, what were your expectations for future instructors from your native country?

All participants in the study highlighted the need for future secondary instructors to have a solid understanding of the essential frameworks of mathematics, and a few professors went further to note instructors must push forward and require students to investigate mathematics at a profound level. Professor D from Singapore noted the need for secondary instructors to possess basic math skills. He expressed, "They [Singapore] emphasized 'ordinary mathematics,' a focus on foundational skills and pedagogical content knowledge. They always knew how to teach to specific students and common errors they would encounter."

Professor C from Japan not only noted the need for future educators to have a grasp on the fundamentals of mathematics but also detailed the necessity for the variation of specific skills and their purpose. He stated:

> Teachers coming from the teacher preparation program from Japan know the basics of mathematics, so they have an advantage there, and so you have a program that can focus more on content knowledge and content knowledge for teaching mathematics but also designing sufficiency. If you do not know the strong foundations of mathematics, you don't even know the ins and outs of how to teach mathematics in a way to help. In the middle school, teachers focus more on mathematics teaching and problem-solving. Uh, in the high schools, they focus more on preparation for university entrance examination, where they focus more on mathematics procedure. Due to high-stakes testing without the use of a graphing calculator, their basic math skills must be better.

Professor B from China extended this theory to relay the vast importance of encouraging all math instructors at varying grade levels to look at the subject from its greatest depth. He framed his perspective:

> I think the very first, formal, and important aspect is that we expect our prospective teachers to have a very solid understanding of advanced and secondary mathematics. The training received at the elementary, secondary, and college level is rigorous, advanced, and in-depth. I remember when I taught the master degree courses in China, working with future teachers, I was trying to incorporate ways to convince them of why as future math teachers, they needed to know advanced mathematics. They needed to learn a lot of college or advanced mathematics. So, students would tend to think if I'm teaching elementary or middle school mathematics, if I know those well, I can teach, right? So, I had to think hard for myself first of ways to make them understand why they needed to

> go farther. I wanted them to see the bigger picture, to know more, to know deeper. Just knowing elementary and middle school math for them would not be enough for them to be a good math teacher; they need to know the content knowledge, which is extremely important.

While Professor A from Canada highlighted the need for a deep understanding of mathematics, she slightly deviated from the others by discussing an approach that fosters a sincere appreciation for mathematics as a whole. She shared:

> My expectations would be that they kind of learn to love math and see it in a different way than they have seen it before, so when they go out to classrooms, they're not putting out the front that anxiety or fear of mathematics to their students. With post-secondary teachers, within Canada and the United States, they pretty much decided to be post-secondary teachers because they love math, and there's a lot less math anxiety, but more of math is a step-by-step process for them. I want them to see that math can be very creative. You just don't have to lecture to students; you can actually get them engaged into meaningful activities… Kids are really smart, they can figure things out. Just putting across the fact that even though the typical secondary teacher as seen a lot of lecture-based instruction, there are much more powerful ways to teach than that.

Interview question three (MM). Please describe the teaching philosophy you imparted on future teachers from your native country concerning how a student's cognitive ability affects mathematical performance.

All instructors unanimously responded while a student's cognitive ability is a factor it does not hinder anyone from learning or improving understanding of mathematics. Professor D from Singapore asserted, “In Singapore, we emphasize the student’s cognitive ability very much.” He summarized every student is encouraged to use meta-cognition strategies which include self-reflection, determining the reasonableness of answers, and finding alternative ways to solve problems. Every student is thought capable of doing this with effort. Professor C from Japan echoed this response by declaring, “Any student can be better at math, with hard work and effort. In Japan if you’re not doing well, maybe you need to study more, so that is a huge difference.”

Professor B from China relied on statistics and Chinese philosophical beliefs to expand on theories of how a student’s intellectual capabilities affect mathematical performance in the classroom. He explained, according to popular belief surrounding the applications of a bell curve, there are always limits to the intellectual functioning of the population as a whole. However, this ideology does not keep students from finding success in mathematics. He discussed:

> You know the English saying, “The early bird, gets the worm?” The Chinese way of saying that is, the literal saying is, “The dumb bird, but if you fly early, you can get the worm.” Also, the other very important traditional belief is, um, practice makes perfect, or hard work can compensate for you being not smart or bright. So, that is an extremely important aspect of the Chinese education system for math in particular. Only a small population can be really good in math, so for the rest of us, you just have to work hard, and do a lot of practice, and it’s very natural, very normal.

A subtheme that emerged within this question focused on student engagement. Professor B from China went further to assert while student effort and practice can lead to further or increased achievement, including students to participate mathematically can be just as beneficial for improvement. He noted, “In theory, even in some traditional, or Confucian teachings say, you’ve got to teach or organize your teaching according to different learning styles and learning ability. That’s a commonly agreed principle.” He continued this thought by mentioning teachers must work toward including those students who have difficulty or lack motivation. Professor A from Canada laterally communicated this thought by stating, “If you keep students engaged, they will find mathematical success.”

Interview question four (DL). As a post-secondary instructor from your native country, please describe the process you use to instruct secondary educators in establishing their daily learning objectives and the steps necessary to complete their typical daily lessons.

Participants’ responses were aligned and pointed to deeper learning being a requirement and focus when establishing the process of instruction. Professor C from Japan discussed teaching mathematics required progression from thinking like a student to transforming into an educator, with a broader view of mathematics and seeing it “differently.” He further elaborated, “You have to shift them to ideas of learning mathematics through experiences and gaining a holistic perspective, rather than memorizing facts and procedures.”

Professor B from China and Professor A from Canada expanded on this thought and mirrored their feedback to reflect a direct correlation between teaching pedagogical content knowledge to deepen student understanding of mathematics and enhance future lessons in a secondary education setting. Professor A from Canada discussed:

> I'll be pretty explicit about how research shows that students learn better through active learning and group problem solving and the standards of mathematical practice, and so these are the ways that I want them to be working on mathematics in the classroom.

While Professor B from China and Professor D from Singapore were clear to weigh in on the importance of group work and discussion, the professors explained time is limited to cover content in classrooms, and these interactions must be concise and well-organized by the teacher with very direct student outcomes.

Professor B from China and Professor D from Singapore were precise in describing typical daily lessons. Professor D from Singapore described a structure of a lesson that includes questions directed to students, lecture, demonstration, and an in-class activity followed by a discussion that tends to be more inclusive of each student's learning. While Professor B from China mentioned these same elements being essential in theory, he was very direct in stating that due to short class periods, typically 45 minutes, time could not be wasted and ultimately resulted in teacher-led activities with less of a focus on group work. Both discussed the importance of assigning homework at the end of the class period. Professor D from Singapore revealed, "Homework is always assigned on a daily basis. They expect it; it is embedded in them 100%. You have to do homework. Your job is a student; you are there to learn."

Interview question five (DL & FS). From the perspective of a post-secondary instructor, is there a specific type of problem, or problems, that you encourage secondary instructors to embed in assignments to guide students in meeting those objectives?

Once again, all professors came to a consensus concerning the need to embed problems requiring higher-order and critical-thinking skills. The most prominent theme for this question was focused on deeper learning as an absolute in the grander scheme of mathematical problem-solving. Professor A from Canada and Professor C from Japan both remarked about the ability to transform a problem's complexity to provoke an enlightened mathematical experience. Professor A from Canada referred to literature she used in a methods course she taught in Canada, *Adapting and Expending Secondary Mathematics Activities*, by Prestage and Perk (2001), which allowed secondary educators to modify the problems in their curriculum to make them more "meaningful" and "accessible" to all students. Professor C from Japan stated, "Many people discuss a type of problem, but if you bring an interesting problem to the students it can become a series of problems that is more important to think about it on varying levels to reach everyone."

Professor B from China and Professor D from Singapore revealed multiple types of problems are assigned and systematically placed within a specific sequence to achieve optimal learning outcomes. Professor D from Singapore quite clearly noted creative and open-ended problems were a suggestion for whole-class discussion to promote higher-order thinking, while the teacher was there to help as a facilitator. When assigning homework, or work to be done independently or outside of the classroom, Professor D noted, "Basic problems were assigned to reinforce fundamental mathematics." Professor B's statements were parallel regarding the order of specific types of problems. Professor

B went further to discuss “word problems,” as described in the English-speaking world, are considered traditional classics. He elaborated:

> While some researchers and radical progressives have sought to abandon these problems in the past, passing them off as artificial and irrelevant, they still have a great deal of value. They teach us to analyze the basics structures of word problems—the variables, what’s given, what’s not given, and how you are going to connect them—with an equation, a formula, a graph, or whatever. That’s the basics; I mean we’re seeing that nowadays that those are the most relevant or important, so if you don’t know those, how can you do something more complicated like designing a program or solving a real-life stock-market problem? You have to do something more complicated. We do value tradition, the routine types, but at the same time, the teachers know well, which is the easier one, which is the harder one, so they choose problems with purpose. However, foundational skills should not be discounted in value. Everything in the Chinese education system is a focus on the foundations—the foundations of knowledge, concepts, and the foundation of skills. That takes some time to practice and acquire these skills. You have to practice; practice in order to be fluent. The fluency is exceptionally important.

Interview question six (DL). In your expert opinion and through observation, please describe the optimal structure of the students’ learning experiences and overall atmosphere in a typical secondary mathematics classroom within your native country.

The subtheme of student engagement once again emerged from this question. All instructors from Eastern Asian regions concisely revealed time was of the essence, and focus must strictly be placed only on academic matters. Professor D from Singapore expressed, "Very little time is wasted, due to quality classroom management skills." He went further to elaborate this was due to the school culture and teacher expectations, and all teachers can improve in this area. Professor B from China continued to explain the reforms of the Chinese educational system were put in place to make the focus of a lesson more enjoyable and exploratory. He expounded:

> So instead of going through traditional routines, they might start the class with some interesting real-world problem to promote curiosity and to explore, in order to have a classroom discussion. This happens so much more than the old time. However, I still believe that whatever open-ended approach or activities they do, first of all, it has to serve a purpose. You cannot do whatever it is you want to do just for fun and make it interesting; you have to make sure your open project has to be well-guided by the teacher. The teacher has to keep a close eye on everything that is happening and make sure—more like scripted, so that way, they can make sure that everything is going toward the desired direction. Then, on the other hand, this kind of process is very short. Very quickly, the teacher brings a closure to the activity and discusses how the activity applies to the new concept being taught. Then right away, the teacher usually turns to a lecture. You hardly see a teacher spend 45 minutes walking around during an activity.

While Professor C from Japan highlighted the need to go in-depth within the realm of mathematics, he also discussed the thought process of the students and their mindset toward math in general. Professor C from Japan noted:

> In the Japanese classroom, they can appreciate the problem and knowledge, even though they may not be able to solve it… "I attempted, I tried it." The Japanese classroom is more open, more culture, welcoming, more inviting. "I tried it; I can do it."

In contrast to the other interviewees, Professor A from Canada disclosed, "As an instructor at the university level, I don't feel I have enough experience within the public school setting to have an opinion concerning the atmosphere."

Interview question seven (DL & C). Please explain how you instruct secondary teachers to determine the curriculum and any supplemental resources used in a typical daily lesson.

The participants from Eastern Asian cultures appeared to share a similarity in their responses to this question focused on a sense of cohesiveness on a national scale and a dedication to intensely working within the curriculum integral to the educational process. Professor D from Singapore relayed the country has not only a national curriculum but also a national mathematics syllabus for each level. Professor D from Singapore went on to explain that due to having a textbook officially issued by the school or the head of the department provided from a nationally selected collection, this is not an issue. He noted textbooks should include higher-order thinking skills and the emphasis on Western ideas of collaborative learning.

However, Professor D stressed the importance of determining which textbook best fits the curriculum used in the classroom. Specifically, Professor D stated, "They [the district and administrators] need to match the philosophy of the curriculum. Curriculum does not support every type of pedagogical philosophy, so you cannot let them [teachers] select the textbook to match the curriculum. It is quite professional work."

While Professor C mentioned Japan's national curriculum, he disclosed the need for secondary educators to "build on" to the curriculum. He contrasted the United States' methods of covering a vast amount of material and standards to Japan's concept of focusing on a standard and exploring the underlying root of the problem to derive a greater sense of meaning and accessibility for all students. He stated, "Japan goes in greater detail into how to study mathematics, design curriculum to make the student think deeper, which the teacher must do every day." He delineated the difference between educators teaching to a textbook from teaching with the textbook when he disclosed, "You're not going to go farther; you're going to go deeper over similar topics to build a deeper understanding. Mathematics is not a bunch of processes and procedures; it's more like a way of thinking."

Professor B from China explained education has heavily influenced Chinese society, and most often, educators are led to rely on the classics for direction and mastery. He revealed:

> Partially because of the traditional Chinese culture, we value authority, we value experts, from the Chinese classic work such as Confucius, Tzu, all of the classic works, all of the famous philosophers. They have only a few works left, so we all try to read and read into it and think hard. The Chinese belief is, how do you read a book? You know a book doesn't have to be thick—you need to start with the same book, and read and read carefully, think hard, and then think about it—then that same book becomes much thicker. Like thicker in your head, because there's

> a lot of thinking and a lot of ideas, lot of questions going on. Then you keep thinking, and in the end, the book becomes thicker again. You have to think everything through. So that kind of reading philosophy toward the classic work applies to teachers preparing to teach mathematics. The textbook becomes very thick. The authors, the writers, can't say everything, everywhere. They just try to put the basic script – all the parts of the mathematical style.

Due to a weighty reliance on the literature passed down through the centuries, Professor B from China explained national standards in China are relatively new in comparison to the substantial influence of the classics.

Professor B from China expounded on this thought by bringing awareness to the fact that China has always had nationally issued textbooks and teaching guides, eliminating the need for supplemental resources. He clarified the common conception teachers have toward textbooks that instructors are to spend a great deal of time studying the textbooks finding their "own subtle comprehensive understanding," and a large portion of their time should be used to come up with their examples and practice problems. He made sure to elaborate teachers are only assigned one to three classes to teach, typically the same class and grade level, and the remainder of the workday is meant to focus on grading, professional development, and sculpting future lessons. Also, state-distributed teaching guides are manufactured to explain to educators the most difficult concepts for students to grasp and how teachers should address certain student difficulties.

Due to multiple reforms through the 1980s to the early 2000s, Professor B talked about China's desire to incorporate practices from the Western world centered on mathematical collaboration and promoting individual interests. He discussed that because of China's openness to research and readiness to embrace different educational philosophies throughout this time, more autonomy was given at the provincial level to make mathematics more engaging to students. The change of philosophies did cause backlash from senior educators and high-ranking mathematicians who were concerned about the loss of the tradition of mathematics, the rigor, and the depth in exchange to make mathematics more exciting. He explained, "I think if I speak for most of the mathematicians who are old-school; they would say math is hard. You can never really, really learn math just by making it fun or real-world relevant." He went on to express that at any time a student delves deeper into any discipline, or subject area, the content will become more abstract and may not be considered enjoyable. He said, "You have to experience the suffering; you know this hardship, the struggles, the challenge, and keep trying."

The reference to the "Western World's" concept of making mathematics more engaging and decentralized was found in Professor A's response. When speaking about Canada's curriculum, she clarified the standards are written at a provincial level, rather than the national level. She also noted supplemental curriculums are used at the secondary level, specifically Geogebra. Professor A also mentioned the integration of Desmos, an interactive site which utilizes a graphing calculator and displays open forums for educators and students to share graphing projects.

Interview question eight (C & PG). In your experience, what types of professional development do you encourage secondary instructors from your native country to take part in to improve their effectiveness as educators, and how often does this typically occur?

Once again, the respondents from China, Japan, and Singapore closely aligned in their perspectives based on the foundational underpinnings of professional development. The feedback given by the respective professors consistently pertained to a daily focus on professional growth not by individual secondary mathematics educators, but rather as a collective unit of instructors reaching a common goal of superior educational outcomes for all students within the school system. Exceptional educational instructors are seen as leaders in these specific school systems, which according to the interviewees, drives instructors to go beyond the classroom and continue researching the optimal path for greater learning outcomes on a local and international platform.

Professor D from Singapore relayed professional development is an area in which Singapore excels. Professor D described professional development activities are held on a national scale, as a group of 20 to 30 schools form a "cluster" to hold conferences, but also growing as an educator is a concurrent focus on a local scale. He explained most Heads of Department, or administrators at each school, form teaching academies made up of master teachers willing to assist colleagues and maintain a well-rounded knowledge of the most current educational research locally and abroad, as well as contribute to the research. Concerning the relationship between the teachers and Heads of Department, he added, "In Singapore, we all find teachers' self-reflection and the ability to learn from their colleagues and their exchanges are their most important resources." He was clear to address a key component of why this relationship works based on the essential principle that master and novice teachers and every type in-between must work together to grow into their expertise.

Professor C from Japan also noted the desire to use colleagues and administrators to gain skills in the area of mathematics education by utilizing a common tool known as lesson study. He explained rather than outsourcing lesson study takes place within the school system, most commonly an ongoing, daily process of designing and crafting lessons implemented every month, at the very least, for observation. The school district typically identifies the area of focus and works toward improvement throughout the year.

Similarly, Professor B from China reiterated professional development is an ongoing and daily practice, mainly due to only a few classes taught during the school day. He emphasized the theory secondary mathematics instructors are seen as experts; they must be prepared by having all grading, planning, and collaboration completed to prepare for their lessons fully. Parallel to the statements made by Professor D from Singapore and Professor C from Japan, professional growth is a group effort, one which binds the spectrum of expert to novice teachers. Professor B from China disclosed students at all grade levels typically stay in one stationary room, and teachers, even at the elementary level, exercise expertise and focus in one content area per grade level; the instructors go from room to room to teach. The area gives way for a communal office space for content-specific educators to gather and collaborate. Professor B specified research groups are formed within these shared spaces according to the subject taught, and everyone participates in daily discussion and research, understood as an integral entity of teaching.

While Professor C from Japan and Professor D from Singapore discussed the concept of peer evaluation of one's instruction, or lesson study from within the district, Professor B from China elaborated concerning the more extreme extent of competition China places on exemplary lessons and teaching. He explained:

> When it comes to professional development, they have school-wide teacher demonstration. They have district-wide demonstration, sometimes the competition. We're really big into competition, like sports. We have an honor, like if you're nationally certified. You write a good lesson; you provide a video, those types of things, you know. But in China right now, there's at least two major incentives for teachers to be engaged in various active planning and teaching—for one, it is the competition. Competition is if you get, uh, like if you get first prize award, second place—that is a big thing. That adds a lot of weight to your resume, which affects your promotion, your being certified, you getting tenured—really, really crucial to those.

He continued to discuss, typically, exceptional teachers who win awards and competitions through their demonstration of exemplary teaching stay in classroom teaching at the same school where they started their careers. They continue to grow and develop their skills within teacher research groups.

The sense of community and cohesiveness found at the local and provincial levels by the professors from Eastern Asian did not appear to be present in the response given by Professor A from Canada. However, Professor A compared and contrasted professional development and how it applied to Canada and the United States. She found a resemblance between the two countries and discussed teachers had math conferences at the regional level and continued their coursework at the university level. She revealed, "Programs had a huge international focus in mathematics, where in the United States, we typically focus on American researchers." She went on to state a master's degree is not a requirement for Canadian teachers. She distinguished, "It was more like they [Canadians] were coming to learn more, not to get a bump in salary. They wanted to be more of a leader, but ultimately, I think it was more of personal interest, more than a requirement."

Interview question nine (MM). In your expert opinion, with respect to your native country, what is the value placed on education, specifically mathematics education?

All participants disclosed mathematics education is of value to their nations; however, respondents from Eastern Asian countries were very direct to state an immense value and attention are placed on mathematics education. In the competitively driven systems of China, Singapore, and Japan, supremacy in mathematics is a way to stand out as a student and set oneself apart from the rest. Professor D from Singapore and Professor B from China both divulged the immense pressures placed on students to succeed academically and the government's role in alleviating pressure. Professor B from China explained the emphasis placed on students to do well at a very young age to be competitive and receive admission to the top schools in the area. He specified:

> In the most recent picture, the government definitely wants to relieve the burden on Chinese students, and partly because the national entrance exams, because in China, there is a lot of pressure. At the first level, we have the reforms, trying to make math easier and more interesting. But, there is always this virtual reality in

> math, which is no matter what the government says, there are only those limited spots in the good schools, colleges, or universities that represent the population. If he or she cannot get into a good university, who is going to give them a good job, so to the contrary, the burden actually gets heavier. Mathematics holistically was a big, big deal. It's a way to demonstrate that you're a genius, that you're smart, so many elementary, middle, and high schools – that was a way to recruit students—to look at their math scores, their awards, to see if they won any prizes —things like that.

Professor D from Singapore mirrored this concept of the importance of mathematics is due to testing and mentioned:

> That is related to culture, expectations of the society, and the education system. For example, they have high-stake national tests that determine students' future, so in Singapore, other students, they want to leave primary school in year six . . . They have pressure on their shoulders also to accept the challenge and work harder, which is quite clear.

However, both professors also expanded on a love of mathematics felt by the nations. Professor D from Singapore explained this notion by stating, "I think that Singapore students learn from the heart; they have a good overall learning attitude; of course, there are exceptions. But overall, students learn mathematics quite seriously; they put a lot of emphasis on mathematics." Professor B from China divulged differences he noted between the Chinese and Western world:

> Chinese tradition is math-focused—the way we talk about math; we talk about math ideas, the thinking habits of mind, logical reasoning; they're all very normal

> in the Chinese math system. Over there, people, kids, parents of kids—they are not as scared of math as the Americans. It's a cultural issue, rather than a belief issue, because if you believe you can do it, you can do it well. But in many ways, the American culture toward math is you assume math is hard. I cannot do math. I don't want to do math. How can you ever be good at math without trying?

Summary

Throughout the past decade, when taking part in testing, students from Canada, China, Japan, and Singapore have earned mean scores which surpass the international average on the PISA and the TIMSS in the area of mathematics (NCES, 2019a). For this reason, candidates who taught at the post-secondary level in the area of mathematics education were selected to take part in this qualitative study to determine the contributing factors leading to mathematical achievement in these nations. Four participants were selected, three males and one female, one each from the respective countries mentioned to present their views concerning math education at the tertiary level. Interviews were conducted by phone and Skype to unveil the pedagogy and instructional processes relayed to secondary educators of mathematics in the formative years of their teacher education programs.

Throughout the interviews, five themes emerged from the data including mathematical mindset, professional growth, cohesiveness, foundational skills, and deeper learning. Of these themes, interviewees often cited a significant relevance and attention toward deeper learning as a source of pedagogical and instructional approach. Hattie (2017) mentioned "deep learning" as an integral facet of a multi-pronged approach to precision teaching when an educator must consider in what phase of the learning process students find themselves. Hattie (2017) posed the conjecture that students must move through specific phases of the learning process (surface, deep, and transfer) to show true mathematical growth.

The findings and conclusions drawn from this study are discussed in Chapter Five. Based on the research questions, the findings from the qualitative data are presented and compared to the research that was examined in the review of literature. Implications for practice and recommendations for future research are detailed.

Chapter Five: Summary and Conclusions

Paul Ernest (2016) stated, "Mathematics education is a complex, multi-disciplinary field of study which treats a wide range of diverse but interrelated areas" (p. 37). Even though the United States has made improvements in the area of mathematics throughout the past decade according to national and international exams, the nation continues to significantly trail behind the countries represented in this study (Canada, China, Japan, and Singapore) despite spending more per student and proportionally graduating less (Tucker, 2011). The many entities of mathematics instruction must be addressed and closely examined to best serve all populations of students (SRI International, 2009).

The current study was designed to identify the essential foundations of mathematics education in Canada, China, Japan, and Singapore and to elicit the perceptions of four randomly selected post-secondary mathematics educators from countries with a high rate of proficiency in the area of mathematics. The objective of this phenomenological inquiry was to discover similarities and differences within these educational systems to glean a greater knowledge of what comprises their success. Phone interviews allowed for data collection and transcription, which led to the emergence of multiple themes. In Chapter Five, the findings are detailed by summarizing the responses from participants while extrapolating the data for patterns and discoveries. Conclusions were reached and led to the implications for practice and suggestions for future research.

Findings

The following section connects the literature presented in Chapter Two with the participants' responses from Chapter Four. The data obtained from the interview questions are analyzed, and the emerging themes from the previous chapter are discussed using the same acronyms. The themes revealed from the data are as follows: Mathematical Mindset (MM), Professional Growth (PG), Cohesiveness (C), Foundational Skills (FS), and Deeper Learning (DL). The responses derived from this query are found to correlate with scholarly research detailing the essential components needed to increase mathematics achievement among secondary student populations.

Interview question one (FS, DL, & MM). As an expert in your field, what characteristic(s) can be attributed to the academic achievement of secondary students in the area of mathematics in your native country?

Overall, this question resulted in the greatest divergence among participants' responses, although the data revealed similarities which can be accredited to the determined themes. Three out of four professors discussed their respective country's curriculum and standards to be the footing for mathematical success. Professor A from Canada cited the repetitive nature of foundational skills across grade levels throughout math textbooks. In slight contrast, Professor B from China and Professor D from Japan both mentioned strong ties to a narrow, albeit deeply rigorous curriculum, for all students mandated at the federal level and adhering to a highly coherent format.

These findings coincided with research-based data derived from Boaler and Staples (2008) in the case of *Railside High*, which implemented multiple reforms, including an intense and well-focused curriculum highlighting critical thinking and problem solving, to provide equity and promote high mathematical achievement among the entire school population. In conjunction with these results, a multitude of researchers determined many successful nations that receive recognition for superior scores on international mathematics exams have been found to possess clear national standards and a strong, data-driven curriculum (Li & Kaiser, 2011; OECD, 2016a; Tucker, 2011).

Additionally, all professors from Eastern Asia cited the need for instructors to have a deep content knowledge of mathematics and a true understanding of mathematical pedagogy. Ultimately, this provides instructors the ability to provide the proper instruction to cultivate student mathematical comprehension. Many researchers have specified mathematics pedagogy and the underlying determinant of content knowledge are accurate predictors of student success in the classroom (Hattie, 2017; Matthews, 2013; SRI International, 2009).

Professor A from Canada, Professor B from China, and Professor D from Singapore expressed students and educators have a love or at least an appreciation for mathematics within the realm of education. Professor A and Professor D discussed educators' passion and an overall positive attitude from students toward the subject. Professor B explained mathematics as a way of life and a necessity, nothing to be feared. Data led to the same conclusion; educators who demonstrate a positive, outward attitude and motivate students in the mathematics classroom are more likely to engage students and assist them in meeting arduous standards and objectives (Bonner, 2014; Kelly & Yuan, 2016; Usta, 2016; You et al., 2016).

Interview question two (DL & FS). As a post-secondary instructor, what were your expectations for future instructors from your native country?

According to several authors, an educator's pedagogical approach to mathematics should be balanced and address procedural knowledge along with a deeper context of problem-solving (Codding et al., 2016: Hattie, 2017; Larson & Kanold, 2016). All professors disclosed the need for prospective secondary educators to possess strong foundational skills in the area of mathematics upon entering the teaching program. However, Professor A from Canada, Professor B from China, and Professor C from Japan discussed the optimal desire would revolve around the theory that secondary educators embed and foster deeper learning in the area of mathematics. The previous research mirrors the perceptions of post-secondary instructors.

Interview question three (MM). Please describe the teaching philosophy you imparted on future teachers from your native country concerning how students' cognitive ability affects their mathematical performance.

Unilaterally, all respondents agreed students' cognitive ability does impact their initial comprehension, but each professor clearly stated all students are capable of improving their functionality in the subject of mathematics. The respondents echoed similar beliefs that through hard work and perseverance, students can find mathematical success. Boaler (2016) and various researchers in the field (Kalaycıoğlu, 2015; Westenskow et al., 2017) reported congruent outcomes; students considered to have a growth mindset show increased achievement, incorporating the belief that practicing complex mathematical problems can lead to overall improvement.

Interview question four (DL). As a post-secondary instructor from your native country, please describe the process you use to instruct secondary educators in establishing their daily learning objectives and the steps necessary to complete typical daily lessons.

All participants conceptually concurred the daily learning objectives must center upon engaging students in critical thinking and building a greater knowledge of mathematics. While three out of four professors embedded collaborative learning in their responses, Professor B from China and Professor D from Singapore presented a very concise approach that embedded specific steps of such lessons and emphasized the need for organization and homework. These thoughts align with data from many leading authorities within the research community; quality mathematics instruction must place deeper learning as a goal for each lesson or objective to increase students' mathematical comprehension (Boaler & Staples, 2008; Hattie, 2017; NCTM, 2014; SRI International, 2009).

Interview question five (DL & FS). From the perspective of a post-secondary instructor, is there a specific type of problem, or problems, that you encourage secondary instructors to embed in assignments to guide students in meeting those objectives?

A pattern from post-secondary participants emerged consolidating a need for exercises stressing foundational mastery and higher-order thinking skills to reach all students and bring about intrigue. Parallel to Hattie's (2017) work, the post-secondary instructors were careful to describe how a problem, or problems, must be presented to transform learning. Hattie (2017) recognized the need for students to be introduced to basic skills and procedures, eventually moving toward a stage in which the students can delve further into a more complex and abstract way of thinking. Eventually, students should be able to apply concept mastery to new situations (Hattie, 2017). However, Professor B from China and Professor D from Singapore were clear to delineate foundational skills are typically practiced during homework exercises to build fluency and mathematical awareness.

Interview question six (DL). In your expert opinion and through observation, please describe the optimal structure of the students' learning experiences and overall atmosphere in a typical secondary mathematics classroom within your native country.

While the theme of deeper learning was a focal point of this question, the subtheme of student engagement emerged. All professors from Eastern Asia revealed due to time constraints, it is imperative the teacher and students be in tune with each other, and everyone takes part in the learning process. Professor C from Japan went further to elaborate concerning the facilitation of self-encouragement as a culture of the Japanese classroom—students attempting the problem, despite the possibility they may not have the correct solution, is personally powerful to the students. According to Areepattamannil (2014), the mathematical successes of the studied countries are not surprising; students who have intrinsic motivation and a positive view of mathematics are more likely to take part in mathematical activities and achieve at a higher level.

In contrast, Professor A from Canada did not feel she had enough experience to discuss the atmosphere within the public-school setting. This instance may expose a disconnect in the relationship between post-secondary and public secondary institutions. However, due to the small scale of the population sample, this may be a random occurrence.

Interview question seven (DL & C). Please explain how you instruct secondary teachers to determine the curriculum and any supplemental resources used in a typical daily lesson.

A majority of the professors once again focused on the need to integrate collaboration and deeper learning into the curriculum, although a sense of cohesiveness emerged from the responses. Post-secondary instructors expounded on their thoughts about curriculum and supplemental resources, recognizing a need for those who educate young people to be on the same page, teaching the same standards, at the same grade level, while utilizing research-based material. To do so, educational frameworks and curriculum at the national level were regarded as being essential. This reflection coincides with a 1996 study by Peak (as cited in Özer & Sezer, 2014); the TIMSS assessment revealed a national curriculum was considered to be the leading factor of a country's academic achievement in comparison to other nations.

Interview question eight (C & PG). In your experience, what types of professional development do you encourage secondary instructors from your native country to take part in to improve their effectiveness as educators, and how often does this typically occur?

Professor A from Canada relayed the perception of a resemblance between the professional development of secondary instructors from Canada and the United States, consisting of annual conferences and school-sponsored workshops. The main distinction remained that Canadian universities and school systems feel comfortable seeking expertise in mathematics education from international sources; meanwhile, the United States often remains loyal to American researchers and studies. However, no literature could be found to substantiate this finding; it is important to reiterate that literature pertaining to Canada's provincial education systems are also lacking, especially comparative studies (Campbell, 2017; Tucker, 2011).

All post-secondary instructors from Eastern Asia were unified in concluding professional development is never-ending, often a daily routine embedded in collaboration with colleagues who are considered to be experts in the field, thorough planning through scholarly research, and reflection. Respondents felt due to the fact secondary instructors often share a collective space this perpetuates the feeling of a joint effort toward students' mathematical success. Similar studies mirror the ideology that professional development should include consistent, data-driven collaboration with colleagues who have proven to be successful educators, with all members working toward a common goal (Baete & Hochbein, 2014; Basque & Bouchamma, 2016; Boaler & Staples, 2008; Julie, 2014; Schmoker, 2006; Schoenfeld, 2014).

Interview question nine (MM). In your expert opinion, with respect to your native country, what is the value placed on education, specifically mathematics education?

Participants from each country discussed a positive outlook toward mathematics on a national scale. The professors from China, Japan, and Singapore all placed a great emphasis on the value and necessity for mathematics in one's life. Whether it be a way to differentiate one's ability from another's, or a profound desire to take part in a subject to obtain a feeling of enlightenment, all respondents stated mathematics fulfilled this objective for most students. Schoenfeld (2016) reflected on this thought by implying a student's mathematical experiences should not only be informative, but the entire scope of comprehension and understanding should transform the student's attitude concerning the ability to apply mathematics and its inherent merit in life.

Conclusions

This study was guided by the following open-ended questions to discover the causes that have led Canada, China, Japan, and Singapore to their current mathematical success. An analysis of the perceptual data gleaned from post-secondary educators who have taught in the respective countries is provided. Conclusions were drawn based on the synthesis of information derived from scholarly research and the respondents.

Research question one. What leading factors do post-secondary mathematics instructors attribute to the academic achievement of secondary students, based upon the performance of secondary educators within the countries studied?

The interviewees harmoniously decided mathematical achievement stems from multiple aspects, ranging in complexity, brought about by expert educators, which aligns with the view discussed by Ernest (2016). Researchers time and time again have stated there is no one precise solution to increase mathematical achievement among students but rather many factors that must be addressed to bring about the transformation of the educational process (Boaler & Staples, 2008; Hattie, 2017; SRI International, 2009; Tucker, 2011). However, it appears the data expressed by the professors from China, Japan, and Singapore were more homogenous than those relayed by Professor A from Canada. The difference is not necessarily surprising, as studies have shown cultural patterns may impact educational systems, not always stemming from a regional or national perspective, but also from within a student's home and school district (Cheng Yong, 2015; Leung et al., 2014).

The themes of foundational skills and most significantly, deeper learning, continually surfaced throughout the interviews as the main source of a successful mathematics program when embedded into a focused curriculum with instruction based on mathematical pedagogy. Much like Hattie (2017), as discussed in *Visible Learning*, the participants agreed there must be a balance of procedural knowledge and critical problem solving, and an instructor must have a sound understanding of mathematical pedagogy to determine at what point it is best to implement these different modes of instruction. Also, respondents were clear the curriculum must facilitate options for both.

Another theme connected to mathematical success and frequently discussed by all professors pertained to students having a *mathematical mindset*, coined by Jo Boaler (2016). The mathematical mindset formulates around the conception any student can build upon his or her ability as it pertains to mathematics; a student can progress when consistent practice and hard work are applied to thinking critically about problem-solving. While Professor A from Canada consistently discussed this ideology from the educator's perspective, professors from Eastern Asian countries most often related this mindset to students within the secondary setting.

Research question two. How do post-secondary mathematics instructors, from the countries studied, describe the preparation for secondary instructors' pedagogical approach?

Foundational skills were paramount to professors from China, Japan, and Singapore when the instructors discussed the necessary content skills possessed by prospective teachers entering initial teacher programs at the post-secondary level. From their perceptions as current post-secondary educators in the United States with backgrounds of teaching in their respective native countries, all Eastern Asian instructors explicitly stated foundational skills, along with a deep understanding of mathematics, are lacking in American post-secondary students entering mathematics education programs. This revelation is not new to the post-secondary world, as Houston and Yonghong (2016) discovered at least half of students entering college required remedial coursework before advancing to credited, prerequisite mathematics courses.

Professor B from China, Professor C from Japan, and Professor D from Singapore were direct in drawing a stark contrast to the level of mathematical knowledge held by prospective candidates majoring in programs for mathematics educators from the previously mentioned countries. Not only did these students have a strong command of basic math understanding, but their knowledge concerning the inner-workings of mathematics and complex problem solving was vast. Based on this account, post-secondary instructors from these countries were able to go further and immerse students in a program focused on a deep understanding of mathematical pedagogy, which includes how to relay the curriculum, content, and instruction to meet student needs (Fan, 2014). Studies have cited a teacher’s level of cognition can have a positive impact on student achievement, and this may represent one factor that contributes to the overwhelming mathematical success of China, Japan, and Singapore (Basque & Bouchamma, 2016; Goldhaber & Walch, 2014; Hanushek et al., 2014; Reckase et al., 2015).

The subtheme of student engagement surfaced throughout the interview process from all participants. Each respondent reflected and examined the instructional approach put forth in post-secondary programs from the countries studied, and discussed at the heart of the lesson was an open-ended problem meant to transform students’ conceptual understanding of mathematics. The interviewees did not go into great detail or specifics concerning the problem but did remark the exercise should allow for critical thinking, collaboration in problem solving, and increased student interest in the topic.

Research question three. How do post-secondary mathematics teachers characterize the social climate of the typical secondary classroom, among the countries studied?

Based on responses collected from interviews, the social and organizational structure appeared to differ between Western and Eastern perspectives. Participants from Eastern Asia were similar in their responses, which aligned to a clear organizational method brought about by a concentration on lesson planning and a profound mindset that all students can increase their mathematical proficiency, thus creating a more equitable environment. From the Western perspective of Canada, the professor did not feel she had enough experience within the secondary mathematics classroom to provide a sufficient account of the social climate. According to Fadlelmula, Cakiroglu, and Sungur (2015), the area of social climate within the mathematics classroom requires additional research and leaves many questions unanswered.

Research question four. Among the countries studied, how do post-secondary educators depict the structure of the curriculum and additional materials used by secondary instructors in the mathematics classroom?

A sense of cohesiveness emerged from the commentary provided by professors from China, Japan, and Singapore. An integral aspect of their educational systems focuses on the use of a national curriculum and adherence to similar core values surrounding educational philosophies. The post-secondary instructors from Eastern Asia conveyed the need for the text to derive from research-based methodologies and for secondary educators to strictly study the text to bring a broader scope to their instruction. Many studies have congruently attributed the mathematical proficiency of Eastern Asia to a unified national curriculum, with well-defined standards and a precise plan for implementation (Li & Kaiser, 2011; OECD, 2016a; Tucker, 2011).

Professor A from Canada diverged from this perspective, remarking instructors use the provincially designed curriculum with supplementary texts. While textbooks from Canada appeared to adhere to a spiral design, it was difficult to differentiate the fundamental differences across provinces without closer examination. When comparing Canada to its Eastern counterparts within this study, it is a distinct possibility to point to a cohesive and coherent national curriculum as being a key difference between doing well and having a superior reign of continued mathematical achievement.

Research question five. Among the countries studied, how do post-secondary instructors summarize the initial and ongoing professional development secondary instructors receive throughout their careers?

Ultimately, the perspectives concerning professional development differed between the Eastern and Western countries. Professor B from China, Professor C from Japan, and Professor D from Singapore all expressed a need for mathematics instructors to gather collectively regularly, with at least one expert in the field, to discuss research-based pedagogy to address data-driven weaknesses among students or districts. The constant emphasis placed on teachers to grow as professionals and better their school systems as a whole was apparent; so much so that competitions in these countries often motivate teachers to polish their craft. A theme of cohesiveness embodied the educational communities built within these countries, bringing about concepts like lesson study, which was originally solidified in Japan and currently resonates worldwide (Mincu, 2015; Takahashi, 2015). The ability for teachers within these countries to gather together to bring about positive change in their instructional pedagogy may point to one facet that could comprise their mathematical proficiency on a global scale.

Professor A from Canada drew a likeness of professional development to that of the United States and concluded the similarities lie in annual conferences and attending post-secondary courses. In contrast, Professor A distinguished that unlike the United States, Canada relies on international research more so than its southern counterpart. When comparing scores on international assessments (NCES, 2019a, 2019b), the infrequency and lack of cohesion found within professional development may be an underlying determinant of Canada's less successful results than those of Eastern Asian countries. At the same time, Canada's desire to seek sources and knowledge from the international community may set them apart in terms of greater mathematical success than the United States (Tucker, 2011). Due to the scarcity of empirical data reviewing Canada's mathematical proficiency, as noted by Tucker (2011), there is no evidence to substantiate this conclusion.

Implications for Practice

According to data from national and international exams, the United States is failing to compete educationally on a global scale and produce students who are mathematically proficient at a reasonable rate (Larson & Kanold, 2016; NCES, 2015; OECD, 2016a). Due to the vastly changing economy creating an evolution of needs in the 21st-century American workforce, a new era of educational reform, especially in the area of mathematics, must transpire to facilitate a quality educational experience at the highest level for all students (Larson & Kanold, 2016; NCTM, 2014; OECD, 2016a; Tucker, 2011). The following recommendations have been provided based upon the findings of this study to challenge the status quo within the American education system and bring about the demand for positive change in the educational process for students, educators, administrators, and various stakeholders as it pertains to secondary mathematics programs.

Cohesive and coherent standards and curriculum. Ideally, a national set of standards integrating a balanced approach of procedural mathematics and critical-thinking skills, along with incorporating collaborative approaches to higher-order problem solving, would be the most advantageous solution for all students. Like the Asian countries from this study, national standards may allow students to learn in an equitable environment and bridge the gap between achievers and non-achievers. Additionally, by aligning standards across the nation, school districts may readily examine and develop an appropriate curriculum linking mathematical knowledge to real-world scenarios which address a reasonably coherent pattern of mathematical development.

A previous attempt to reform the United States educational standards, known as *Common Core State Standards of Mathematics*, had initial bipartisan support yet eventually failed (Larson & Kanold, 2016). Larson and Kanold (2016) distinguished the following arguments made within and from outside the educational community:

(1) Federal versus local control was largely misinterpreted;

(2) a lack of effective professional development to introduce the standards to educators and administrators;

(3) inconsistent methods and standards of assessing mathematical proficiency per state; and

(4) perceptions in the media based on opinion, rather than data. (pp. 43-57)

It is prudent to learn from these missteps and attempt groundbreaking reform once again to encourage an equitable, 21st-century education for students.

While states are not necessarily considering revitalizing this initiative, school districts can advance this agenda at the local level. Similar to *Railside High,* school leaders and educators can work together to create and implement a high-quality, rigorous curriculum, promoting complex problem solving and collaboration taught to all students (Boaler & Staples, 2008). Also, instructors can work toward aligning assessments to the curriculum to determine learning gains made by students, closing achievement gaps across student populations.

Initiatives centering upon a mathematical mindset. State departments of education should implement federal and state initiatives which promote a strong need and desire for mathematics in one's scholastic and professional career. By discussing a positive view of mathematics in all entities of life throughout various media outlets, the discussions surrounding mathematics can transcend conceptions of difficulty and hopelessness and begin with vigor and the notion of striving toward exceptional achievement. Providing a productive outlook toward mathematics can begin at the grassroots level by organizing math nights and competitions to engage students in the positive outlets of mathematics. Instructors should also develop and raise awareness for the excitement mathematics can bring by embedding activities and exercises that encourage intrigue and nurture students' mathematical skills from multiple entry points.

Productive professional development. While participants for this study and researchers target the need for post-secondary mathematics programs with strong content and instructional pedagogy, professional development continues to be an essential element for an educator's continued growth (Baete & Hochbein, 2014; Fan, 2014; Julie, 2014; Schoenfeld, 2014). Professional development should include a data-driven effort to collaborate with educators, especially those with expertise and superior results in increasing student achievement. Instructors should frequently meet, with a specific area of focus supported by data collected from the classroom, while utilizing research-based pedagogy to improve mathematical achievement. Whether using a lesson study model as suggested by Professor C from Japan or Professional Learning Communities as detailed by Schmoker (2006), it is imperative the collective group have precise objectives, conduct observations to collect data, and take time for reflection with oversight from experts to assess students' needs.

Recommendations for Future Research

As this study contributes to the body of existing research, it is limited in its capacity; by no means has it exhausted each definitive aspect that supports the exceptional secondary mathematics education all students deserve. There are many opportunities through which researchers, administrators, and educators alike could expand upon the search for greater outcomes and student achievement in the area of mathematics. It is critical at this point when technological advances are rapidly revolutionizing the world, and with the need to apply solutions to the ever-changing environment is present, that researchers look toward future studies to transform the presentation of mathematics to meet the growing needs of students.

This investigation was conducted to derive the factors that contribute to a highly successful mathematics program by gleaning the perceptions of post-secondary educators who teach prospective math instructors from top-ranking countries around the world. Due to the scope and geographical location of this research, the study was limited to four international participants. Future research could be expanded to include more members of the secondary and post-secondary mathematics education community. Inquiries concerning elements that promote mathematical success across multiple nations could be studied using a mixed-methods approach, analyzing the perceptions of educators in comparison to data collected from within secondary classrooms to uncover a more well-rounded account of the phenomenon that relates to their mathematical affluence.

While the findings of this project conclusively noted the need for (1) cohesive standards and curriculum to create equity among student populations, (2) cultivation of a growth mindset toward mathematics, and (3) provisions for effective professional development, research-driven data specific to these dynamics were lacking from particular countries. Tucker (2011) revealed while Canada resembles the United States population, the country earns consistently higher marks on international assessments in the area of mathematics.

Due to the structure of Canada's education system, the only country within this study to prescribe to functioning individually at the provincial level, it is difficult to determine the unique features of their mathematical success across regions, which do not function at a commensurate level (OECD, 2016b; Tucker, 2011; Vashchyshyn &

Chernoff, 2016). Further studies comparing individual provinces within Canada could result in determining the precise factor that contributes to their success. Similar studies could be completed within the United States, as equity across the 50 diverse educational systems appears to be a matter of debate in the research community (Boaler & Staples, 2008; Nasir et al., 2014; NCTM, 2014; OECD, 2016a; Tucker, 2011).

Integrated into this study were various frameworks (Matthews, 2013; Schoenfeld, 2016; SRI International, 2009). Within SRI International's (2009) framework, RTI was addressed, but due to a lack of existing research among the countries studied, the topic was ultimately not included in the inquiry. Response to Intervention appears to be a Western concept, as the participants from this study conveyed individual students with difficulties receive attention during class time by the instructor or during after-school tutoring sessions. Further qualitative and quantitative measures delving into this topic would provide greater scope to provide a quality education for all students within the regular mathematics classroom. In conjunction with further research into RTI, each entity embedded in this study (e.g., curriculum, pedagogy and instructional approach, teacher knowledge and expectations, and organizational and social climate) has the possibility to stand alone in a qualitative and/or quantitative model to compare these attributes globally and provide a clear, more precise picture of what represents a successful mathematics education program.

Summary

This qualitative study was designed to elicit the perceptions of four post-secondary educators who have taught in the countries of Canada, China, Japan, and Singapore. The countries mentioned above have proven to demonstrate sustained proficiency in the area of mathematics based on international examinations such as the PISA and TIMSS. The researcher sought to derive expertise from these instructors to unveil the phenomenon of mathematical excellence that exists among these countries. By doing so, educators, administrators, and various stakeholders could enact reforms and initiatives based on the findings to increase mathematical achievement among students within any educational jurisdiction.

Scholarly literature was gathered and presented. A diverse set of frameworks from Matthews (2013), Schoenfeld (2014), and SRI International (2009) were utilized to design the study centering on curriculum, pedagogy and instructional approach, teacher knowledge and expectations, and organizational and social climate as possible factors that support students' mathematical proficiency. Participants were selected from a random, homogenous sample. Open-ended interviews were conducted to extract further knowledge based on the four respondents' teaching experience at the post-secondary level within mathematics education programs from the researched countries. Responses were recorded and transcribed. Content analysis was used to extrapolate various themes that emerged, and commonalities in responses were found (Fraenkel et al., 2015).

In their interviews, all participants continually emphasized a need for foundational learning, coupled with an even greater presence of deeper learning. Integrating this schema promotes critical thinking and problem solving while allowing students to collaborate to increase their mathematical understanding and identify any misconceptions in their thinking. Respondents were clear to state all students are capable of increasing their mathematical success when allowed to explore mathematical concepts and problems that facilitate complex ideas. Participants espoused allowing students access to problems at varying levels, which improves student engagement. These findings correlate with the research reviewed in Chapter Two (Hattie, 2017; NCTM, 2014; SRI International, 2009)

Subtle differences were found among the participants' responses that led to the conclusions found in the present literature of Chapter Two concerning the delineation between the "Western" and "Eastern" perspectives toward mathematics education (Li & Kaiser, 2011). The congruity among the professors from China, Japan, and Singapore stressed the value of having national standards and a curriculum which allows a cohesive and coherent format to introduce mathematical concepts in a logical sequence. Also, the respondents placed an extreme significance on continued professional development by adhering to measurable goals for student improvement, implementing research-based pedagogy, and collaborating with colleagues, including expert mathematics instructors.

The evidence presented in this study gives credence to changes that could be enacted in any or all school systems within the United States to increase mathematical achievement. For school leaders and educators striving to leap toward gaining equity in mathematical programs across the nation, further research is required to collect quantitative and qualitative data verifying the impact of these factors in American school districts and globally. For future students in many nations, it is imperative as a society to seek a united vision in every education system and standard, followed by an intense focus on a balanced mathematics curriculum fostering procedural knowledge and deeper learning. Lastly, a conscious desire for all administrators and educators to place continued growth as professionals at the forefront is essential to improve education as a whole.

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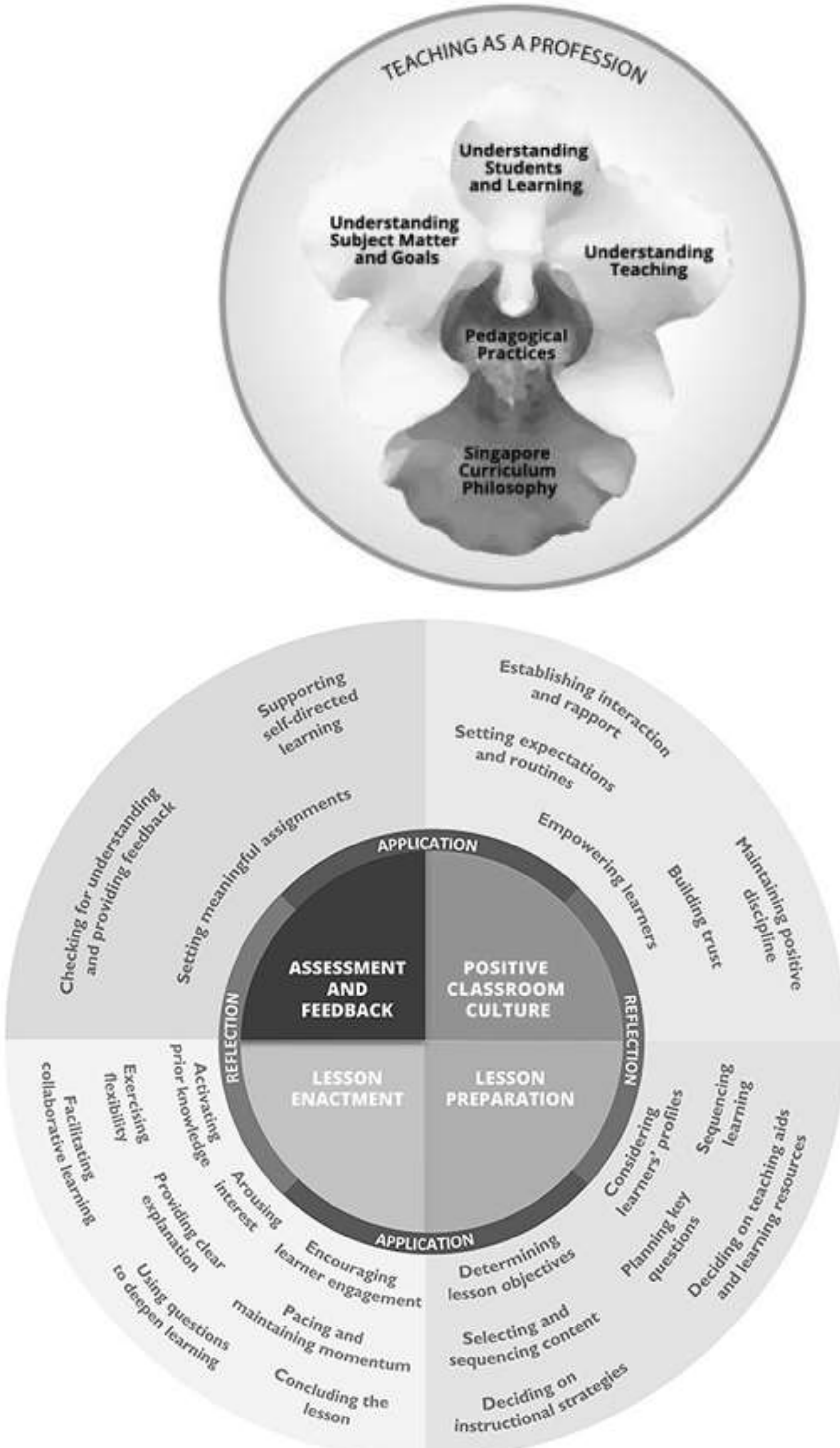
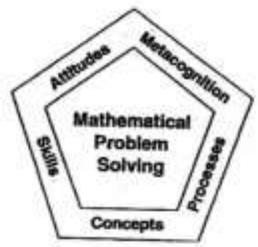
Appendix A

The Singapore Teaching Practice



Appendix B

Singapore's Evolution of the School Mathematics Curriculum Framework



Primary Goal: Mathematical Problem Solving

Component	1991- 2000	2001- 2006	2007- 2012	2013 - beyond
Concepts	- Numerical - Geometrical - Algebraic - Statistical	- Numerical - Geometrical - Algebraic - Statistical	- Numerical - Geometrical - Algebraic - Statistical - Probabilistic - Analytical	- Numerical - Algebraic - Geometrical - Statistical - Probabilistic - Analytical
Skills	- Estimation & Approximation - Mental calculation - Communication - Use of mathematical tools - Arithmetic manipulation - Algebraic manipulation - Handling data	- Estimation & Approximation - Mental calculation - Communication - Use of mathematical tools - Arithmetic manipulation - Algebraic manipulation - Handling data	- Numerical calculation - Algebraic manipulation - Spatial visualization - Data analysis - Measurement - Use of mathematical tools - Estimation	- Numerical calculation - Algebraic manipulation - Spatial visualization - Data analysis - Measurement - Use of mathematical tools - Estimation
Attitudes	- Appreciation - Interest - Confidence	- Appreciation - Interest - Confidence - Perseverance	- Beliefs - Appreciation - Interest - Confidence - Perseverance	- Beliefs - Interest - Appreciation - Confidence - Perseverance
Meta cognition	- Monitoring one's own thinking	- Monitoring one's own thinking	- Monitoring of one's own thinking - Self-regulation of learning	- Monitoring of one's own thinking - Self-regulation of learning
Processes	- Heuristics - Deductive reasoning - Inductive reasoning	- Heuristics - Thinking Skills	- Reasoning, communication and connections - Applications and modeling - Heuristics - Thinking Skills	- Reasoning, communication and connections - Applications and modeling - Thinking skills and heuristics

Appendix C

Interview Questions

1. As an expert in your field, what characteristic(s) can be attributed to the academic achievement of secondary students in the area of mathematics in your native country?

2. As a post-secondary instructor, what were your expectations for future instructors from your native country?

3. Please describe the teaching philosophy you imparted on future teachers from your native country concerning how a student's cognitive ability affects their mathematical performance.

4. As a post-secondary instructor from your native country, please describe the process you use to instruct secondary educators in establishing their daily learning objectives and the steps necessary to complete their typical daily lessons.

5. From the perspective of a post-secondary instructor, is there a specific type of problem, or problems, that you encourage secondary instructors to embed in assignments to guide students in meeting those objectives.

6. In your expert opinion and through observation, please describe the optimal structure of the student's learning experiences and overall atmosphere in a typical secondary mathematics classroom within your native country.

7. Please explain how you instruct secondary teachers to determine the curriculum and any supplemental resources used in a typical daily lesson.

8. In your experience, what types of professional development do you encourage secondary instructors from your native country to take part in to improve their effectiveness as an educator, and how often does this typically occur?

9. In your expert opinion, with respect to your native country, what is the value placed on education, specifically mathematics education?

Appendix D

Recruitment Letter to Participants

Dear Professor:

This letter is an invitation for post-secondary mathematics instructors to participate in a study I am currently conducting in partial fulfillment of my doctoral degree from Lindenwood University under the supervision of Dr. Julie Williams. This study is focused on the leading factors that contribute to mathematical success in top-ranking countries around the world by analyzing the perceptions of post-secondary mathematics instructors in these countries.

As determined by experts and international assessments, your country is considered to be among the best and most knowledgeable in achieving results in the field of mathematics education. The pedagogy and methodology warrant further examination and discussion. Educational stakeholders, especially educators and administrators, could greatly benefit by learning from those who excel in teaching mathematics to secondary youth. As an educator, it is with great privilege that I seek the opinions and pedagogy that inspire expert secondary mathematics instructors in the classroom.

Li and Kaiser (2011) defined an expert secondary mathematics instructor as in possession of all of the following qualities: a) has taught longer than seven years; b) prepares for multiple outcomes of students' understanding and solutions; c) provides quality feedback and questioning in a timely manner to provoke students' comprehension; and d) implements activities that elicit critical and conceptual mathematical thinking. Participation in this study is voluntary.

Assurances of confidentiality and anonymity of the participating educator and academic institution are of the upmost importance, and no identifiable information will be released in conjunction with this body of research. The results of the study will be revealed through the dissertation and will be published by Lindenwood University. Thank you in advance for your assistance in this process and your wealth of contributions to education as a whole; I ask that you read the letter of informed consent and respond to Kendra Snow at xxx-xxxx to set up a convenient time to conduct an interview pertaining to this body of research.

Educationally yours,

Kendra Snow
Doctoral Student

Appendix E

Letter of Informed Consent

LINDENWOOD

Research Study Consent Form

Educators' Perceptions Concerning the Leading Factors of Mathematics Achievement in Top-Ranking Nations Around the World

Before reading this consent form, please know:

- Your decision to participate is your choice
- You will have time to think about the study
- You will be able to withdraw from this study at any time
- You are free to ask questions about the study at any time

After reading this consent form, we hope that you will know:

- Why we are conducting this study
- What you will be required to do
- What are the possible risks and benefits of the study
- What alternatives are available, if the study involves treatment or therapy
- What to do if you have questions or concerns during the study

Basic information about this study:

- We are interested in learning about the elements that contribute to students' mathematical achievement in the countries of Canada, China, Japan, and Singapore.
- You will be asked to set aside 30 minutes for an interview by phone or Skype, depending upon your preference.
- Risks of participation include any risks encountered in daily life.

Appendix F

LINDENWOOD

Research Study Consent Form

Educators' Perceptions Concerning the Leading Factors of Mathematics Achievement in Top-Ranking Nations Around the World

You are asked to participate in a research study being conducted by Kendra Snow under the guidance of Dr. Julie Williams at Lindenwood University. Being in a research study is voluntary, and you are free to stop at any time. Before you choose to participate, you are free to discuss this research study with family, friends, or a physician. Do not feel like you must join this study until all of your questions or concerns are answered. If you decide to participate, you will be asked to sign this form.

Why is this research being conducted?
We are conducting this study to collect data to potentially guide school districts or classroom instructors on appropriate interventions and reform to increase performance in mathematics, especially at the secondary level. We will be asking about three other people to answer these questions.

What am I being asked to do?
A consent form will be sent to you and needs to be signed before the primary investigator continues to move forward to the data collection phase. Once this form is signed, the primary investigator will contact you through email or by phone, depending on preference, to arrange a time at your convenience to conduct an interview through Skype or by phone. At the end of the interview, you will be asked if you are interested in participating in an additional interview if further inquiry is required at a later date and time. You will be notified if this is a possibility, and it will be brief, no longer than 10 minutes.

How long will I be in this study?

This study will last no longer than a year.

Who is supporting this study?

This study is not funded by a grant or funding agency.

What are the risks of this study?

- Privacy and Confidentiality**:** We will not be collecting any information that will identify you.

What are the benefits of this study?

You will receive no direct benefits for completing this survey. We hope what we learn may benefit other people in the future.

What if I do not choose to participate in this research?

It is always your choice to participate in this study. You may withdraw at any time. You may choose not to answer any questions or perform tasks that make you uncomfortable. If you decide to withdraw, you will not receive any penalty or loss of benefits. If you would like to withdraw from a study, please use the contact information found at the end of this form.

What if new information becomes available about the study?

During the course of this study, we may find information that could be important to you and your decision to participate in this research. We will notify you as soon as possible if such information becomes available.

How will you keep my information private?

We will do everything we can to protect your privacy. We do not intend to include information that could identify you in any publication or presentation. Any information we collect will be stored by the researcher in a secure location. The only people who will be able to see your data include members of the research team, qualified staff of Lindenwood University, and representatives of state or federal agencies.

How can I withdraw from this study?

Notify the research team immediately if you would like to withdraw from this research study.

Who can I contact with questions or concerns?

If you have any questions about your rights as a participant in this research or concerns about the study, or if you feel under any pressure to enroll or to continue to participate in this study, you may contact the Lindenwood University Institutional Review Board Director, Michael Leary, at (636) 949-4730 or mleary@lindenwood.edu. You can contact the researcher, Kendra Snow, directly at xxx-xxxx. You may also contact Dr. Julie Williams directly at xxx-xxxx.

I have read this consent form and have been given the opportunity to ask questions. I will also be given a copy of this consent form for my records. I consent to my participation in the research described above.

______________________________ ______________
Participant's Signature **Date**

____
Participant's Printed Name

______________________________ ______________
Signature of Principal Investigator or Designee **Date**

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Investigator or Designee Printed Name

Appendix G

IRB Approval

LINDENWOOD

LINDENWOOD UNIVERSITY ST.CHARLES, MISSOURI

DATE:	April 25, 2018
TO:	Kendra Snow
FROM:	Lindenwood University Institutional Review Board
STUDY TITLE:	[1111623-3] Educators' Perceptions Concerning the Leading Factors of Mathematics Achievement in Top-Ranking Nations Around the World
IRB REFERENCE #:	
SUBMISSION TYPE:	Amendment/Modification
ACTION:	APPROVED
APPROVAL DATE:	April 25, 2018
EXPIRATION DATE:	January 29, 2019
REVIEW TYPE:	Expedited Review

Thank you for your submission of Amendment/Modification materials for this research project. Lindenwood University Institutional Review Board has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a study design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

This submission has received Expedited Review based on the applicable federal regulation.

Please remember that informed consent is a process beginning with a description of the study and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the study via a dialogue between the researcher and research participant. Federal regulations require each participant receive a copy of the signed consent document.

Please note that any revision to previously approved materials must be approved by this office prior to initiation. Please use the appropriate revision forms for this procedure.

All SERIOUS and UNEXPECTED adverse events must be reported to this office. Please use the appropriate adverse event forms for this procedure. All FDA and sponsor reporting requirements should also be followed.

All NON-COMPLIANCE issues or COMPLAINTS regarding this project must be reported promptly to the IRB.

This project has been determined to be a Minimal Risk project. Based on the risks, this project requires continuing review by this committee on an annual basis. Please use the completion/amendment form for this procedure. Your documentation for continuing review must be received with sufficient time for review and continued approval before the expiration date of January 29, 2019.

Please note that all research records must be retained for a minimum of three years.

Vita

Kendra Snow currently serves as a middle school mathematics instructor at Mansfield R-IV School District. Collectively, she has taught for nine years for Mansfield Schools, 10 years overall, with a concentration in the areas of special education, history, and mathematics. Kendra earned a Master's degree in Special Education Administration from William Woods University and a Bachelor's degree in Special Education-Cross Categorical from Missouri State University.